Chapter2:
Digital Image Fundamentals

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Human Visual Perception

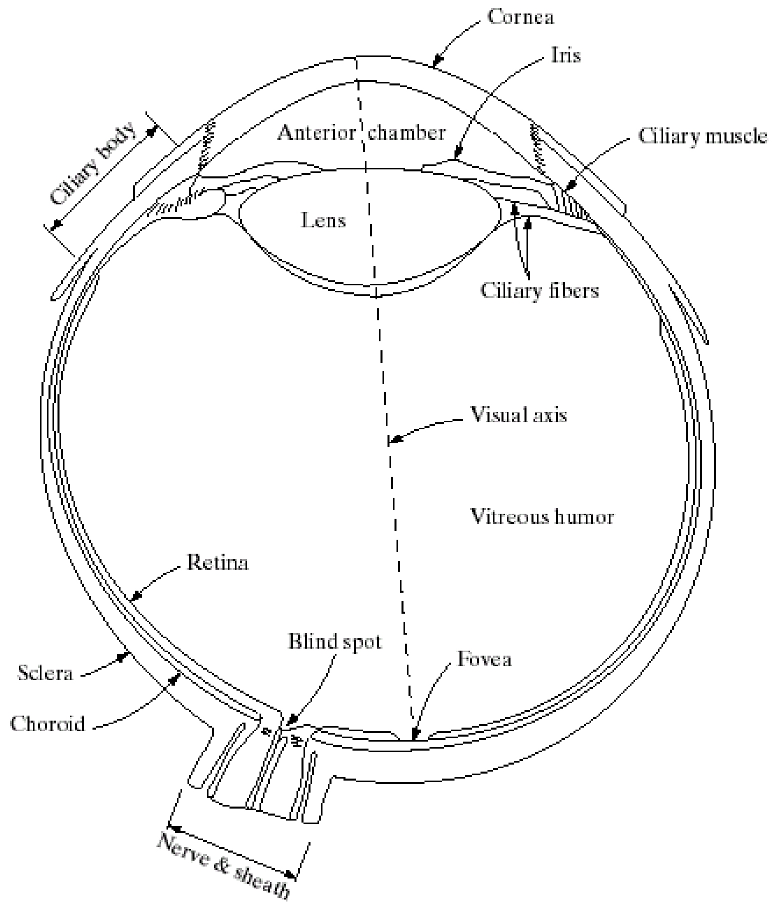


FIGURE 2.1 Simplified diagram of a cross section of the human eye.

The Human Eye

- Diameter: 20 mm
- 3 membranes enclose the eye
  - Cornea & sclera
  - Choroid
  - Retina

The Choroid

- The choroid contains blood vessels for eye nutrition and is heavily pigmented to reduce extraneous light entrance and backscatter.

- It is divided into the ciliary body and the iris diaphragm, which controls the amount of light that enters the pupil (2 mm ~ 8 mm).

The Lens

- The lens is made up of fibrous cells and is suspended by fibers that attach it to the ciliary body.

- It is slightly yellow and absorbs approx. 8% of the visible light spectrum.

The Retina

- The retina lines the entire posterior portion.
- Discrete light receptors are distributed over the surface of the retina:
  - cones (6-7 million per eye) and
  - rods (75-150 million per eye)

Cones

- Cones are located in the fovea and are sensitive to color.
- Each one is connected to its own nerve end.
- Cone vision is called *photopic* (or bright-light vision).

Rods

- Rods are giving a general, overall picture of the field of view and are not involved in color vision.
- Several rods are connected to a single nerve and are sensitive to low levels of illumination (*scotopic* or dim-light vision).

Receptor Distribution

- The distribution of receptors is radially symmetric about the fovea.
- Cones are most dense in the center of the fovea while rods increase in density from the center out to approximately 20% off axis and then decrease.

Cones & Rods

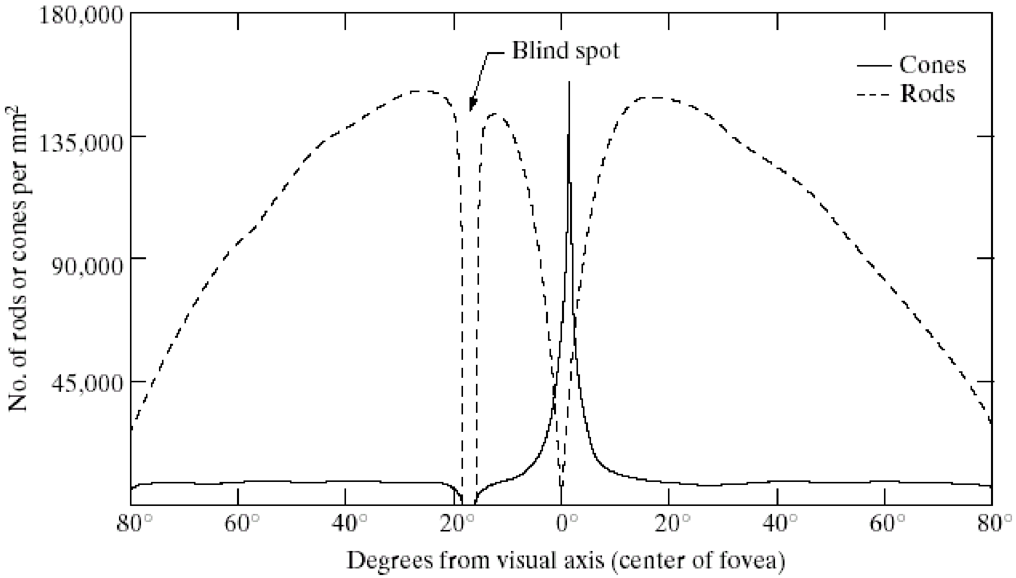


FIGURE 2.2 Distribution of rods and cones in the retina.

The Fovea

- The fovea is circular (1.5 mm in diameter) but can be assumed to be a square sensor array (1.5 mm x 1.5 mm).
- The density of cones: 150,000 elements/mm^2 ~ 337,000 for the fovea.
- A CCD imaging chip of medium resolution needs 5 mm x 5 mm for this number of elements

Image Formation in the Eye

- The eye lens (if compared to an optical lens) is flexible.
- It gets controlled by the fibers of the ciliary body and to focus on distant objects it gets flatter (and vice versa).

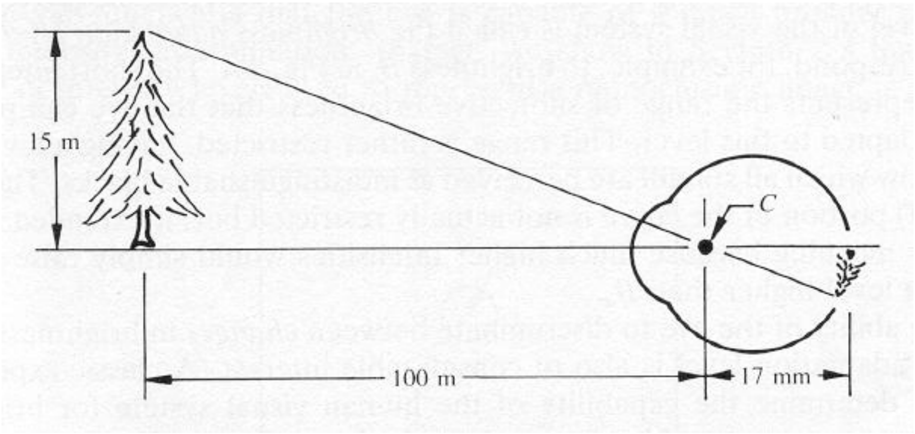
Image Formation in the Eye

- Distance between the center of the lens and the retina (*focal length*):
  - varies from 17 mm to 14 mm (refractive power of lens goes from minimum to maximum).

- Objects farther than 3 m use minimum refractive lens powers (and vice versa).

Image Formation in the Eye

- Example:
  - Calculation of retinal image of an object



$$\frac{15}{100} = \frac{x}{17}$$

$$x = 2.55mm$$

Image Formation in the Eye

- Perception takes place by the relative excitation of light receptors.
- These receptors transform radiant energy into electrical impulses that are ultimately decoded by the brain.

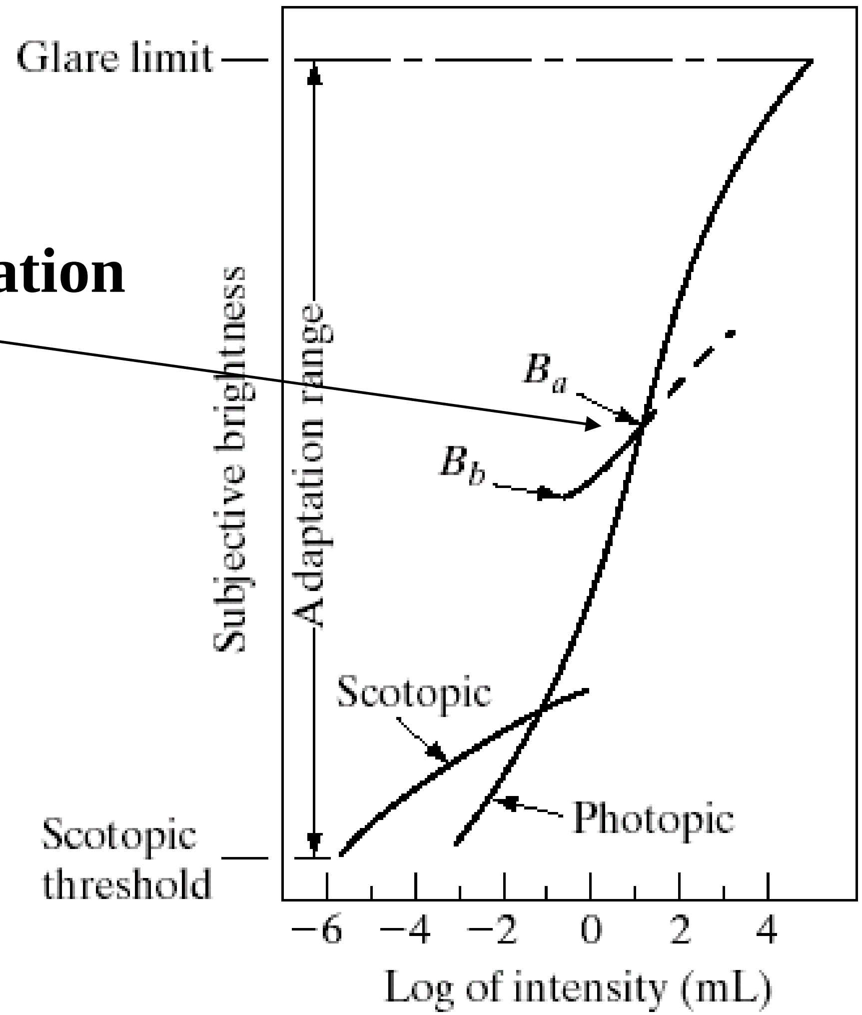
Brightness Adaptation & Discrimination

- Range of light intensity levels to which HVS (human visual system) can adapt: on the order of 10^{10}.
- Subjective brightness (i.e. intensity as perceived by the HVS) is a logarithmic function of the light intensity incident on the eye.

Illustration

FIGURE 2.4 Range of subjective brightness sensations showing a particular adaptation level.

Brightness adaptation



Brightness Adaptation & Discrimination

- The HVS cannot operate over such a range simultaneously.

- For any given set of conditions, the current sensitivity level of HVS is called the brightness adaptation level.

Brightness Adaptation & Discrimination

- The eye also discriminates between changes in brightness at any specific adaptation level.

$$\frac{\Delta I_c}{I} \rightarrow \text{Weber ratio}$$

Where: ΔI_c: the increment of illumination discriminable 50% of the time and

I : background illumination

Experiments on Discrimination

The ability of the eye to discriminate between *changes* in light Intensity at any specific adaptation level

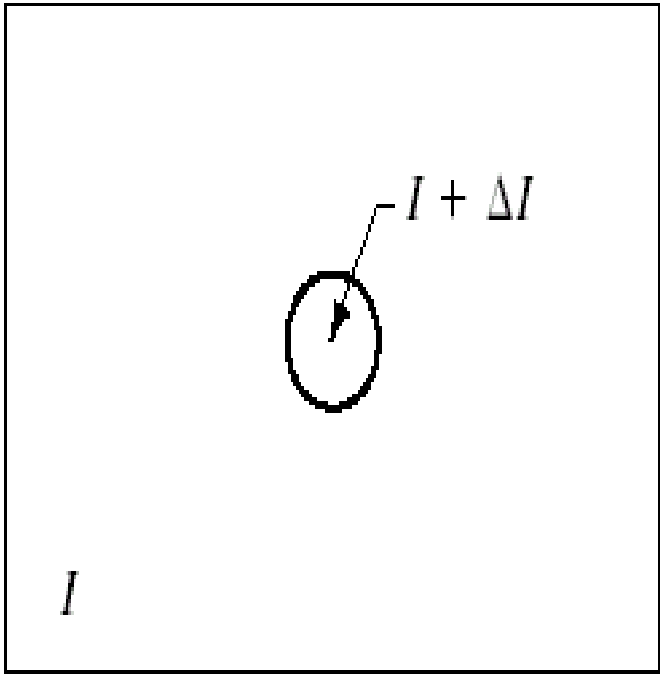
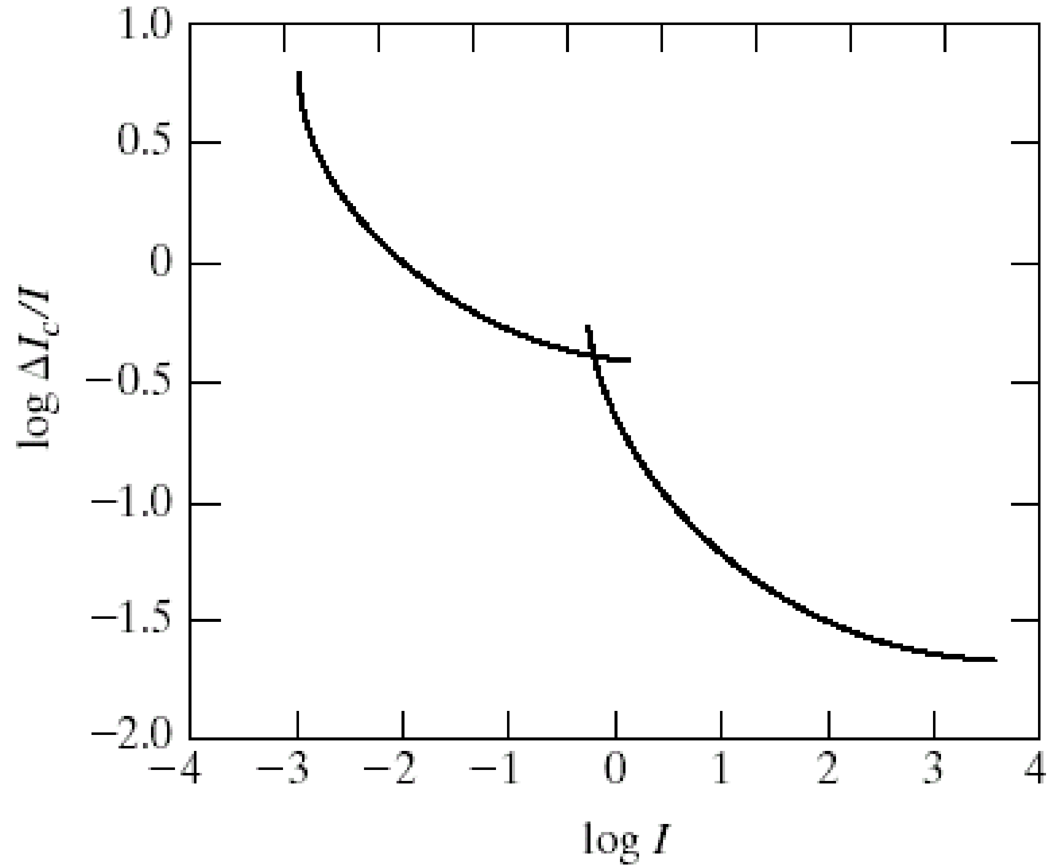


FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

Discriminate

FIGURE 2.6
Typical Weber ratio as a function of intensity.



Brightness Adaptation & Discrimination

- Small values of Weber ratio mean good brightness discrimination (and vice versa).
- At low levels of illumination brightness discrimination is poor (rods) and it improves significantly as background illumination increases (cones).

Brightness Adaptation & Discrimination

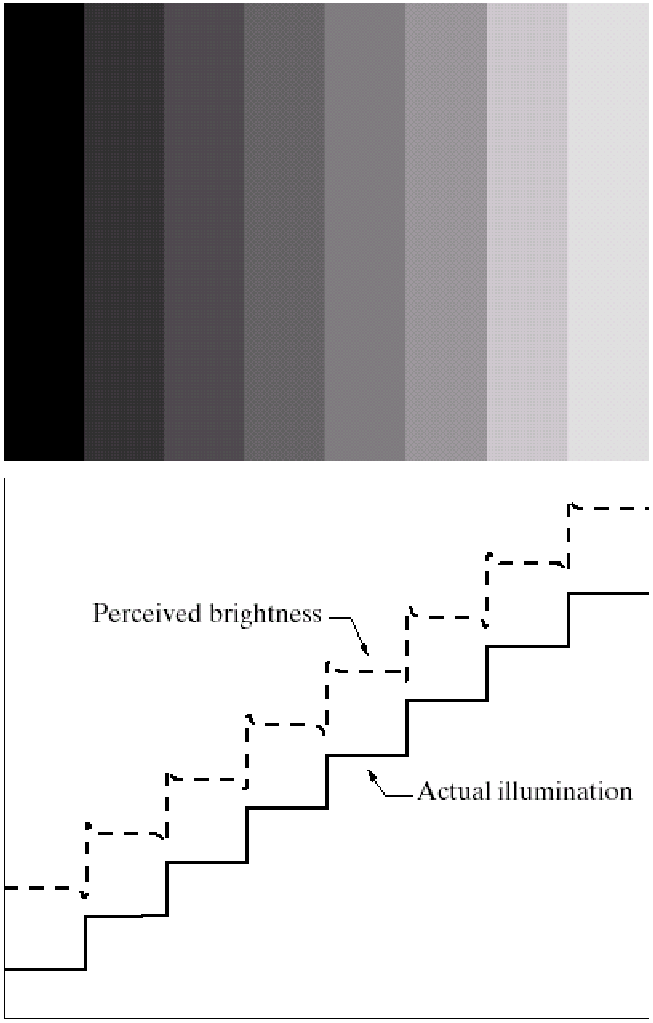
- The typical observer can discern one to two dozen different intensity changes
  - i.e. the number of different intensities a person can see at any one point in a monochrome image

Brightness Adaptation & Discrimination

- Overall intensity discrimination is broad due to different set of incremental changes to be detected at each new adaptation level.

- Perceived brightness is not a simple function of intensity
  - Scalloped effect, Mach band pattern
  - Simultaneous contrast

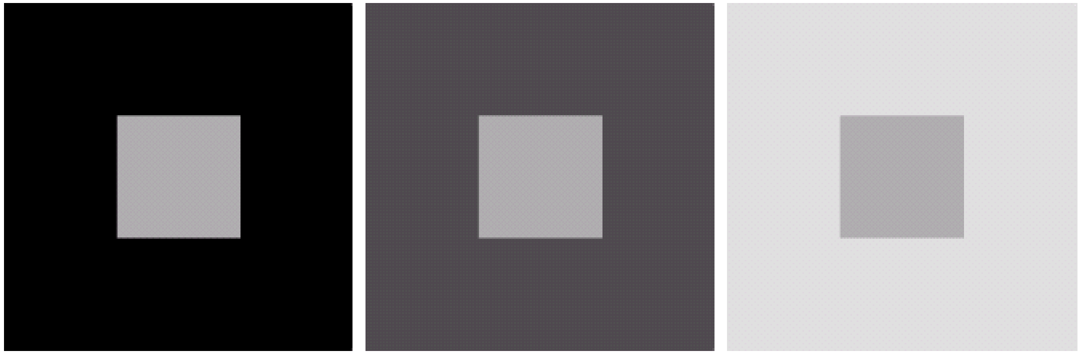
Perceived Brightness



a
b

FIGURE 2.7
(a) An example showing that perceived brightness is not a simple function of intensity. The relative vertical positions between the two profiles in (b) have no special significance; they were chosen for clarity.

Simultaneous Contrast



Illusions

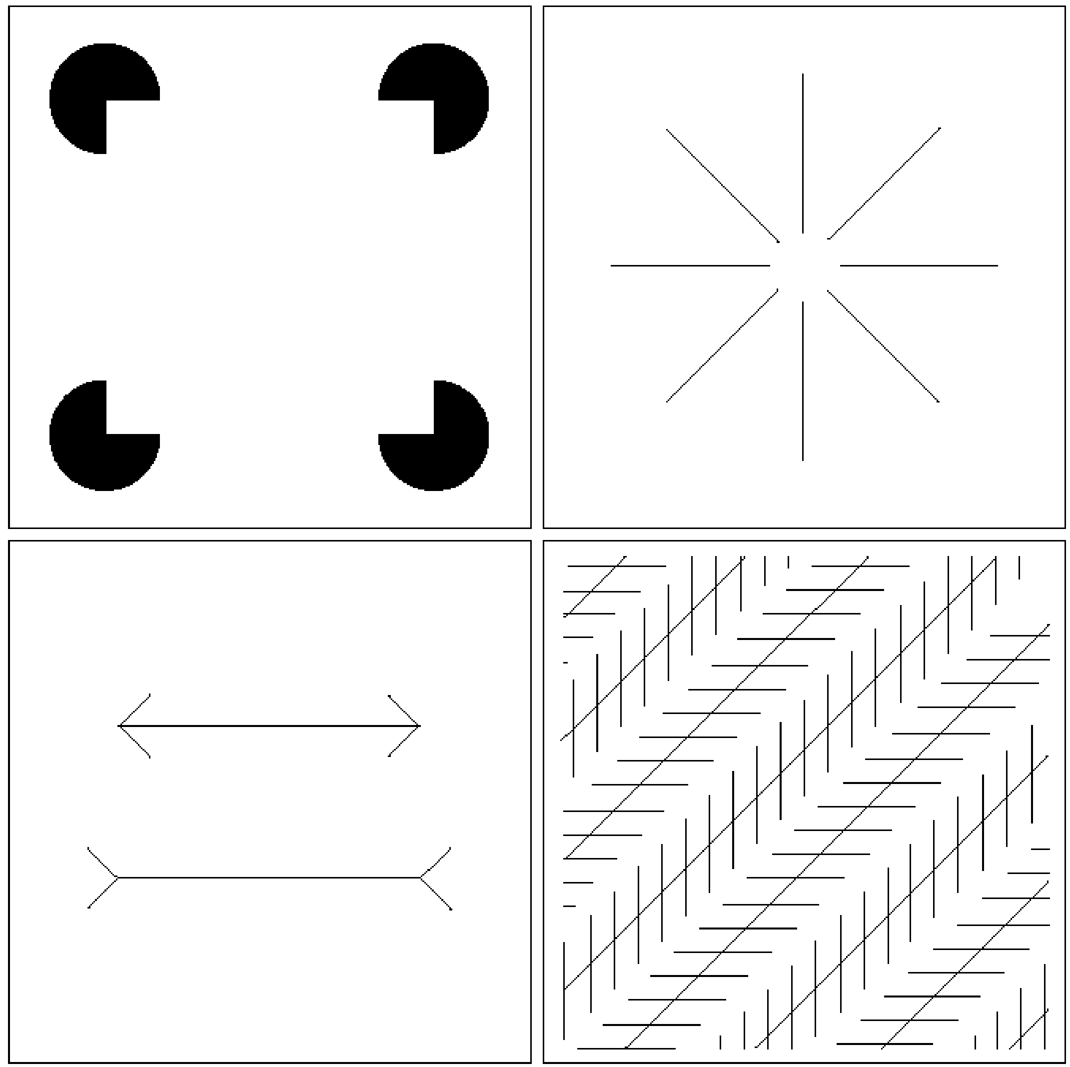


FIGURE 2.9 Some well-known optical illusions.

Light and the Electromagnetic Spectrum

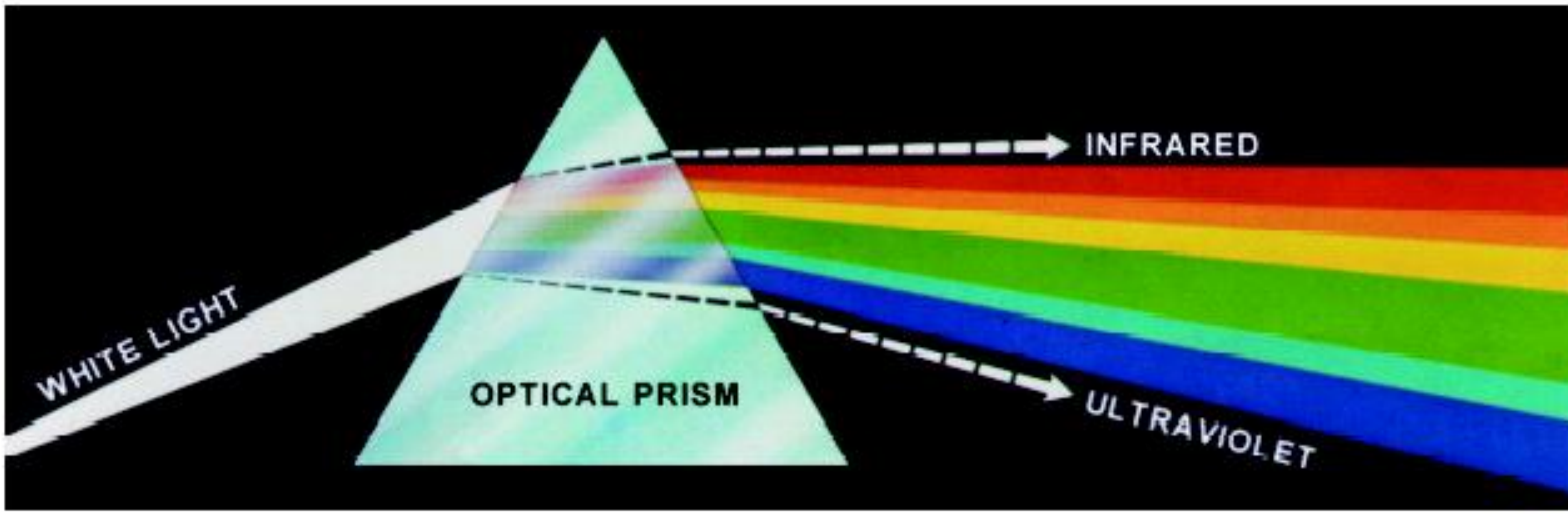


FIGURE 6.1 Color spectrum seen by passing white light through a prism. (Courtesy of the General Electric Co., Lamp Business Division.)

Violet, Blue, Green, Yellow, Orange, and Red

Blends smoothly into the next.

Light and Electromagnetic (EM) Spectrum

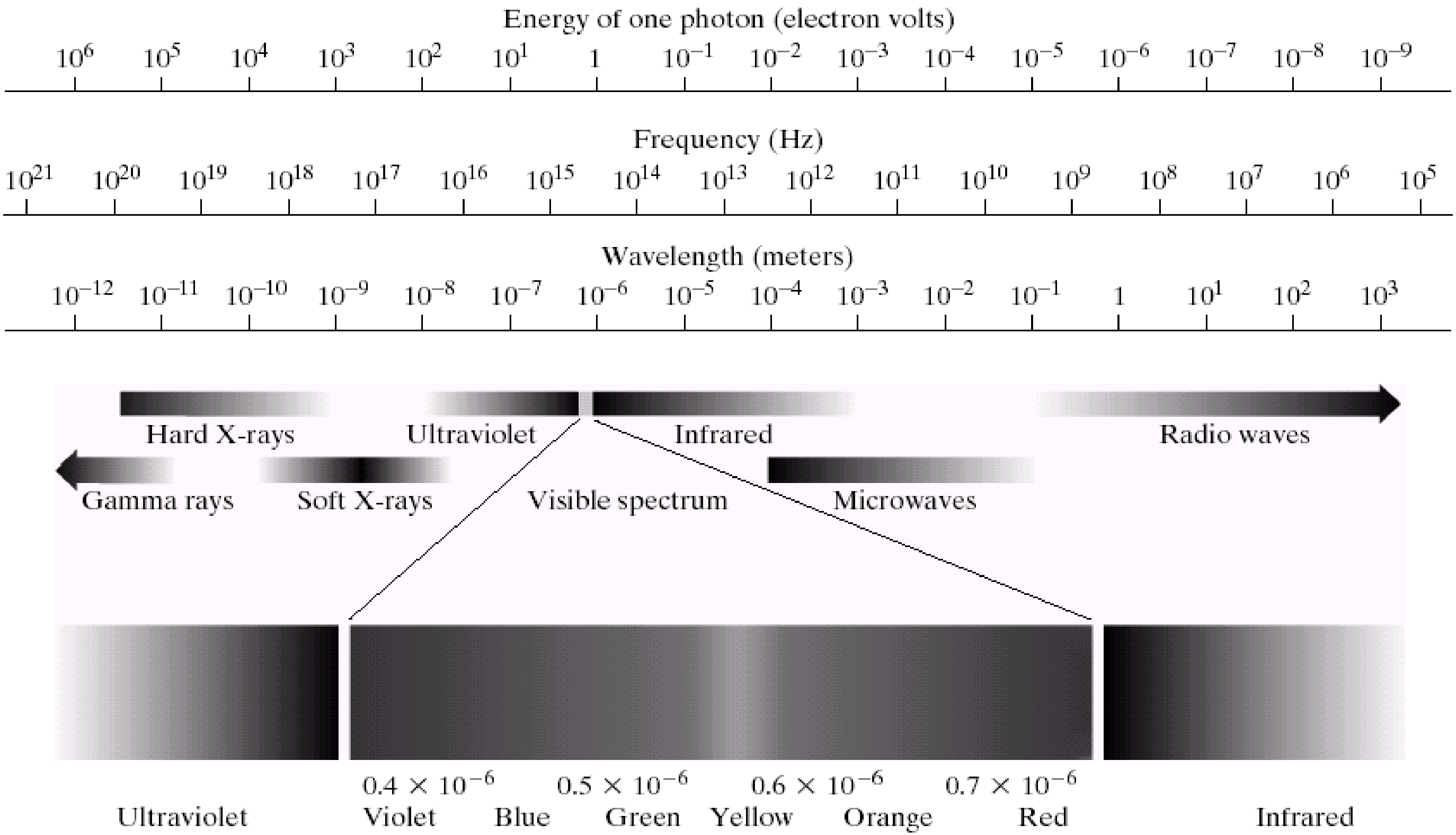


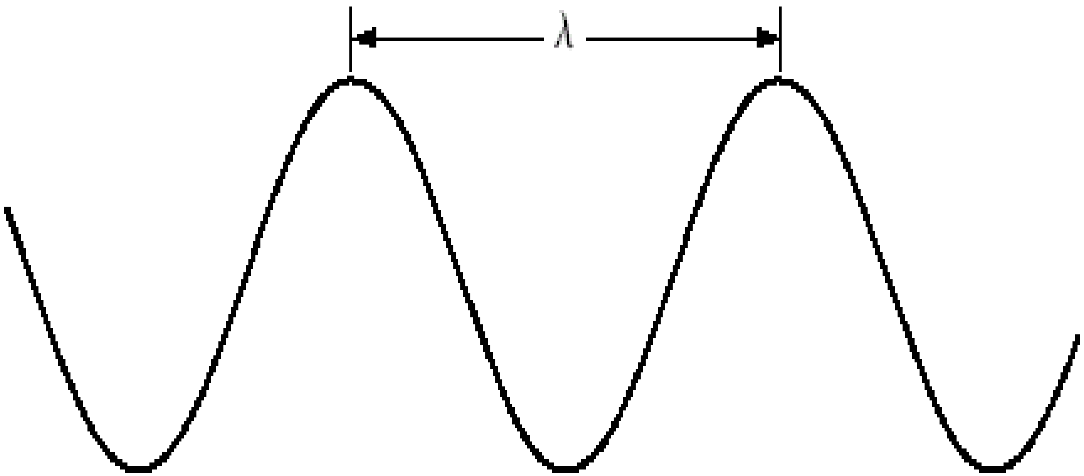
FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

Properties of EM

- Wavelength (λ) and frequency (v)
  - $\lambda = c / v$ (c is the speed of light)
- Electromagnetic Energy and Frequency
  - $E = hv$ (h is Planck's constant)
- Unit:
  - λ: meters; v: Hertz; E: electron-volt

Properties of EM

FIGURE 2.11
Graphical representation of one wavelength.



- **Wave:**
- **A stream of massless particles**
- **Each massless particle contains a certain amount of energy.**
- **Each bundle of energy is called a *photon***

Light partition

- A particular type of EM radiation that can be seen and sensed by the human eye.
  - Range from approximately 0.43µm (violet) to 0.79 µm (red)
  - Violet, blue, green, yellow, orange, red
  - Blend smoothly

Light reflectance properties

- A body that reflects light and is relatively balanced in all visible wavelengths
  - appears white to the observer.
- A body that favors reflectance in a limited range of the visible spectrum
  - exhibits some shades of color.
- Achromatic or monochromatic light:
  - the only attribute is intensity--Gray-level
  - Black to Gray to White

Chromatic light

- Radiance (Watts: W)
  - The total amount of energy that flows from the light source.
- Luminance (lumen: lm)
  - A measure of the amount of energy an observer perceives from a light source.
  - Example: Far Infrared Region
- Brightness
  - Subjective descriptor of light perception that is practically impossible to measure.

Other EM Spectrum: Short-wavelength End

- Gamma rays
  - Medical Imaging
  - Astronomical Imaging
  - Radiation in nuclear environments
- Hard Rays
  - Industrial Applications
- Soft Rays
  - Chest X-Ray (shorter wavelength)
  - Dental X-Ray (lower energy end)
- Ultraviolet

Other EM Spectrum: Long-wavelength End

- Infrared region:
  - Heat-Signatures
  - Near-infrared
  - Far-Infrared
- Microwave
- Radio wave
  - AM and FM and TV
  - Stellar bodies

Note

- The wavelength of an EM wave required to 'see' an object must be of the same size as or smaller than the object.
- Energy radiated by EM waves is not the only method for image generation. (sound, electron beams)

Image Sensing and Acquisition

- "Illumination"
  - EM waves
  - Less traditional sources, like ultrasound
- "Scene"
  - Familiar objects
  - Image a source, like acquiring images of the sun
- Reflectance and Transmission

Three principal Sensors

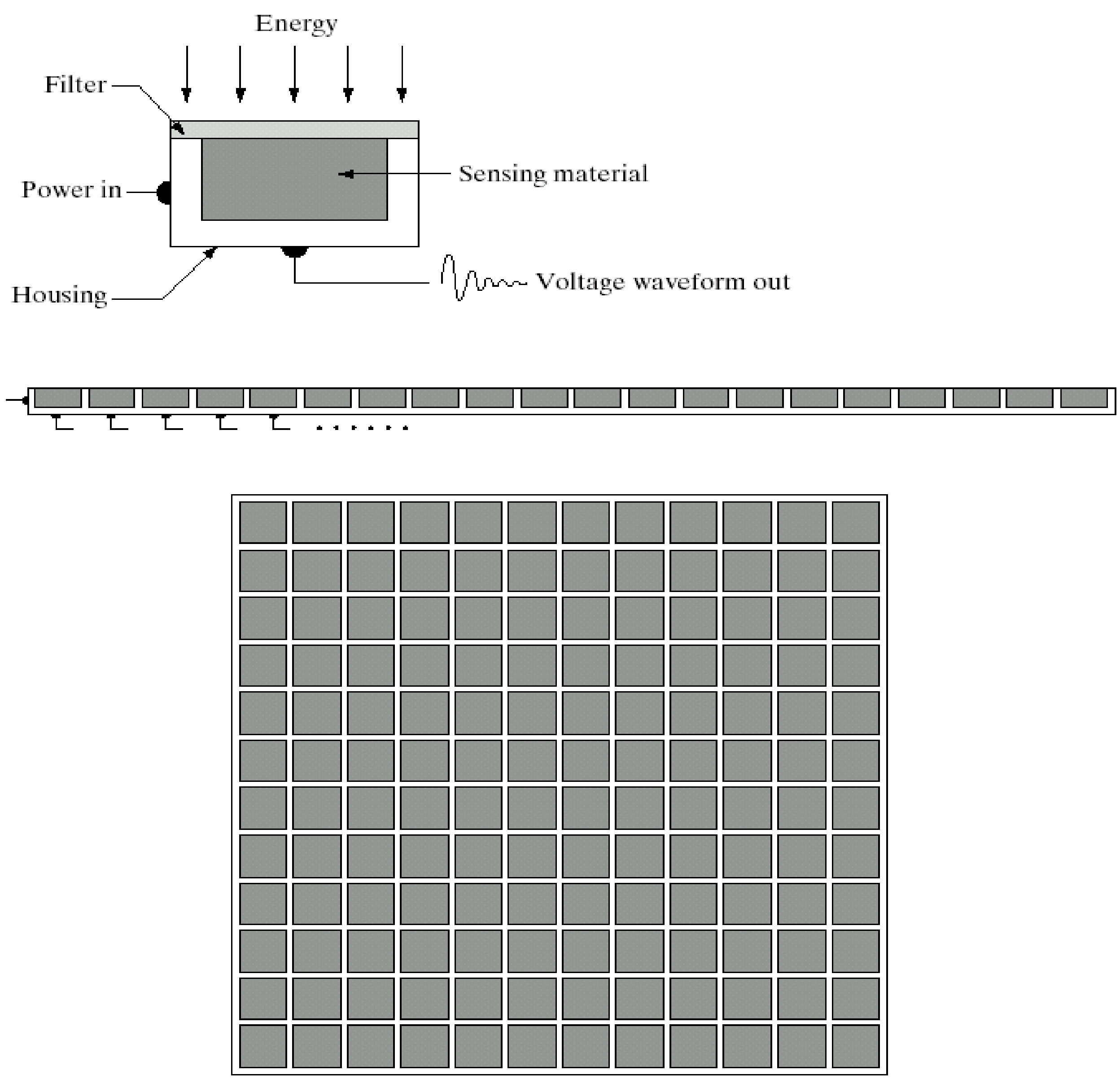


FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image acquisition using a single sensor: Microdensitomers

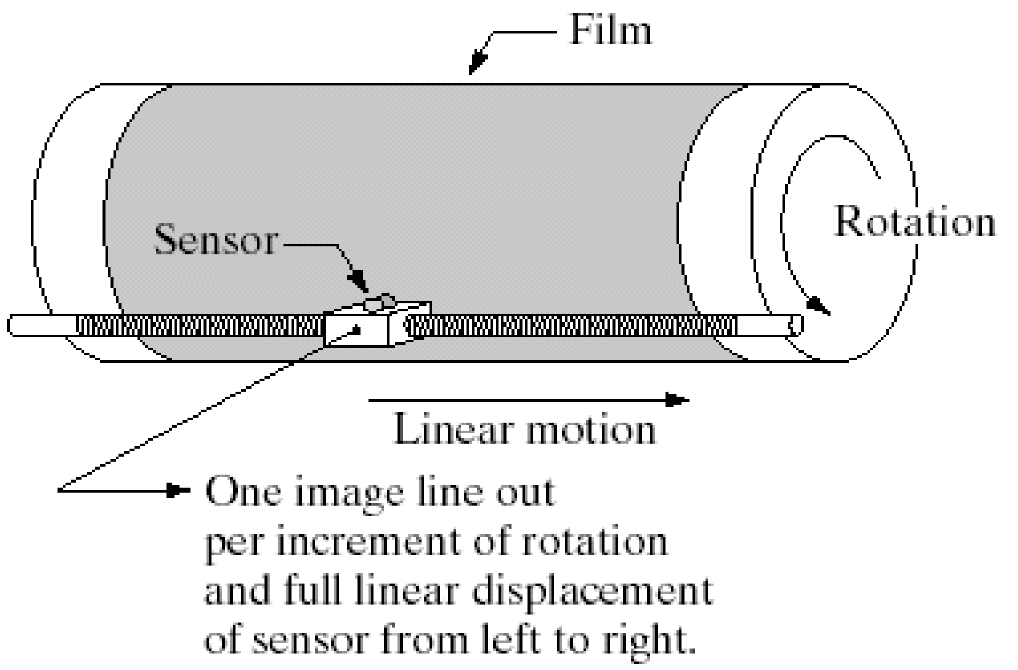
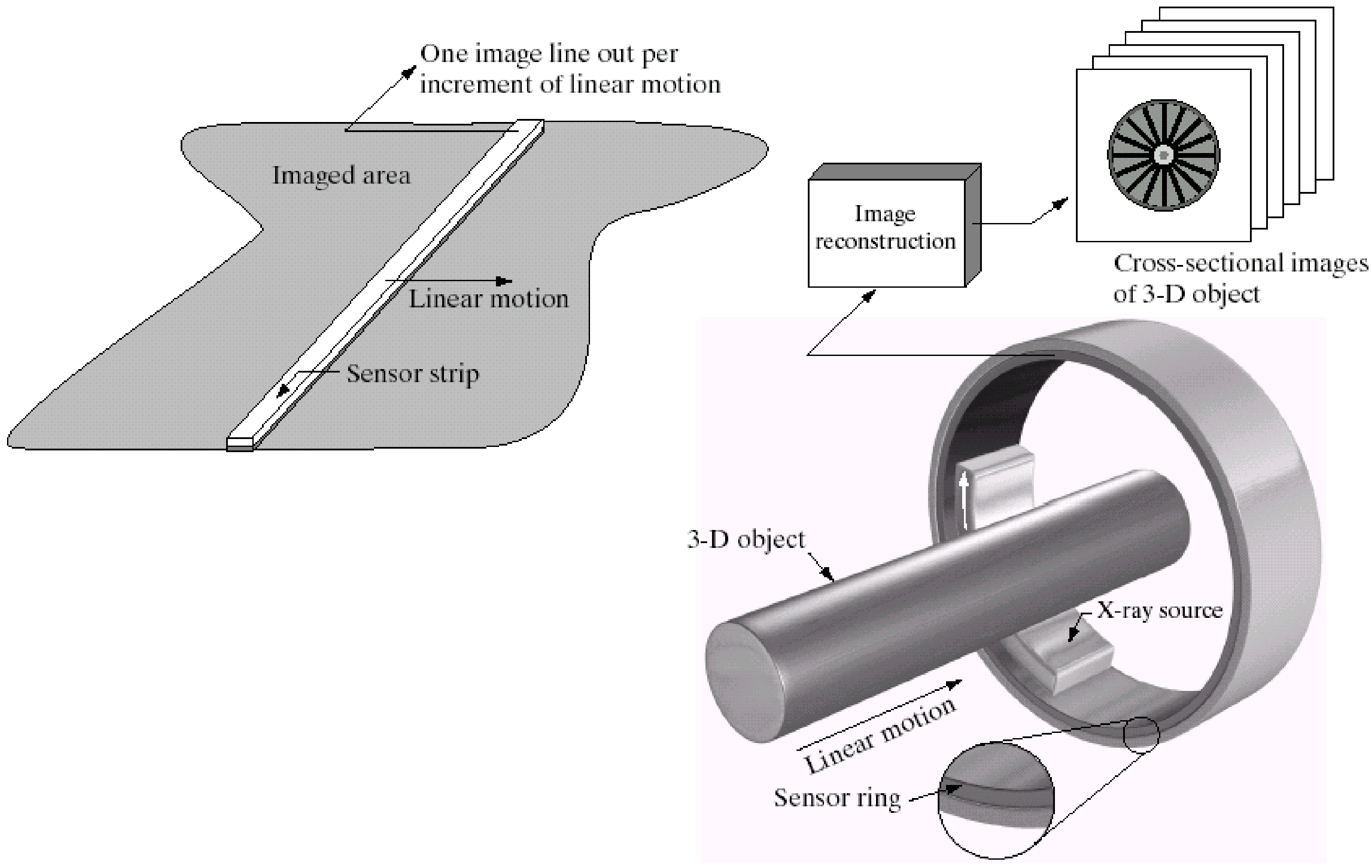


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

Image acquisition using sensor strips



a b

FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image acquisition using sensor array

- CCD (charge-coupled devices)
  - Digital Cameras
  - Light Sensing Instruments
- The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor.
  - Reduce low noise image.
- A complete image can be obtained by focusing the energy pattern onto the surface of the array.

Image acquisition using sensor array

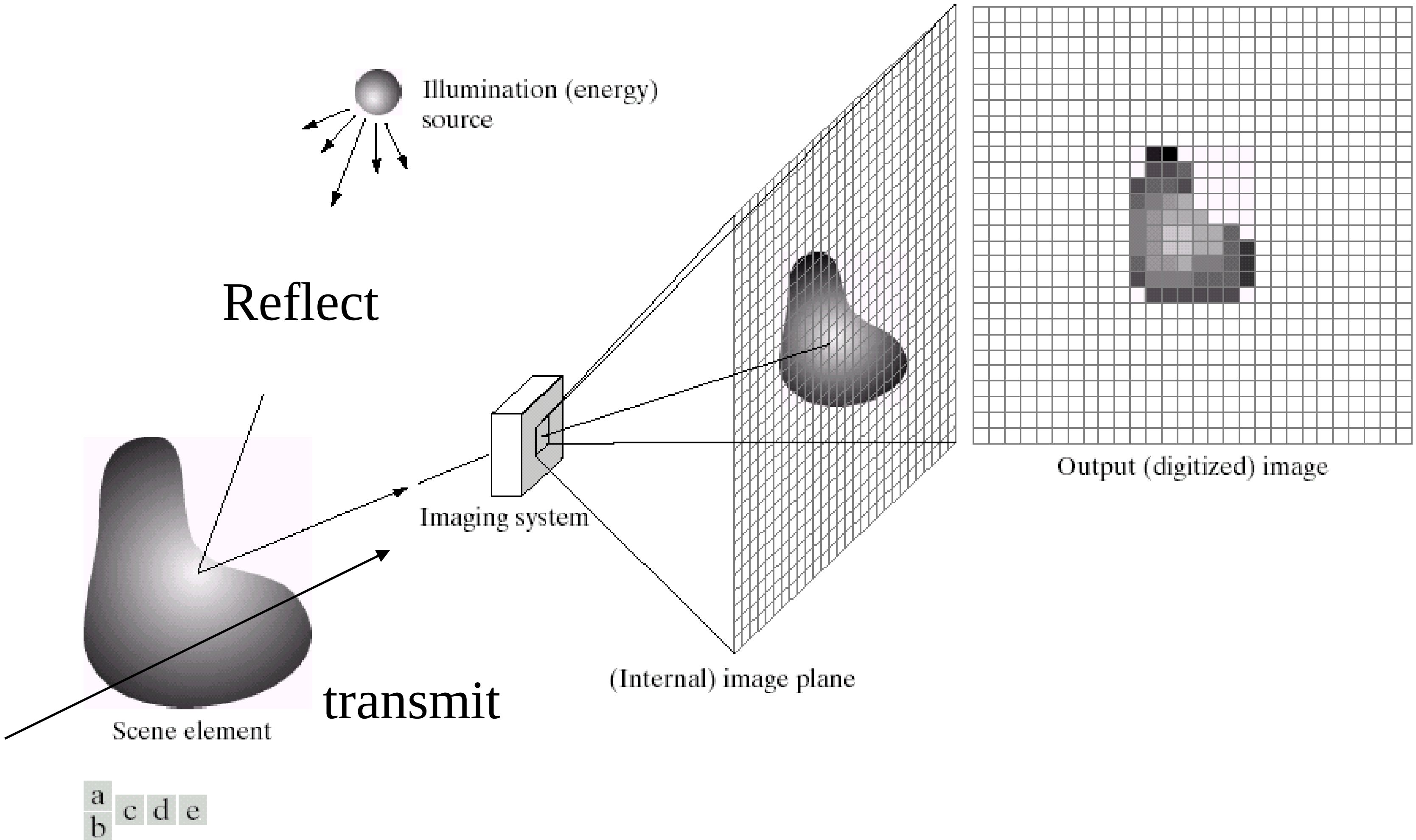


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Model

- Image: a 2-D light-intensity function $f(x,y)$
- The value of f at (x,y) → the intensity (brightness) of the image at that point
- $0 < f(x,y) < \infty$

A Simple Image Model

- Nature of f(x,y):
  - The amount of source light incident on the scene being viewed
  - The amount of light reflected by the objects in the scene

A Simple Image Model

- Illumination & reflectance components:
  - Illumination: $i(x,y)$
  - Reflectance: $r(x,y)$
  - $f(x,y) = i(x,y) \cdot r(x,y)$
  - $0 < i(x,y) < \infty$

    and $0 < r(x,y) < 1$

    (from total absorption to total reflectance)

A Simple Image Model

- Sample values of r(x,y):
  - 0.01: black velvet
  - 0.93: snow

- Sample values of i(x,y):
  - 9000 foot-candles: sunny day
  - 1000 foot-candles: cloudy day
  - 0.01 foot-candles: full moon

A Simple Image Model

- Intensity of a monochrome image f at (x_o,y_o): gray level l of the image at that point

$$l=f(x_o, y_o)$$

- $L_{min} \le l \le L_{max}$
  - Where L_{min}: positive
    L_{max}: finite

A Simple Image Model

- In practice:
  - $L_{min} = i_{min}\ r_{min}$ and
  - $L_{max} = i_{max}\ r_{max}$
- E.g. for indoor image processing:
  - $L_{min} \approx 10$ $\quad L_{max} \approx 1000$
- $[L_{min}, L_{max}]$: gray scale
  - Often shifted to $[0,L-1]$ → $l=0$: black
    $l=L-1$: white

Sampling & Quantization

- The spatial and amplitude digitization of f(x,y) is called:
  - image sampling when it refers to spatial coordinates (x,y) and
  - gray-level quantization when it refers to the amplitude.

Digital Image

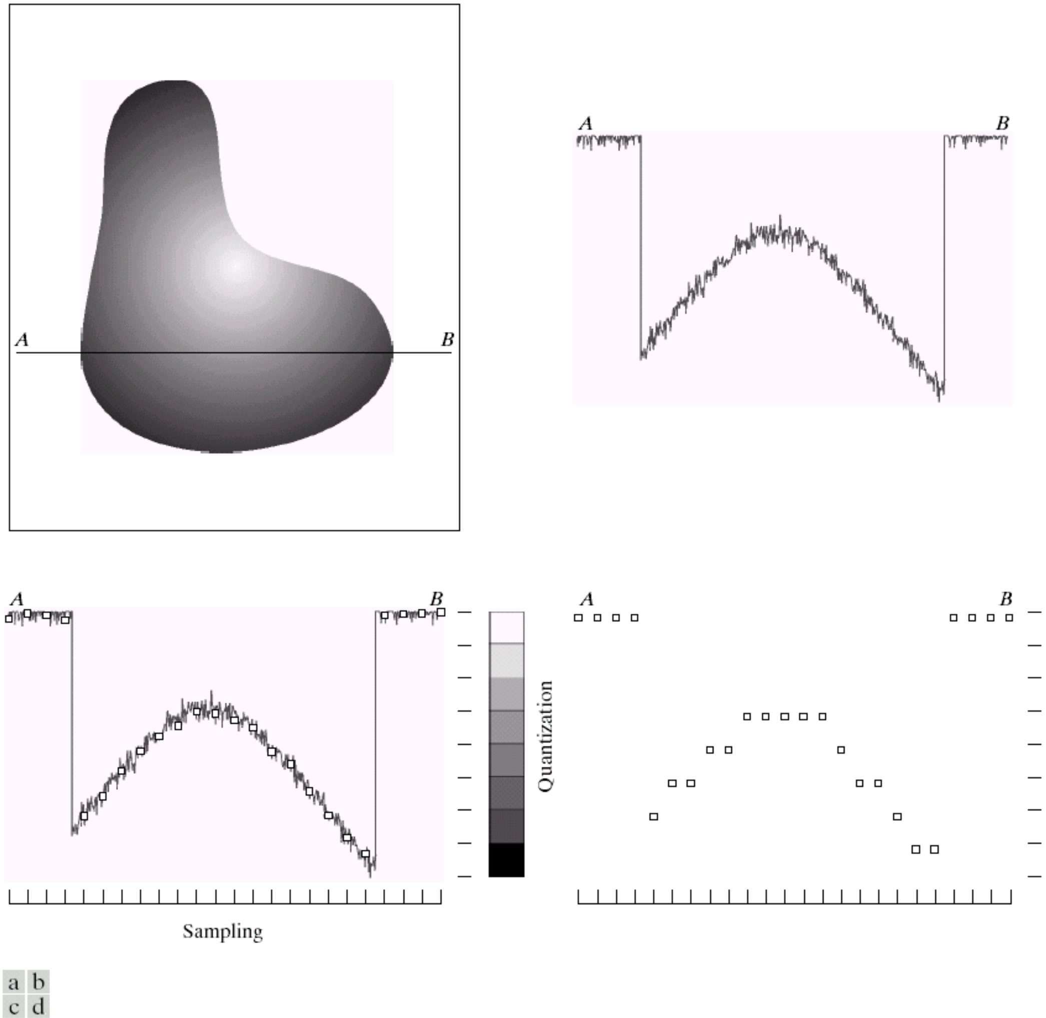


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sampling and Quantization

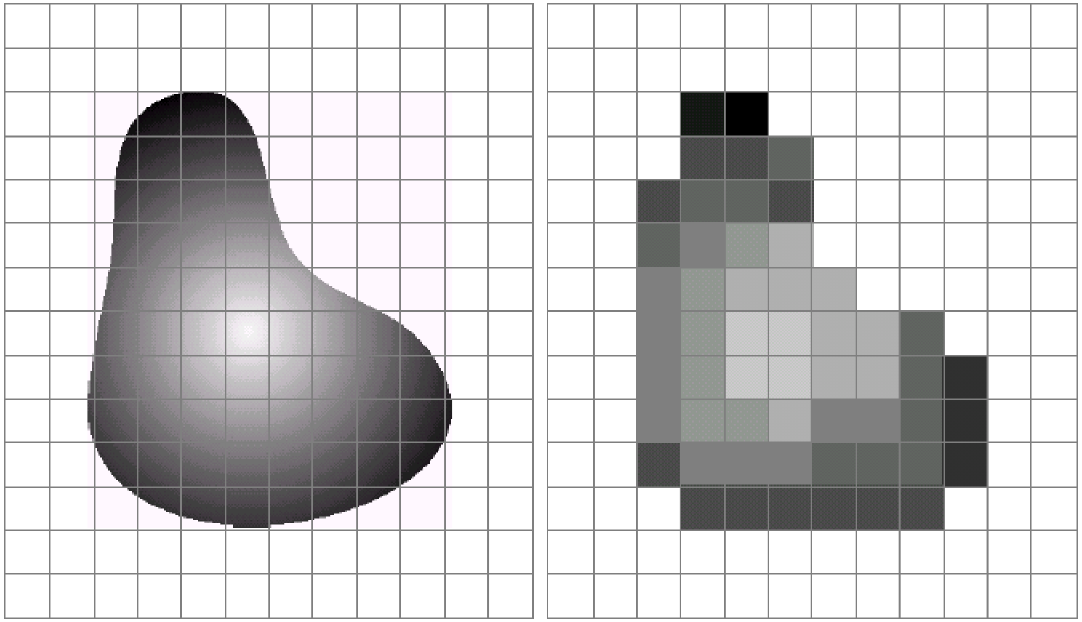


FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

A Digital Image

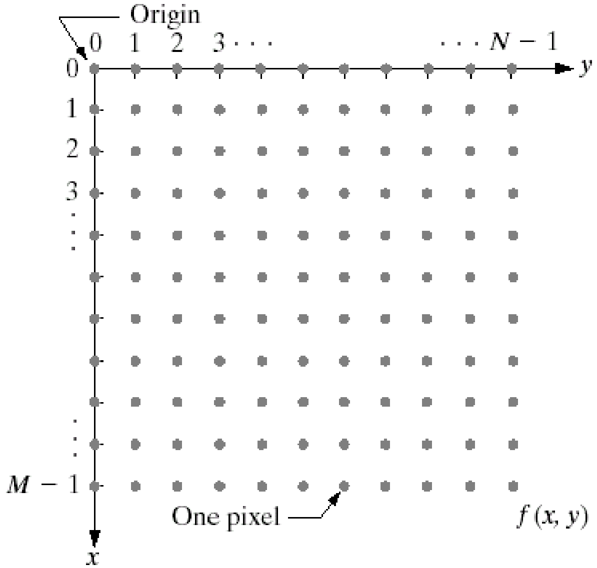


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

Sampling & Quantization

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,M-1) \\ f(1,0) & \dots & \dots & f(1,M-1) \\ \dots & \dots & \dots & \dots \\ f(N-1,0) & f(N-1,1) & \dots & f(N-1,M-1) \end{bmatrix}$$

Digital Image

Image Elements
(Pixels)

Sampling & Quantization

- Important terms for future discussion:
  - Z: set of real integers
  - R: set of real numbers

Sampling & Quantization

- Sampling: partitioning xy plane into a grid
  - the coordinate of the center of each grid is a pair of elements from the Cartesian product Z x Z (Z^2)
- Z^2 is the set of all ordered pairs of elements (a,b) with a and b being integers from Z.

Sampling & Quantization

- f(x,y) is a digital image if:
  - (x,y) are integers from Z^2 and
  - f is a function that assigns a gray-level value (from R) to each distinct pair of coordinates (x,y) [quantization]
- Gray levels are usually integers
  - then Z replaces R

Sampling & Quantization

- The digitization process requires decisions about:
  - values for N,M (where N x M: the image array)

    and

  - the number of discrete gray levels allowed for each pixel.

Sampling & Quantization

- Usually, in DIP these quantities are integer powers of two:

$N=2^n$ $M=2^m$ and $G=2^k$

number of gray levels

- Another assumption is that the discrete levels are equally spaced between 0 and L-1 in the gray scale.

Examples

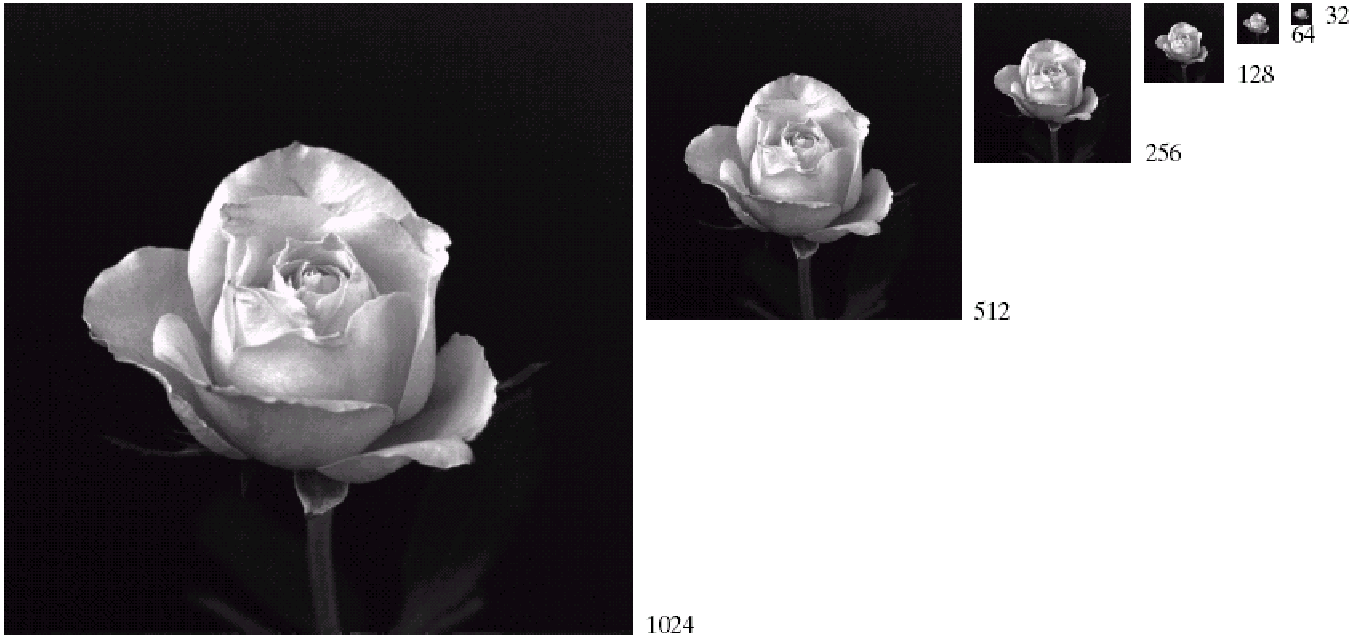


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

Examples

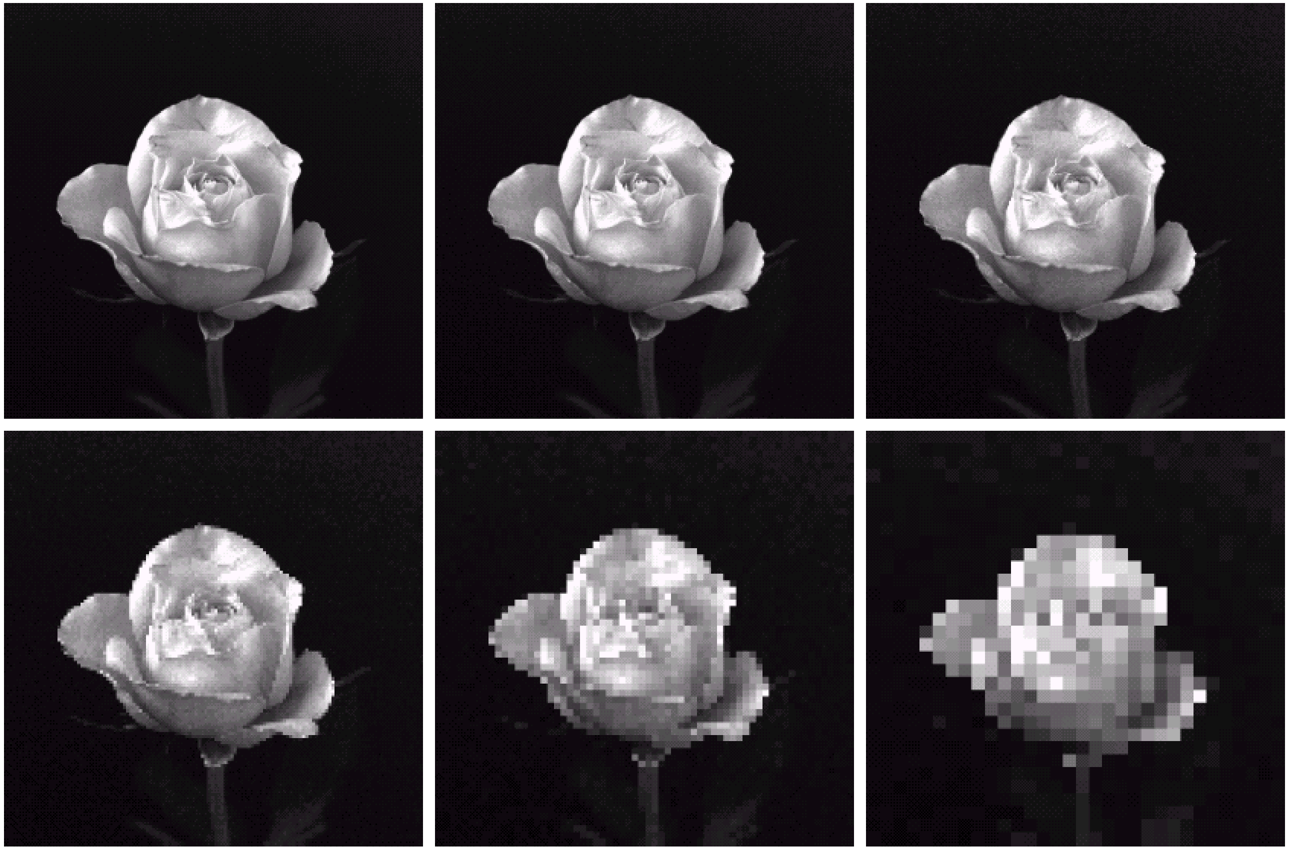
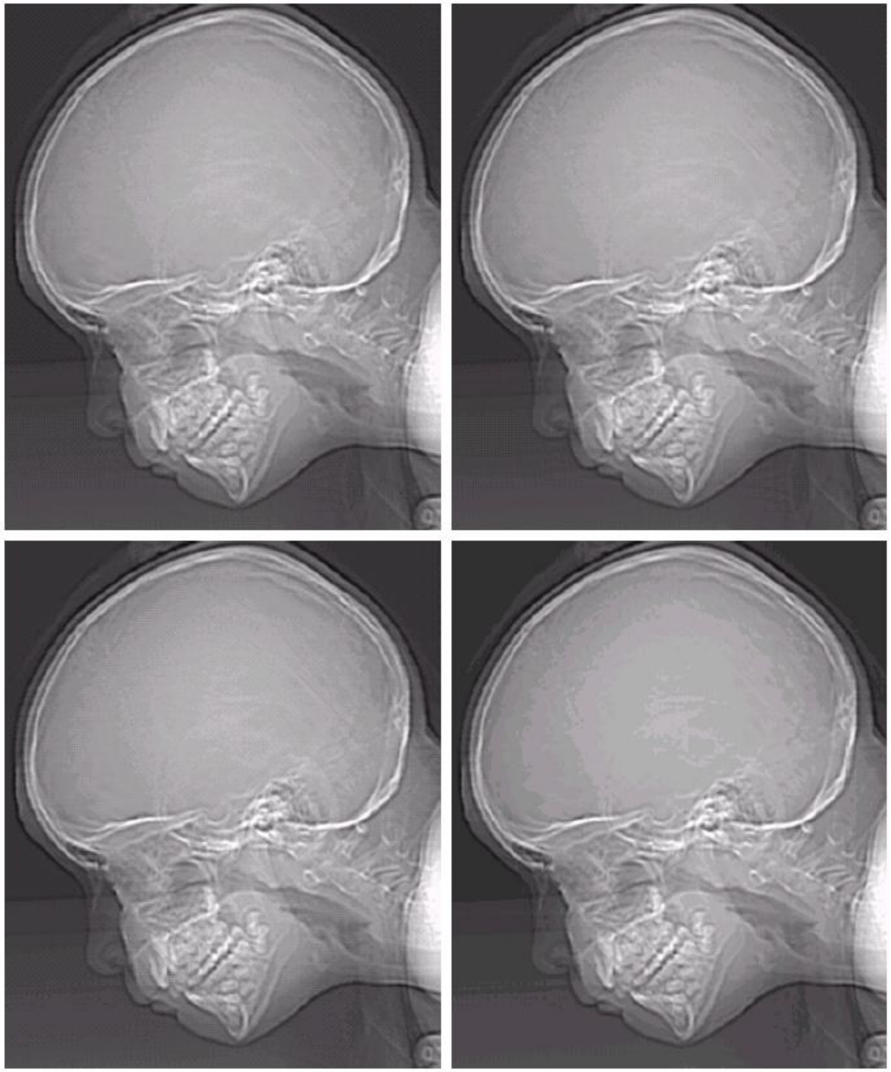


FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

Examples



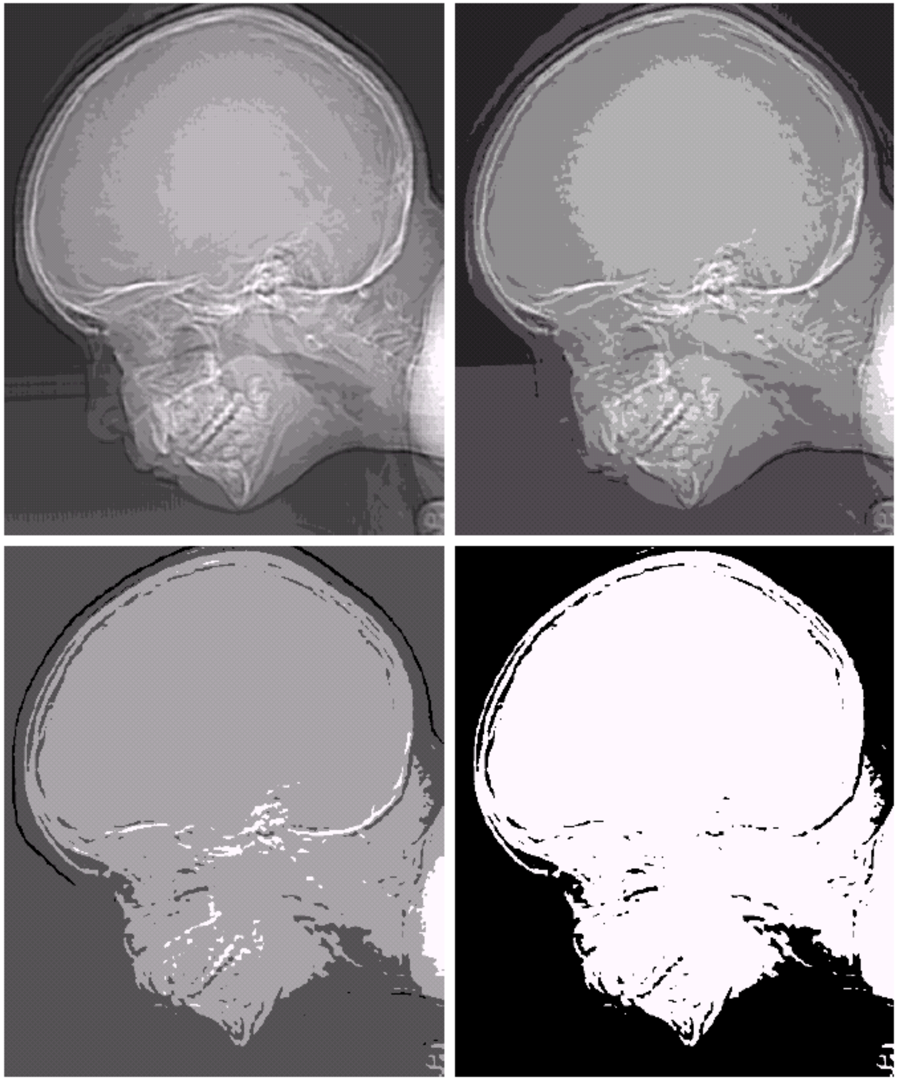
a	b
c	d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Examples

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Sampling & Quantization

- If b is the number of bits required to store a digitized image then:
  - $b = N \times M \times k$ (if $M=N$, then $b=N^2k$)

Storage

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Sampling & Quantization

- How many samples and gray levels are required for a good approximation?
  - Resolution (the degree of discernible detail) of an image depends on sample number and gray level number.
  - i.e. the more these parameters are increased, the closer the digitized array approximates the original image.

Sampling & Quantization

- How many samples and gray levels are required for a good approximation? (cont.)
  - But: storage & processing requirements increase rapidly as a function of N, M, and k

Sampling & Quantization

- Different versions (images) of the same object can be generated through:
  - Varying N, M numbers
  - Varying k (number of bits)
  - Varying both

Sampling & Quantization

- Isopreference curves (in the Nm plane)
  - Each point: image having values of N and k equal to the coordinates of this point
  - Points lying on an isopreference curve correspond to images of equal subjective quality.

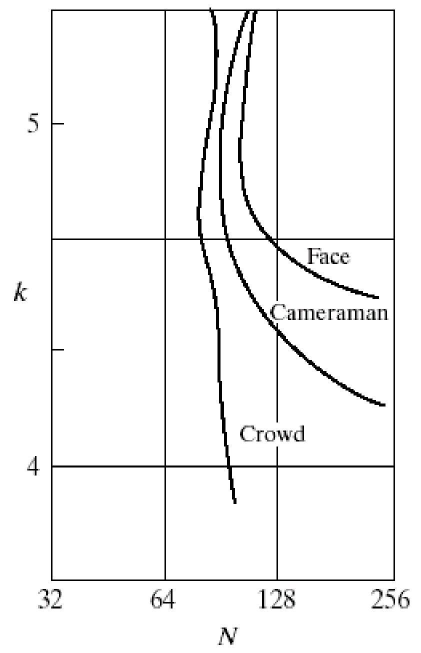
Examples

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Isopreference Curves

FIGURE 2.23 Representative isopreference curves for the three types of images in Fig. 2.22.



Sampling & Quantization

- Conclusions:
  - Quality of images increases as N & k increase
  - Sometimes, for fixed N, the quality improved by decreasing k (increased contrast)
  - For images with large amounts of detail, few gray levels are needed

Nonuniform Sampling & Quantization

- An adaptive sampling scheme can improve the appearance of an image, where the sampling would consider the characteristics of the image.
  - i.e. fine sampling in the neighborhood of sharp gray-level transitions (e.g. boundaries)
  - Coarse sampling in relatively smooth regions
- Considerations: boundary detection, detail content

Nonuniform Sampling & Quantization

- Similarly: nonuniform quantization process
- In this case:
  - few gray levels in the neighborhood of boundaries
  - more in regions of smooth gray-level variations (reducing thus false contours)

Some Basic Relationships Between Pixels

- Definitions:
  - $f(x,y)$: digital image
  - Pixels: q, p
  - Subset of pixels of $f(x,y)$: S

Neighbors of a Pixel

- A pixel p at (x,y) has 2 horizontal and 2 vertical neighbors:
  - (x+1,y), (x-1,y), (x,y+1), (x,y-1)
  - This set of pixels is called the 4-neighbors of p: $N_4(p)$

Neighbors of a Pixel

- The 4 diagonal neighbors of p are: ($N_D(p)$)
  - (x+1,y+1), (x+1,y-1), (x-1,y+1), (x-1,y-1)

- $N_4(p) + N_D(p) \rightarrow N_8(p)$: the 8-neighbors of p

Connectivity

- Connectivity between pixels is important:
  - Because it is used in establishing boundaries of objects and components of regions in an image

Connectivity

- Two pixels are connected if:
  - They are neighbors (i.e. adjacent in some sense -- e.g. $N_4(p)$, $N_8(p)$, …)
  - Their gray levels satisfy a specified criterion of similarity (e.g. equality, …)
- V is the set of gray-level values used to define adjacency (e.g. V={1} for adjacency of pixels of value 1)

Adjacency

- We consider three types of adjacency:
  - 4-adjacency: two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$
  - 8-adjacency : p & q are 8- adjacent if q is in the set $N_8(p)$

Adjacency

- The third type of adjacency:
  - m-adjacency: p & q with values from V are m-adjacent if
    - q is in $N_4(p)$ or
    - q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels with values from V

Adjacency

- Mixed adjacency is a modification of 8-adjacency and is used to eliminate the multiple path connections that often arise when 8-adjacency is used.

$$
\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}
\qquad\qquad
\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}
\qquad\qquad
\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}
$$

Adjacency

- Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.

Path

- A path (curve) from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels:
  - (x_0,y_0), (x_1,y_1), …, (x_n,y_n)
  - where $(x_0,y_0)=(x,y)$, $(x_n,y_n)=(s,t)$, and (x_i,y_i) is adjacent to (x_{i-1},y_{i-1}), for $1 \le i \le n$; **n is the length of the path.**
- If $(x_o, y_o) = (x_n, y_n)$: a closed path

Paths

- 4-, 8-, m-paths can be defined depending on the type of adjacency specified.
- If p,q $\in$ S, then q is connected to p in S if there is a path from p to q consisting entirely of pixels in S.

Connectivity

- For any pixel p in S, the set of pixels in S that are connected to p is a connected component of S.

- If S has only one connected component then S is called a connected set.

Boundary

- R a subset of pixels: R is a region if R is a connected set.
- Its boundary (border, contour) is the set of pixels in R that have at least one neighbor not in R
- Edge can be the region boundary (in binary images)

Distance Measures

- For pixels p,q,z with coordinates (x,y), (s,t), (u,v), D is a distance function or metric if:
  - $D(p,q) \geq 0$ $\quad$ ($D(p,q)=0$ iff $p=q$)
  - $D(p,q) = D(q,p)$ and
  - $D(p,z) \leq D(p,q) + D(q,z)$

Distance Measures

- Euclidean distance:
  - $D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$
  - Points (pixels) having a distance less than or equal to r from (x,y) are contained in a disk of radius r centered at (x,y).

Distance Measures

- D_4 distance (city-block distance):
  - $D_4(p,q) = |x-s| + |y-t|$
  - forms a diamond centered at (x,y)
  - e.g. pixels with $D_4 \leq 2$ from p

$$
\begin{array}{ccccc}
 & & 2 & & \\
 & 2 & 1 & 2 & \\
2 & 1 & 0 & 1 & 2 \\
 & 2 & 1 & 2 & \\
 & & 2 & &
\end{array}
$$

$D_4 = 1$ are the 4-neighbors of p

Distance Measures

- D_8 distance (chessboard distance):
  - $D_8(p,q) = \max(|x-s|,|y-t|)$
  - Forms a square centered at p
  - e.g. pixels with $D_8 \leq 2$ from p

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

$D_8 = 1$ are the 8-neighbors of p

Distance Measures

- D_4 and D_8 distances between p and q are independent of any paths that exist between the points because these distances involve only the coordinates of the points (regardless of whether a connected path exists between them).

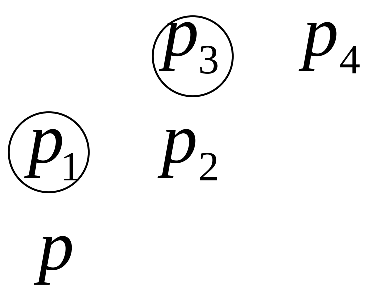
Distance Measures

- However, for m-connectivity the value of the distance (length of path) between two pixels depends on the values of the pixels along the path and those of their neighbors.

Distance Measures

- e.g. assume $p, p_2, p_4 = 1$

  p_1, p_3 = can have either 0 or 1



If only connectivity of pixels valued 1 is allowed, and p_1 and p_3 are 0, the m-distance between p and p_4 is 2.

If either p_1 or p_3 is 1, the distance is 3.

If both p_1 and p_3 are 1, the distance is 4 ($pp_1p_2p_3p_4$)