Delamination buckling growth in laminated composites using layerwise-interface element

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In this paper, a numerical investigation on the buckling of composite laminates containing delamination, under in-plane compressive loads, is presented. For this purpose, delamination propagation is modeled using the softening behavior of interface elements. The full layerwise plate theory is applied for approximating the displacement field of laminates and the interface elements are considered as a numerical layer between any two adjacent layers where the delamination is expected to propagate. A non-linear computer code was developed to handle the numerical procedure of delamination buckling growth in composite laminates using layerwise-interface elements. The load/displacement behavior and the contours of embedded and through-the-width delamination propagation for composite laminates are presented. It is shown that delamination growth can be well predicted using this layerwise-interface elements with decohesive law.

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1. Introduction

Delamination is a common failure mode in layered composite materials which may result from the manufacturing imperfections, lay-up geometries, edge effects and various loadings. The presence of the delamination can cause significant reduction in stiffness and strength of the laminate under compressive loads; hence, a clear understanding of the compressive failure behavior of the laminate is extremely important. Many investigations were already performed on the buckling induced delamination propagation of composite laminates.

Whitcomb and Shivakumar [1] studied the delamination growth due to the local buckling of a composite plate with square and rectangular embedded delaminations. They applied the fracture mechanics approach for predicting the delamination growth using a virtual crack closure technique (VCCT) in order to calculate the total strain energy release rate. They showed that, whether a delamination grows in the load direction or perpendicular to the load direction, it depends on the laminate aspect ratio, the strain level and the absolute size of the delamination. Nilsson et al. [2] studied delamination buckling and growth of slender composite panels using both numerical and experimental methods. The investigated panels were cross-ply laminates under compression loading and containing embedded delaminations in different depths. For all delamination depths, the analysis showed a drastic increase in the energy release rate when global buckling takes place. Riccio et al. [3] investigated the compressive behavior of carbon fiber/epoxy laminated composite panels containing through-the-width and embedded delaminations. They also used a modified VCCT for predicting the delamination growth. They studied the effect of trough-the-thickness location of delamination on the stability or instability of delamination growth. Lachaud et al. [4] also used the VCC integral to simulate the propagation of delamination caused by the local buckling, on thermoset and thermoplastic carbon/fiber composite laminates having embedded delaminations. They also performed experiments to verify the obtained results.

Tafreshi and Oswald [5] developed finite element models to study the global, local and mixed-mode buckling behavior of composite plates with embedded delaminations. They applied a displacement field to the laminate that arises from the Mindlin/Kirchhoff plate theory and they also used the modified crack closure technique to calculate the strain energy release rate in order to predict the delamination growth. In the case of global buckling, they performed a parametric study to investigate the influence of the delamination size, shape and stacking sequence on the buckling load. It is worth to mention that delamination growth analyses using fracture mechanics approaches are very sensitive to the size and shape of the elements around the delamination front and they need special considerations of boundary movement problems in the step by step crack propagation process.

Hwang and Liu [6] performed experiments to study the buckling loads, buckling modes, post-buckling behavior and delamina-

tion growth of the delaminated unidirectional carbon/epoxy laminates, containing strip shaped delaminations. They investigated the effect of delamination length and its through-the-thickness location on the buckling load. Zhang and Wang [7,8] presented a layerwise B-spline finite strip method with consideration of delamination kinematics to study the buckling and post-buckling behavior of delaminated composite laminates. For the prediction of delamination propagation, they employed a fracture mechanics approach which is based on the energy release rate criterion. Aslan and Sahin [9] studied the effects of delamination size and through-the-thickness location on the critical buckling load and compressive failure load of the E-glass/epoxy composite laminates containing multiple through-the-width delaminations. The experimental and numerical study performed for the [0/90/90/0], cross-ply laminates with and without delaminations. The lengths of delaminations were different for different depths. Suemasu et al. [10] presented a numerical study to investigate the compressive behavior of carbon fiber reinforced plastic laminated plates containing multiple embedded delaminations. They used 20 node brick elements for discretizing the composite layers and incorporated cohesive elements for modeling the delamination. They investigated the effects of interlaminar toughness on the buckling mode and delamination growth stability. Kyoung and Kim [11] presented an analytical method to determine the delamination buckling and growth of one-dimensional beam-plate. They also investigated the effects of delamination length and location on the buckling load and delamination growth. De Borst and Remmers [12] used mezo level approach to study the local buckling growth of a delaminated layer. Wagner et al. [13] proposed a finite element method to simulate the delamination propagation in plate strips and plates with circular embedded delamination. They used interface elements in the regions that the delamination is expected to propagate. The used interface element formulation was based on the plasticity theory.

The objective of this paper is to employ an interface element incorporating with layerwise theory to study the delamination buckling and growth of laminated composites containing initial through-the-width or embedded delamination. The fracture mechanics approaches can not easily predict the simultaneous delamination initiation and propagation, therefore, the interface element concept in conjunction with layerwise elements (layerwise theory of laminates) is used to determine the delamination growth and the fracture mechanics concept is employed indirectly in this approach. For this purpose, a finite thickness interface element model is used to simulate the progressive delamination. A bilinear constitutive equation relates the stress and strains of the element. The initiation and propagation of the delamination is modeled through the softening behavior of the interface layer. The present investigation is based on the formulation of the full layerwise plate theory to approximate the displacement field of the layers and the interface element is considered as a thin film numerical layer between any two adjacent layers. A non-linear layerwise finite element code was also developed to handle the numerical procedure of delamination buckling growth in composite laminates.

2. Layerwise finite element formulation

In this study the composite laminate is discretized by 8-noded multilayer elements as shown in Fig. 1, where the full layerwise plate theory is used to approximate the displacement of each node and each material layer. Each node of the element has three degrees of freedom. In contrast with the 3-D solid elements, the layerwise elements provide the capability of discretizing the structure by a 2-D mesh. Therefore, the amount of input data is reduced and the number of operations for constructing the element stiffness matrix is decreased. The interface element concept in the layerwise finite element method is considered as a numerical layer of the model inserted between any two adjacent layers, which the delamination is expected to propagate as typically shown in Fig. 3. The displacement field of the laminate can be approximated using 2-D interpolation functions multiplying by through-the-thickness interpolation functions and is written as [16,19]:

\[
\begin{align*}
\mathbf{u}(x,y,z) &= \sum_{j=1}^{n} \mathbf{U}_j(x,y)\mathbf{H}_j(z) \\
\mathbf{v}(x,y,z) &= \sum_{j=1}^{n} \mathbf{V}_j(x,y)\mathbf{H}_j(z) \\
\mathbf{w}(x,y,z) &= \sum_{j=1}^{n} \mathbf{W}_j(x,y)\mathbf{H}_j(z)
\end{align*}
\]

In this equation, \( n \) is the number of nodes in the thickness direction and \( \mathbf{U}_j, \mathbf{V}_j \) and \( \mathbf{W}_j \) are the in-plane approximate of the nodal values of displacement components, \( u, v \) and \( w \) respectively. For quadratic variation of the displacement, three nodes are considered through-the-thickness of each numerical layer, \( N_x \). The number of numerical layers can be greater than, equal to or less than the material layers of the composite laminate. \( \mathbf{H}_j \) is through-the-thickness quadratic interpolation function of the \( j \)th node and is defined by Van Hoa and Feng [18]:

\[
\begin{align*}
H_{2k-1}(z) &= G_1^k(z) \\
H_{2k}(z) &= G_2^k(z), \quad \forall z \leq z_{2k} \\
H_{2k+1}(z) &= G_3^k(z), \quad k = 1, 2, \ldots, N_x
\end{align*}
\]

where

\[
\begin{align*}
G_1^k &= \left( 1 - \frac{z}{h_k} \right) \left( 1 - \frac{2z}{h_k} \right) = \frac{1}{2} (1 - \zeta)(1 - \zeta) \\
G_2^k &= 4 \frac{z}{h_k} \left( 1 - \frac{z}{h_k} \right) = (1 + \zeta)(1 - \zeta) \quad \text{which} \quad z = z - z_0^k \\
G_3^k &= -\frac{z}{h_k} \left( 1 - \frac{2z}{h_k} \right) = \frac{1}{2} \zeta(1 + \zeta)
\end{align*}
\]

and \( z_0^k \) denotes the \( z \)-coordinate of bottom of the \( k \)th numerical layer. This equation can be written in the extended form as:

\[
\begin{align*}
\mathbf{u} &= \sum_{j=1}^{n} \sum_{i=1}^{8} \mathbf{U}_j N_i(\zeta, \eta) \mathbf{H}_j(\zeta) \\
\mathbf{v} &= \sum_{j=1}^{n} \sum_{i=1}^{8} \mathbf{V}_j N_i(\zeta, \eta) \mathbf{H}_j(\zeta) \\
\mathbf{w} &= \sum_{j=1}^{n} \sum_{i=1}^{8} \mathbf{W}_j N_i(\zeta, \eta) \mathbf{H}_j(\zeta)
\end{align*}
\]
In which, \( N_i \) is the in-plane shape function of the \( i \)th node and \( \xi, \eta \) and \( \zeta \) are local coordinates. According to this layerwise definition for the displacement field, the non-linear strain/displacement relations of the von-Karman type can be written as:

\[
\begin{align*}
\varepsilon_{xx} &= \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{m} U_i N_{ix} H_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{m} W_i N_{ix} H_j^2 \\
\varepsilon_{yy} &= \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{m} V_i N_{iy} H_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{m} W_i N_{iy} H_j^2 \\
\varepsilon_{zz} &= \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{m} W_i N_{iz} H_j^2 + \left( \sum_{j=1}^{n} \sum_{i=1}^{m} W_i N_{iz} H_j \right) \left( \sum_{k=1}^{n} \sum_{i=1}^{m} W_k N_{iz} H_k \right) \\
\gamma_{xy} &= \sum_{j=1}^{n} \sum_{i=1}^{m} U_i N_{ix} V_j N_{iy} H_j + V_j N_{iy} H_j, \\
\gamma_{yz} &= \sum_{j=1}^{n} \sum_{i=1}^{m} V_i N_{iy} W_j N_{iz} H_j + W_j N_{iz} H_j, \\
\gamma_{xz} &= \sum_{j=1}^{n} \sum_{i=1}^{m} W_i N_{iz} H_j + W_j N_{iz} H_j \\
\end{align*}
\]

\( j = 1, 2, \ldots, n \) (5)

It is worth to mention that the von-Karman non-linear terms are usually sufficient for the modeling of buckling behavior of laminates, especially when they are subjected to damage propagation such as delamination. The matrix form of this equation is:

\[
\{ \varepsilon \} = [\mathbf{B}] \{ \Delta \}
\]

(6)

In which, \( \Delta \) is the displacement components vector and \( \mathbf{B} \) is the matrix of derivatives of the shape functions and is defined by Van Hoa and Feng [18]:

\[
[\mathbf{B}] \{ \Delta \} = [\mathbf{b}_1 \mathbf{b}_2 \ldots \mathbf{b}_m] \begin{bmatrix} \delta_{u1} \\ \delta_{v1} \\ \vdots \\ \delta_{um} \end{bmatrix} = \begin{bmatrix} U_1 \\ V_1 \\ \vdots \\ W_m \end{bmatrix}
\]

(7)

and

\[
[\mathbf{B}] = \begin{bmatrix} N_{ix} H_j & W_i N_{ix} H_j \\ 0 & N_{iy} H_j & N_{iz} H_j \\ 0 & 0 & N_{iz} H_j \end{bmatrix}
\]

(8)

The potential energy equation is defined as:

\[
\Pi = \frac{1}{2} \int \{ \varepsilon \}^T \{ \varepsilon \} dV - T^T \{ \varepsilon \} dF
\]

In which, \( \{ \varepsilon \} \) is the elastic stiffness matrix of the material layer. The tangent stiffness matrix is defined as the second derivative of the potential energy equation:

\[
[\mathbf{K}]^T = \frac{\partial^2 \Pi}{\partial (\{ \varepsilon \})^2} = \int \left[ \frac{\partial \{ \varepsilon \}}{\partial \{ \varepsilon \}} \right]^T [\mathbf{C}] \left[ \frac{\partial \{ \varepsilon \}}{\partial \{ \varepsilon \}} \right] dV + \int \left[ \frac{\partial^2 \{ \varepsilon \}}{\partial (\{ \varepsilon \})^2} \right]^T [\mathbf{C}] \{ \varepsilon \} dV
\]

(10)

In which, the term \( \frac{\partial \{ \varepsilon \}}{\partial \{ \varepsilon \}} \) represents the \( \{ \mathbf{B} \} \) matrix which contains non-linear terms of displacements and can be determined by Eq. (8). The term \( \frac{\partial^2 \{ \varepsilon \}}{\partial (\{ \varepsilon \})^2} \) represents the derivative of matrix \( \{ \mathbf{B} \} \) with respect to displacement vector and can be defined as:

\[
[\mathbf{K}]^T = \begin{bmatrix} 0 & 0 & N_{m,Hx} N_{ix} H_j \\ 0 & 0 & N_{m,Hy} N_{iy} H_j \\ 0 & 0 & N_{m,Hz} N_{iz} H_j \end{bmatrix}
\]

\( j, m = 1, 2, \ldots, n \)

So, the tangent stiffness matrix of each material layer can be obtained by:

\[
[\mathbf{K}]_l^T = \int \mathbf{e}_l^T \left[ \begin{bmatrix} \mathbf{C} \end{bmatrix} \right] dV + \int \mathbf{e}_l^T \left[ \begin{bmatrix} \frac{\partial^2 \{ \varepsilon \}}{\partial (\{ \varepsilon \})^2} \end{bmatrix} \right] \left[ \begin{bmatrix} \mathbf{C} \end{bmatrix} \right] dV
\]

(12)

3. Implementing the interface layer

The used interface layer is similar to an isoparametric 16-node hexahedral solid element with a very small thickness, as shown in Fig. 2. In this figure, \((X, Y, Z)\) denote the global coordinate and the \( O' \) is the central point of the midplane of the element. The thickness of the interface layer is about 1/100 of the laminate thickness and it is inserted as a numerical layer between two adjacent laminate layers. The strain vector, \( \varepsilon \), stress vector, \( \sigma \), and the displacement vector, \( \mathbf{u} \), for the interface layer are defined by:

\[
\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}, \quad \sigma = \mathbf{D} \varepsilon
\]

(13)

In which \( \mathbf{D} \) is the interface elastic-damage stiffness matrix. Because the interface layer links the two adjacent physical layers, the interlaminar components of the strain and stress fields are only considered for this virtually assumed layer. In order to avoid Poisson’s locking and for decoupling of the interlaminar shear stress with the out of plane displacement similar to solid-like interface element stress filed with the zero-thickness interface element, the terms \( w_x \) and \( w_y \) are assumed to be zero [17]. It means that, the displacement variations in the thickness direction are only important in the definition of interface behavior.

The displacement and strain vectors can be approximated as follows:

\[
\mathbf{u} = \sum_{i=1}^{16} \mathbf{N}_i \mathbf{u}_i, \quad \varepsilon = \sum_{i=1}^{16} \mathbf{B}_i \mathbf{u}_i
\]

(14)

In this equation, \( \mathbf{N}_i \) is the shape function of the \( i \)th node and \( \mathbf{B}_i \) is the shape function derivatives matrix which is defined by:

\[
\mathbf{B}_i = \begin{bmatrix} N_{ix} & 0 & 0 \\ 0 & N_{iy} & 0 \\ 0 & 0 & N_{iz} \end{bmatrix}
\]

(15)

![Fig. 2. Finite thickness interface element.](image-url)
Using the finite element approach, the displacement and strain vectors can be approximated from the layerwise formulation with linear through-the-thickness shape function. In this way in Eq. (4), $N_i$ is the shape function of $i$th node and $H_j$ is through-the-thickness linear interpolation function of the $j$th node defined by:

$$H_1(\zeta) = \frac{1 - \zeta}{2}, \quad H_2(\zeta) = \frac{1 + \zeta}{2}$$

Eq. (16)

Fig. 3 shows a typical composite laminate including four material layers and three interface layers, which totally divided into nine numerical layers. For a typical laminate containing two material layers connecting with an interface layer (totally three numerical layers), the global stiffness matrix of the laminate is constructed by assembling of the stiffness matrices of three single numerical layers, as follows:

$$[B]^T[D][B] = 
\begin{bmatrix}
B_{1-1}^T D_{11} B_{1-1} & B_{1-1}^T D_{12} B_{1-2} & B_{1-1}^T D_{13} B_{1-3} & 0 & 0 & 0 \\
B_{1-2}^T D_{21} B_{1-1} & B_{1-2}^T D_{22} B_{1-2} & B_{1-2}^T D_{23} B_{1-3} & 0 & 0 & 0 \\
B_{1-3}^T D_{31} B_{1-1} & B_{1-3}^T D_{32} B_{1-2} & B_{1-3}^T D_{33} B_{1-3} + B_{1-3}^T D_{34} B_{1-4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}$$

Eq. (17)

In this equation, $B$ is the matrix of shape function derivatives for material layers and $\bar{B}$ is the matrix of shape function derivatives for interface layers. The superscripts of $M1$ and $M2$ denote the material layers and $I$ denote the interface layer and $i = 1, 2, \ldots, n$, where $n = 6$ is the number of nodes through-the-thickness of the laminate.

4. Constitutive equation of the interface

For the interface element, an irreversible constitutive law can be incorporated for modeling of an inelastic softening behavior. In this article, a bilinear constitutive law is chosen which is based on the zero-thickness cohesive element presented in Ref. [20] and also is the same as its modified solid-like model presented in Ref. [14]. This constitutive law relates the stresses to strains for any pure fracture or loading mode. In the case of mixed mode loading, the constitutive law relates the effective stresses to the effective strains, as shown in Fig. 4. The effective strain is a positive continuous quantity and when there is not any compressive normal strain, it is equal to the norm of strain vector and is defined by:

$$e_m = \sqrt{(e_x^2) + (e_y^2) + (e_z^2)}$$

Eq. (18)

where the $\max$ operator is: $\max(a, b) = \begin{cases} a & \text{if } a \leq b \\ b & \text{if } a > b \end{cases}$

If $e_z \leq 0$ then: $\gamma_{\text{shear}} = \sqrt{e_x^2 + e_y^2}$

Eq. (19)

The constitutive law contains of three different parts. If the effective strain is less than $e_m^0$, the interface material behaves linearly elastic and no damage is presented in the element, so there isn't any decohesion between layers. When the effective strain reaches the $e_m^0$, the interlaminar damage initiates and after that, the interface stresses decrease linearly. The strain $e_m^0$ refers to complete decohesion.

In order to complete the description of interface behavior, it is necessary to specify the strains, corresponding to initiation and completion of damage. The delamination initiation is predicted by the quadratic failure criterion and the effective strain at delamination onset is defined by:

$\sigma_m = \begin{cases} 0 & \text{if } e_m \leq e_m^I \\ \frac{G_c}{h_0} + (1-d)K & \text{if } e_m > e_m^I \end{cases}$

Fig. 4. Mixed mode bilinear constitutive law.

The effective critical fracture toughness is equal to the area under the effective stress/strain curve and this equivalence is used to calculate the effective critical energy release rate and is calculated by:

$$\bar{G}_c = \frac{g}{h_0}$$

where $h_0$ is the interface element thickness and $G_c$ is the mixed mode critical energy release rate and is calculated by a criterion proposed by Benzegagh and Kenane [15] (B–K criterion) which is defined as:

$$G_c = G_{IIc} + (G_{IIIc} - G_{IIc}) \left( \frac{G_{shear}}{G_f} \right) \eta$$

In which $G_{IIc}$, $G_{IIIc}$ and $G_{shear}$ are the critical strain energy release rates of mode I, II and III respectively and $G_{shear} = G_{IIc} + G_{IIIc}$ and $G_f = G_{IIc} + G_{shear}$ and $\eta$ is an experimental parameter.

By introducing a mixed-mode scalar-valued damage parameter:

$$d = \frac{\varepsilon_m - \varepsilon_m^b}{\varepsilon_m^b - \varepsilon_m^c}$$

The interface constitutive law in elastic and elastic-damage conditions can be defined by:

$$\sigma = D' e$$

In which:

$$D' = \begin{cases} \begin{bmatrix} KI & e_m - e_m^b \ v_m \ a_0 \ e_m^b \ a_0 \ end{bmatrix} & \text{elastic cond.} \\ (1 - d)KI & \text{elastic–damage cond.} \\ KI & e_m - e_m \end{cases}$$

In this equation, $K$ is the penalty stiffness, $I$ is the identity matrix, $d$ is the damage parameter and:

$$I_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{-a_0}{2h_0} \end{bmatrix}$$

To avoid penetration and friction free contact of delaminated faces and considering the closure effect in the opening mode, Eq. (26-c) was introduced.

### 5. Finite element analysis

A non-linear finite element program is developed to handle the numerical procedure. The laminate model is discretized by 2-D eight-node elements. In the thickness direction of each layer, the displacement field is approximated by quadratic shape functions. Each layer of the laminate can be modeled by one or more elements of this type.

The displacement field is approximated by full layerwise laminate plate theory and the geometric non-linearity of von-Karman type is applied to the strain/displacement relations. In order to verify the developed layerwise finite element program, the obtained results are compared with the available numerical and experimental results for a DCB specimen and a laminated beam containing a thorough the width pre-delamination.

The geometry and loading of the DCB specimen is shown in Fig. 5. The specimen has a length of $l = 150$ mm, width of $b = 25.4$ mm and thickness of $2h = 3.05$ mm. The pre-delaminated region has a length of $a_0 = 31.75$ mm. The AS-4/3501-6 graphite/epoxy material is used which its mechanical properties are listed in Table 1.

Material properties of graphite/epoxy.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$E_{11}$ (N/mm$^2$)</td>
<td>139,300</td>
<td>138,000</td>
</tr>
<tr>
<td>$E_{22}$ (N/mm$^2$)</td>
<td>9720</td>
<td>8960</td>
</tr>
<tr>
<td>$G_{12}$ (N/mm$^2$)</td>
<td>5580</td>
<td>7100</td>
</tr>
<tr>
<td>$G_{13}$ (N/mm$^2$)</td>
<td>3450</td>
<td>3446</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.29</td>
<td>0.3</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$G_{23}$ (N/mm$^2$)</td>
<td>0.0876</td>
<td>0.222</td>
</tr>
<tr>
<td>$G_{23}$ (N/mm$^2$)</td>
<td>0.3152</td>
<td>0.444</td>
</tr>
<tr>
<td>$\sigma^*$ (N/mm$^2$)</td>
<td>44.54</td>
<td>51.7</td>
</tr>
<tr>
<td>$\sigma^+$ (N/mm$^2$)</td>
<td>106.9</td>
<td>91.7</td>
</tr>
</tbody>
</table>

Fig. 6 compares the obtained loads/displacements from the present study and reference [14]. This figure shows that, the obtained results from this study are in good agreement with the experimental, analytical, and previously available numerical results. The differences in the linear part of the curve are due to the considered different penalty stiffness values for the numerical elements.

Corigliano and Allix [21] proposed the definition of the penalty stiffness of the 2-D cohesive elements, as a function of the interface virtual thickness, \( t \), and elastic modules of the interface (\( E_{33}, G_{13} \) and \( G_{23} \)) as:

\[
K_{33} = \frac{E_{33}}{t}, \quad K_{11} = \frac{2G_{13}}{t}, \quad K_{22} = \frac{2G_{23}}{t}
\]

and for simplicity it can be assumed that \( K_{11} = K_{22} = K_{33} = \frac{E_{33}}{t} \). For 2-D (zero-thickness) interface models, the penalty stiffness parameter is equal to the slope of the stress/relative displacement curve, whereas for solid-like interface models, this parameter is the slope of stress/strain curve, so the stiffness of solid-like element is \( 1/t \) times of the stiffness of the zero-thickness element and can be equal to \( E_{33} \).

The second example is a composite laminate containing thorough the width pre-delamination. The obtained results for delamination buckling in case of non-growing delamination are compared with those obtained from the ANSYS finite element software. The geometry of the laminate with \([0/45/45//0]\) lay-up is shown in Fig. 8, where “/” is the through-the-thickness location of delamination. The used material properties are also listed in Table 1. The laminate is loaded symmetrically in the ‘x’ direction by applying ends shortening. The boundary conditions are clamped for loaded ends and free for the other ends. The obtained load versus normalized transverse deflection and load versus end-shortening displacement from the present study are compared with the results obtained from ANSYS commercial code in Fig. 9a and bb. These two figures show that, the obtained results from the developed non-linear procedure are in good agreement with those obtained from the ANSYS finite element software.

Fig. 7. Comparison of the results obtained from the present study with Ref. [8] for the \([0/45/45//0]\) lay-up: (a) load versus transverse deflection and (b) load versus end-shortening displacement.

Fig. 8. Geometry dimensions of the specimen.

Fig. 9. Comparison of the results obtained from the present study with ANSYS commercial code for the \([0/45/45//0]\) lay-up: (a) load versus normalized transverse deflection and (b) load versus end-shortening displacement.
6. Numerical models

The first numerical example is a unidirectional laminated composite plate, containing a central through-the-width delamination exposing to a uniform in-plane end-shortening. The laminate is made up of T300/976 graphite/epoxy material, with a [04/012/04] lay-up where “||” symbol indicate the through-the-thickness location of delamination. The material properties, needed for numerical analysis, are presented in Table 1. Also, details of geometry, dimensions and boundary conditions of the laminate, are available in Ref. [8]. Some geometrical information are presented in Fig. 7b. In the present model, the penalty stiffness of the interface layer is considered to be $10^6$ N/mm. Instead of imperfection, concentrated loads equal to 3 N are applied to the nodes of centerline of the upper and lower layers. The obtained numerical results for the variation of normalized compressive load versus central out of plane displacement and also versus the applied axial end-shortening, are illustrated in Fig. 7a and b and compared with the presented results in Ref. [8] and the results show good accordance. These figures also show that, at a load around 210 N/mm the local buckling of the delaminated layers occurs but the base laminate still has very small deflection. By increasing the load, the delamination propagates and the delaminated layers deflect more. At a load around 1350 N/mm, the base laminate buckles and deflects downward and the delamination area becomes very large. The oscillations in the results for the variation of normalized compressive load versus central out of plane displacement, presented in Ref. [8], is due to the use of crack closure method with an elastic analysis for delamination growth. However, a continuous variation of normalized compressive load versus central out of plane displacement was obtained in the present work which is due to the use of interface layer with non-linear constitutive law. In addition, the differences between the results of the present study with those in Ref. [8] especially in the last loading steps could be due to the use of different type of elements and numerical method in which we used layerwise-interface elements however reference [8] used discontinues layerwise in the finite strip method. The advantage of the present method is the capability to predict the embedded delamination growth with any arbitrary shape.

The other series of examples consist of two symmetric cross-ply laminates containing central through-the-width delaminations with different lay-ups and laminated plates with two types of stacking sequences containing central rectangular embedded delamination. For these laminates, the AS-4/3501-6 graphite/epoxy material with the available mechanical properties and listed in Table 1 is used.

![Fig. 10](image-url) (a) Geometry and dimensions and (b) finite element mesh of the model.

6.1. Through-the-width delamination

The laminate geometry, dimensions and typical mesh of the models are illustrated in Fig. 10. The plate is symmetrically loaded by applying displacement at two ends. The boundary conditions are clamped for loaded ends and free for the other edges. An initial transverse perturbation load is also applied on the midline of the models with opposite directions for upper and lower layers.

The selected stacking sequences are [0\textdegree/90\textdegree/90\textdegree//0\textdegree] and [90\textdegree/0\textdegree/0\textdegree//90\textdegree] where “//” is the through-the-thickness location of delamination. Due to the symmetry condition with respect to the \(y\)-axis, half of the laminate has been modeled. The penalty stiffness of the interface elements, \(K\), is considered to be in the order of transverse modulus of elasticity, \(E_{22}\), and therefore is equal to 8960 N/mm\(^2\). Variations of reaction load versus normalized transverse deflection and also versus the applied axial displacement are obtained and illustrated in Figs. 11 and 12 for both lay-ups.

Variations of load versus normalized transverse deflection in Fig. 11a show that for the case of [0\textdegree/90\textdegree/90\textdegree//0\textdegree] lay-up, the buckling mode is of mixed type. The delamination is initially closed, then by increasing the load, the delaminated sub-laminate starts to deflect upward because of the presence of perturbation, but the base laminate is still unaffected. When the load of the upper sub-laminate, reaches its critical value, which is less than the critical load of the base laminate, the delaminated layers buckle but they still can carry load in the post-buckling region. The buckling point has been shown in Fig. 11b as well. Fig. 11a also shows that by further increasing of the load, a mixed-mode buckling phenomenon occurs, the base laminate becomes critical, and the delamination grows at the same time. Therefore, the laminate cannot stand under further load and the load versus displacement curve is dropped to a softening behavior as shown in Fig. 11b.

Fig. 12a shows the variations of reaction load versus normalized transverse deflection for [90\textdegree/0\textdegree/0\textdegree//90\textdegree] laminate. Because of the low bending stiffness of delaminated layers (upper sub-laminate) with the fiber direction of 90\textdegree, it starts bending upward due to the perturbation forces at the early stages of loading, and there isn’t a clear buckling point for this case. This is approved in Fig. 12b by illustrating the load versus in-plane end-shortening displacement. In the early loading steps, the base laminate is not affected significantly. By increasing the applied load, the base laminate starts to buckle and at this point, the upper sub-laminate deflects upward by the constraint of the base laminate, and then the delamination growth is occurred. Therefore, the laminate can’t stand under further increase of load and the load versus displacement curve is dropped to a softening behavior as shown in Fig. 12b.

The contour of damage propagation corresponding to the loading steps of (I), (II) and (III) shown in Fig. 11a for [0\textdegree/90\textdegree/90\textdegree//0\textdegree] laminate, are also illustrated in Fig. 13. In this figure, the dark areas denote the delaminated parts and the hatched areas correspond to the initial defined delamination area. These figures show gradual delamination propagations by increasing the end-shortening at post-buckling stage of the laminate.

![Fig. 11. Results for [0\textdegree/90\textdegree/90\textdegree//0\textdegree] laminate: (a) load versus normalized transverse deflection and (b) load versus end-shortening displacement.](image1)

![Fig. 12. Results for [90\textdegree/0\textdegree/0\textdegree//90\textdegree] laminate: (a) load versus normalized transverse deflection and (b) load versus end-shortening displacement.](image2)
6.2. Embedded delamination

The geometry and dimensions of the laminates containing an initial rectangular embedded delamination are shown in Fig. 14. Two lay-ups of unidirectional ([0/0/0/0/0/0/0/0/0/0/0]) and quasi-isotropic ([0/−45/45/−45/90/0/90/0/−45/45/0/0/−45/0]) are considered for the laminates. The laminate is loaded by applying displacement at one end as shown in Fig. 14 and the boundary conditions are clamped at the laminate edges. In order to prevent the complete failure of the delaminated layers, and to reduce the computational time, a predefined permitted delamination area with the size of 40 mm × 30 mm was selected for the laminates. For this purpose, the behavior of the existed interface elements between this area and the edges of the laminate are forced to be elastic. However, the quasi-isotropic lay-up is un-symmetric about the x- and y-axis, but half of the specimen is only modeled to decrease the computational time. Besides, in order to reduce the computational effort, each group of four material layers is assumed as one numerical layer by means of the laminated element concept.

In the case of unidirectional lay-up, variations of load versus displacement results are shown in Fig. 15a and b. The analyses were performed for two conditions of buckling without delamination propagation (elastic interface elements), and buckling with delamination propagation. Fig. 15a shows that, before occurrence of the buckling, the stationary and propagating delamination behaviors are coincident. A few steps after the buckling point, the delamination grows unsteadily and leads to an abrupt reduction in the curve slope. After that, delamination grows more steadily until it reaches the borders that the propagation has been prevented intentionally (dashed lines in Fig. 17); in this stage the plate starts buckling in the mixed mode condition.

The obtained variations of load versus displacement for quasi-isotropic laminate are illustrated in Fig. 16a and b for both with and without delamination propagation. Fig. 16a shows that, before the point with $N_y = 550$ N/mm, there is no delamination and the buckling behaviors without delamination and with delamination propagation of the laminate, are the same. After this point, delamination starts to propagate and therefore the stiffness of the laminate diminishes as shown by decrease in the slope of the curve in Fig. 16b as well. It is also worth to mention that, before the propagating of delamination, the buckling mode is of global type. But, as the delamination grows and becomes larger, the buckling mode...
Fig. 15. Results of [0/0/0/0] lay-up: (a) load versus normalized transverse deflection and (b) load versus end-shortening displacement.

Fig. 16. Results for [0/-45/45/90/02] lay-up: (a) load versus normalized transverse deflection and (b) load versus end-shortening displacement.

Fig. 17. Damage propagation contours for [0/-45/45/90/02] lay-up at the indicated points in Fig. 15a: (a) at point I, (b) at point II, (c) at point III, and (d) at point IV.
changes to the mixed mode type. To have a better understanding of delamination growth in various loading steps, the contours of the damage propagation for the loading steps, which are pointed in Fig. 16a, are also shown in Fig. 17. In this figure, the dashed lines specify the permitted delamination growth area and dark regions denote the delaminated areas.

7. Conclusion

The buckling delamination growth of composite laminates containing initial through-the-width or embedded delamination was investigated in this study. The delamination growth caused by compressive loads was predicted via considering the softening behavior for interface layers which was implemented in the full layerwise laminated theory. The geometrical non-linearity of the problem was made by von-karman non-linear terms. It was shown that the delamination growth can be well predicted using the layerwise-interface elements with decohesive law. It was also shown that, the buckling mode and delamination growth process, depends on the stacking sequence of the laminates. These figures also show the gradual delamination propagations by increasing the end-shortening at post-buckling stage of the laminates containing initial through-the-width or embedded delamination.

References