Control Configuration Selection

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Control Configuration Selection

LECTURE 1

Introduction

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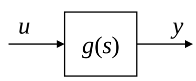
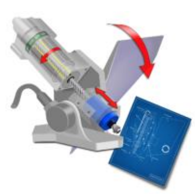
SISO/MIMO Plants

SISO plant:



Examples:

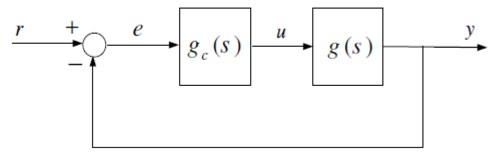
- A dc electric motor



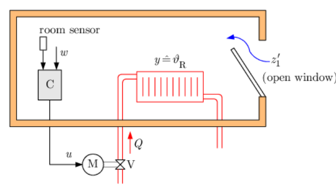
- Temperature of the room
- ?

SISO/MIMO Plants

Closed Loop SISO Control System:



- Temperature controller of the room:

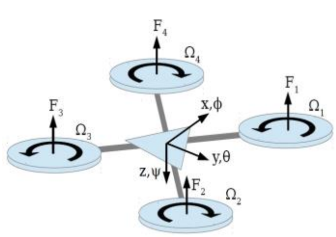


MIMO Plants

Multi Input-Multi Output (MIMO) Plants

Examples:

- Quadrotor



Quadrotor configuration.

$$\ddot{x} = (s_\phi s_\psi + c_\phi s_\theta c_\psi)\frac{F_z}{m},$$

$$\ddot{y} = (-s_\phi c_\psi + c_\phi s_\theta s_\psi)\frac{F_z}{m},$$

$$\ddot{z} = g + (c_\phi c_\theta)\frac{F_z}{m},$$

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\left(\frac{J_y - J_z}{J_x}\right) - \frac{J_r}{J_x}\dot{\theta}\Omega_r + \frac{M_x}{J_x},$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{J_z - J_x}{J_y}\right) + \frac{J_r}{J_y}\dot{\phi}\Omega_r + \frac{M_y}{J_y},$$

$$\ddot{\psi} = \dot{\phi}\dot{\theta}\left(\frac{J_x - J_y}{J_z}\right) + \frac{M_z}{J_z},$$

MIMO Plants

Examples:

A Continuous Stirred Tank Reactor (CSTR)

A continuous stirred tank reactor (CSTR) is used to convert a reactant (A) to a product (B). The reaction is liquid phase, first order and exothermic. Perfect mixing is assumed. A cooling jacket surrounds the reactor to remove the heat of reaction.

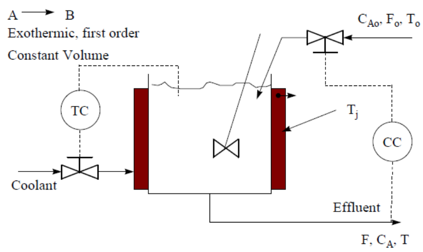
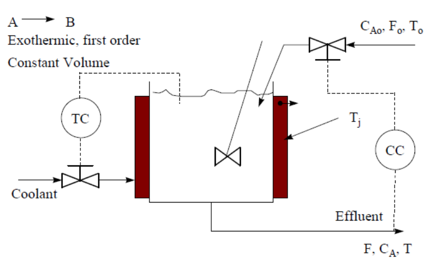


Fig 1 A basic control scheme for a CSTR

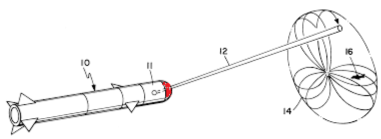
MIMO Plants

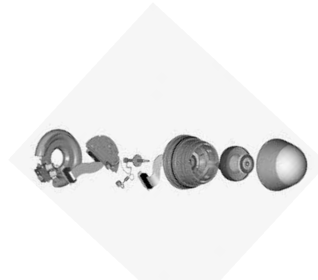


In this system variables of interest (from a control engineers perspective) could be, for example, product composition and temperature of the reacting mass. There will therefore be a composition control loop as well as a temperature control loop. Feed to the reactor is often used to manipulate product composition while temperature is controlled by adding (removing) energy via heating (cooling) coils or jackets. This basic control configuration is demonstrated in Fig (1). 'TC' represents a temperature controller, the mv for this loop being coolant flowrate to the jacket. 'CC' represents the composition controller, the mv being reactant feedrate.

MIMO Plants

- Free Gyro Seeker





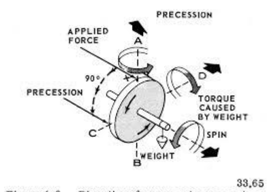


Figure 4-6. — Direction of gyroscopic precession.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{I_r \cos x_3}(T_z + I_R \omega_s x_4)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{1}{I_r}(T_y - I_R \omega_s x_2 \cos x_3)$$

MIMO Plants

- Railway vehicles

Multiple-mass model

The *multiple-mass* model is illustrated in Fig. 1.

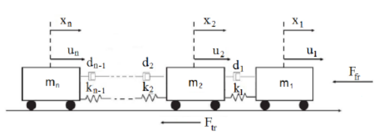


Figure 1: Multiple-mass model

(Chou, Xia & Kayser, 2007) are given by:

$$\begin{cases} m_1\ddot{x}_1 = & u_1 - k_1(x_1 - x_2) - d_1(\dot{x}_1 - \dot{x}_2) - \\ & -m_1(c_0 + c_v\dot{x}_1) - mc_a\dot{x}_1^2 - 9.98\ m_1 \sin\ \theta_1 \\ & -0.004\, m_1 D_1 \\ m_i\ddot{x}_i = & u_i - k_i(x_i - x_{i+1}) - k_{i-1}(x_i - x_{i-1}) \\ & -d_i(\dot{x}_i - \dot{x}_{i+1}) - d_{i-1}(\dot{x}_i - \dot{x}_{i-1}) \\ & -m_i(c_0 + c_v\dot{x}_i) - 9.98\ m_i sin\ \theta_i - 0.004\, m_i D_i \\ & i = 2, \ldots, n-1 \\ m_n\ddot{x}_n = & u_n - k_{n-1}(x_n - x_{n-1}) - d_{n-1}(\dot{x}_n - \dot{x}_{n-1}) \\ & -m_n(c_0 + c_v\dot{x}_n) - 9.98\ m_n sin\ \theta_n \\ & -0.004\, m_n D_n \end{cases} \quad (1)$$

MIMO Plants

Matrix Transfer function representation for Linear MIMO plants:

$$Y(s) = G(s)U(s)$$

$$Y(s) = \begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_l(s) \end{bmatrix} \quad G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1m}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2m}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{l1}(s) & g_{l2}(s) & \cdots & g_{lm}(s) \end{bmatrix} \quad U(s) = \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_m(s) \end{bmatrix}$$

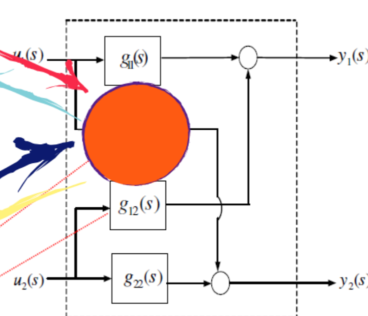
Interaction

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

$$\Downarrow$$

$$y_1(s) = g_{11}(s)u_1(s) + g_{12}(s)u_2(s)$$
$$y_2(s) = g_{21}(s)u_1(s) + g_{22}(s)u_2(s)$$

Or in block diagram representation:



- INTERACTIONS

Interaction

$$u_1(s) = g_{c1}\left[r_1(s) - y_1(s)\right]$$
$$u_2(s) = g_{c2}\left[r_2(s) - y_2(s)\right]$$

where $r_1(s), r_2(s)$ are the reference inputs or the set points.

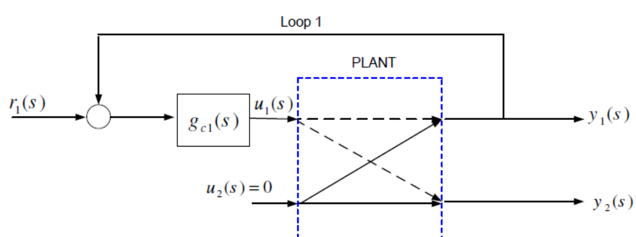
⇩

Consider the following two separate cases:

- One loop closed
- Both loops closed

Interaction

- Only first loop closed: Second loop open and its input kept constant, i.e. zero

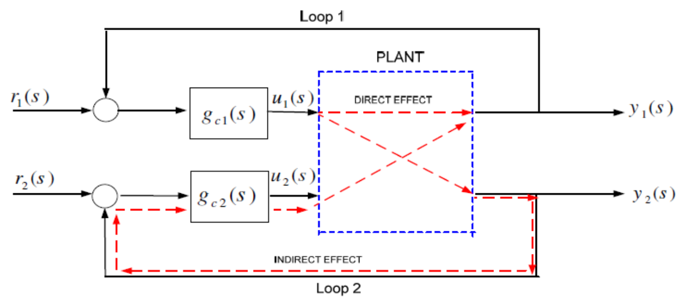


Then,

$$y_1(s) = \frac{g_{11}(s)g_{c1}(s)}{1 + g_{11}(s)g_{c1}(s)} r_1(s)$$
$$y_2(s) = \frac{g_{21}(s)g_{c1}(s)}{1 + g_{11}(s)g_{c1}(s)} r_1(s)$$

⇨ Any Change in the first set point will affect both the outputs under control (first output) and the output under no control (second output).

Interaction



Assume that the plant is under tight control. A change is made in the first set point. The following key observations are made:

- The Direct Effect: The first controller will get the first output to the desired set point.
- The Indirect Effect: The first controller will disturb the second output and the second controller attempts to reject its effects. But changes in the second controller effects the first loop performance. INTERACTION BETWEEN TWO CONTROL LOOPS! 8

Interaction

$$y_1(s) = g_{11}(s)u_1(s) + g_{12}(s)u_2(s)$$
$$y_2(s) = g_{21}(s)u_1(s) + g_{22}(s)u_2(s) \quad \text{and,}$$

$$u_1(s) = g_{c1}\left[r_1(s) - y_1(s)\right]$$
$$u_2(s) = g_{c2}\left[r_2(s) - y_2(s)\right]$$

Gives,

$$\left(1 + g_{11}(s)g_{c1}(s)\right)y_1(s) + \left(g_{12}(s)g_{c2}(s)\right)y_2(s) = \left(g_{11}(s)g_{c1}(s)\right)r_1(s) + \left(g_{12}(s)g_{c2}(s)\right)r_2(s)$$
$$\left(g_{21}(s)g_{c1}(s)\right)y_1(s) + \left(1 + g_{22}(s)g_{c2}(s)\right)y_2(s) = \left(g_{21}(s)g_{c1}(s)\right)r_1(s) + \left(g_{22}(s)g_{c2}(s)\right)r_2(s)$$

And finally, the closed loop transfer function matrix is

$Y(s) = T(s)R(s)$, That is

Interaction

$$y_1(s) = t_{11}(s) r_1(s) + t_{12}(s) r_2(s)$$
$$y_2(s) = t_{21}(s) r_1(s) + t_{22}(s) r_2(s)$$

Where,

$$t_{11}(s) = \frac{g_{11}(s) g_{c1}(s) + g_{c1}(s) g_{c2}(s)\left(g_{11}(s) g_{22}(s) - g_{12}(s) g_{21}(s)\right)}{q(s)}$$

$$t_{12}(s) = \frac{g_{12}(s) g_{c2}(s)}{q(s)}$$

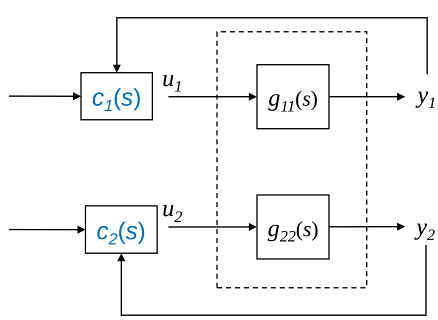
$$t_{21}(s) = \frac{g_{21}(s) g_{c1}(s)}{q(s)}$$

$$t_{22}(s) = \frac{g_{22}(s) g_{c2}(s) + g_{c1}(s) g_{c2}(s)\left(g_{11}(s) g_{22}(s) - g_{12}(s) g_{21}(s)\right)}{q(s)}$$

$$q(s) = \left(1 + g_{11}(s) g_{c1}(s)\right)\left(1 + g_{22}(s) g_{c2}(s)\right) - g_{12}(s) g_{21}(s) g_{c1}(s) g_{c2}(s)$$

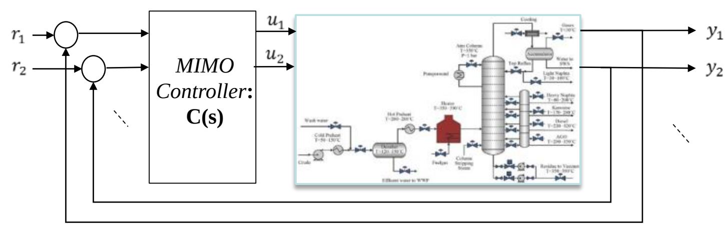
No Interaction

If $g_{12}(s)=g_{21}(s)=0$ then we have a **Decoupled Plant.**

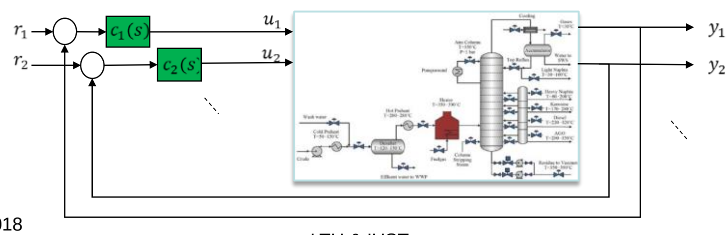


Centralized vs Decentralized Control Configuration

Centralized Control:



Decentralized Control:



Centralized vs Decentralized Control Configuration

Advantages of Decentralized Control Configuration:

- Easy in design and implementation
- Simple tuning
- Robustness in dealing with faults and uncertainties
- Cost effective
- Understandable for operators of the production process

Disadvantages of Decentralized Control Configuration:

- ?

Control **Structure** Design

Control Structure design includes 2 steps:

1. Input-Output Selection
2. Control Configuration Selection (Input –Output Pairing)

A definition for the Input-Output Selection is given in (Van de Wal and De Jager 2001) as:

"Select suitable variables to be manipulated by the controller (plant inputs) and suitable variables y to be supplied to the controller (plant outputs)"

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Input-Output Selection

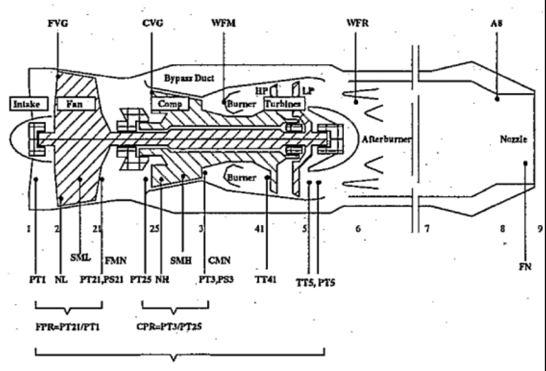
Example: **A JET ENGINE**

The inputs are:

WFM	Main fuel
A8	Nozzle area
FVG	Fan vanes
CVG	Compressor vanes
WFR	Afterburner fuel

The candidate outputs are:

NL	Low pressure rotor speed
NH	High pressure rotor speed
CPR	Compressor pressure ratio, *PT3*/PT25
FPR	Fan pressure ratio, *PT21*/PT1
EPR	Engine pressure ratio, *PT5*/PT1
FMN	Fan mach number, (*PT21-PS21*)/*PS21*
CMN	Comp. mach number, (*PT3-PS3*)/*PS3*
PT21, *PS21*	Fan downstream pressure
PT3, *PS3*	Compressor downstream pressure
PT5	Turbine downstream pressure, total
TT41	Turbine inlet temperature
TT5	Turbine downstream temperature
SML	Fan surge margin, (estimated)
SMH	Compressor surge margin, (estimated)
FN	Net thrust, (estimated)



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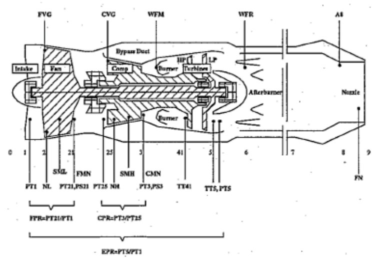
Input-Output Selection

Example: **A JET ENGINE**

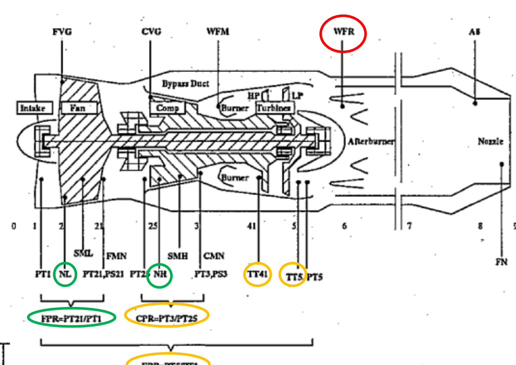
All these outputs are possible to use in a number of different combinations with the available input signals.
The problem is to find the combinations of outputs and inputs that are the most suitable for control design, without actually designing controllers for each case.



I/O Selection tries to obtain:
- Maximum Output Controllability and Robust Stability
- Maximum State Controllability and Observability
- No RHP Zero
- Minimum Interaction

Input-Output Selection

Example: **A JET ENGINE**



Outputs:	lower bound :	condition number:	
NL, NH, FPR, CPR	3.39	6.45	0.53
NL, NH, FPR, TT41	2.68	5.69	0.60
NL, NH, FPR, EPR	2.61	5.05	0.68
NL, NH, FPR, TT5	2.52	5.27	0.65

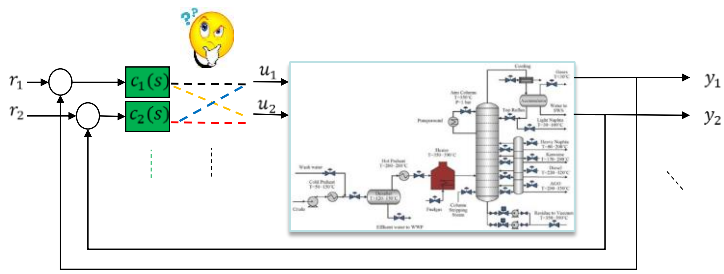
Ref.: H-aring, Melker, and refors. "Application of control structure design methods to a jet engine." *Journal of Guidance, Control, and Dynamics* 24.3 (2001): 510-518.

Control Configuration Selection

Objective:

How can we choose the best **Input/Output pair**, for decentralized control?

Can we use the decentralized control configuration?



Control Configuration Selection

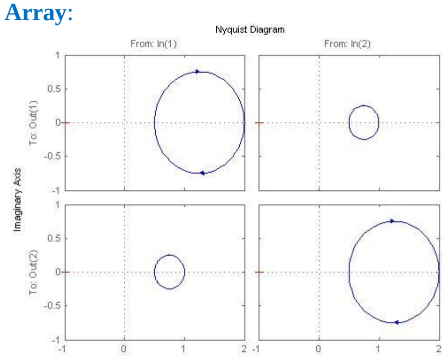
- Can we just use transfer function matrix to find the best I/O pair?

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

Example:

$$G(s) = \begin{bmatrix} \frac{s+2}{2s+1} & \frac{s+1}{2s+1} \\ \frac{s+1}{2s+1} & \frac{s+2}{2s+1} \end{bmatrix}$$

Nyquist Array:

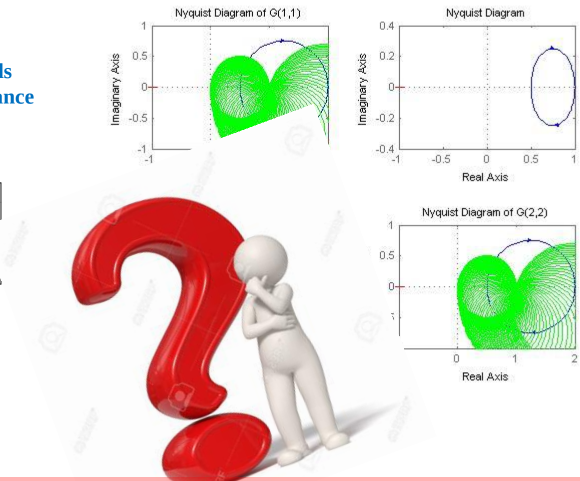


Control Configuration Selection

Example:

- Gershgorin bands
- Diagonal dominance

$$G(s)=\begin{bmatrix}\frac{s+2}{2s+1} & \frac{s+1}{2s+1}\\ \frac{s+1}{2s+1} & \frac{s+2}{2s+1}\end{bmatrix}$$



Can this method consider the interaction (Direct and Indirect effects) in its analysis?

The Quadruple Tank

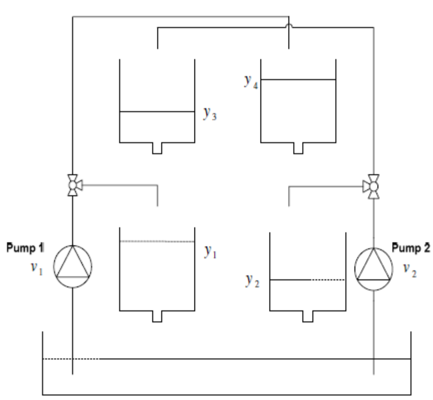


Fig. 2.2 The Quadruple-tank.

$$\dot{h}_1=-\frac{a_1}{A_1}\sqrt{2gh_1}+\frac{a_3}{A_1}\sqrt{2gh_3}+\frac{\gamma_1 k_1}{A_1}v_1$$

$$\dot{h}_2=-\frac{a_2}{A_2}\sqrt{2gh_2}+\frac{a_4}{A_2}\sqrt{2gh_4}+\frac{\gamma_2 k_2}{A_2}v_2$$

$$\dot{h}_3=-\frac{a_3}{A_3}\sqrt{2gh_3}+\frac{(1-\gamma_2)k_2}{A_3}v_2$$

$$\dot{h}_4=-\frac{a_4}{A_4}\sqrt{2gh_4}+\frac{(1-\gamma_1)k_1}{A_4}v_1$$

The Quadruple Tank

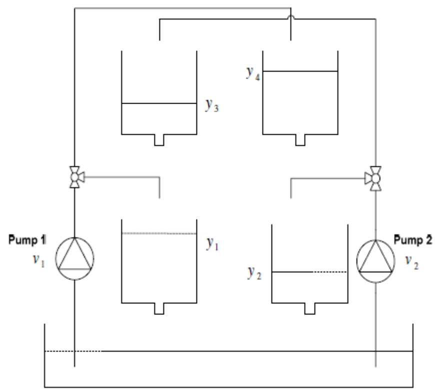


Fig. 2.2 The Quadruple-tank.

$$G(s)=\begin{bmatrix} \dfrac{\gamma_1 T_1 k_1 k_c}{A_1(sT_1+1)} & \dfrac{(1-\gamma_2)T_1 k_2 k_c}{A_1(sT_1+1)(sT_3+1)} \\ \dfrac{(1-\gamma_1)T_2 k_1 k_c}{A_2(sT_2+1)(sT_4+1)} & \dfrac{\gamma_2 T_2 k_2 k_c}{A_2(sT_2+1)} \end{bmatrix}$$

where

$$T_i=\frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$$

$A_1=A_3=28$, $A_2=A_4=32$ (cm^2)

$a_1=a_3=0.071$, $a_2=a_4=0.057$ (cm^2)

$k_c=0.50$ (V/cm)

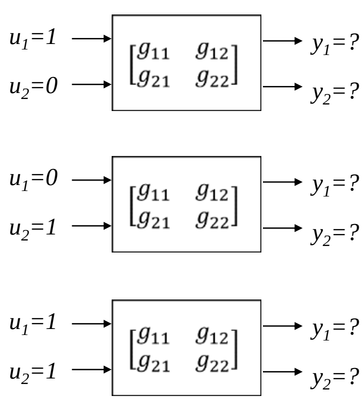
$g=981$ (cm/s^2)

$k_1=k_2=2.9$

$h_1=13.64,\ h_2=16.55,\ h_3=1.91\ \ h_4=1.77$

The Quadruple Tank

Analyze the effect of Inputs on the Outputs using open loop Step response:



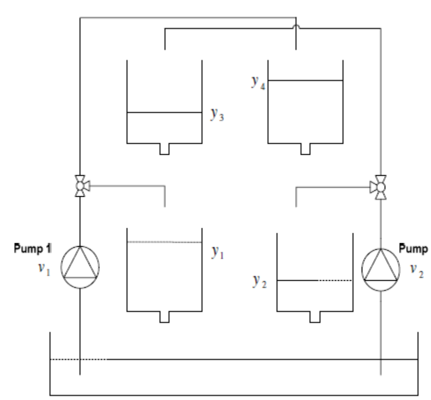


Fig. 2.2 The Quadruple-tank.

Aims of the Course

1. Control the large-scale and/or multivariable plants.
2. Centralized and Decentralized control structures.
3. Control configuration selection in decentralized control structure.
4. Input-Output Pairing strategies:
   - For linear plants.
   - For nonlinear plants.

Learning Outcomes

1. Control the multivariable and large-scale plants based on the decentralized control.
2. Advantages of decentralized control structure.
3. Pairing strategies and corresponding pairing rules.
4. Similarities and differences between the pairing methods.
5. Pairing methods based on the soft-computing algorithms.
6. Pairing result in the presence of uncertainties.
7. Last researches in the field.

Course Evaluation

1. **Course Projects 40%**
2. **Final Exam 40%**
3. **Paper +20%**

Reference Books

- A. Khaki Sedigh, B. Moaveni, **Control Configuration Selection for Multivariable Plants**, Springer, 2009.
- Bijan Moaveni, Vinay Kariwala, **Input-Output Pairing Selection for Design of Decentralized Controller**, 5th chapter in **Plant Wide Control: Recent Advances and Development**, John Wiley and Sons, 2012.
- Sigurd Skogestad, Ian Postlethwaite, **Multivariable feedback control: analysis and design**, John Wiley and Sons, 2005.
- Qing-Guo Wang, **Decoupling Control**, Springer, 2002.
- Jan Marian Maciejowski**, Multivariable feedback design,** Addison-Wesley, 1989.

Conclusion

- Interaction in MIMO Plants
- Centralized and Decentralized Control
- Control Structure Design
    - I/O Selection
    - I/O Pairing