Applications of the current state space model in analyses of saturated induction machines

E. Levi

School of Electrical and Electronic Engineering, Liverpool John Moores University, Byrom Street, Liverpool L3 3AF, UK

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Abstract

Main flux saturation in induction machines plays an important role in a number of operating regimes and therefore has to be accounted for in the process of modelling. The current state space model of a saturated induction machine has gained substantial popularity recently as an excellent tool for transient and steady-state time domain analysis of induction motor drives when the main flux saturation has to be considered. This paper discusses different applications of the model. First, the self-excitation process in single-cage and double-cage (deep-bar) induction generators is addressed. Switch-off and reclosing transients of an induction motor with capacitor reactive power compensation are dealt with next. Two inverter fed induction motor drives, namely, the current-source and resonant parallel inverter fed induction machines, are elaborated further. The last application described here is the rotor flux oriented control of an induction machine. The elaborated cases are illustrated with simulation and experimental results.

Keywords: Induction machines; Saturation modelling

1. Introduction

Classical general theory of electric machines assumes that the non-linear magnetizing curve of a machine can be approximated by a straight line through the origin. Thus, the magnetizing (main, air-gap) flux is supposed to vary linearly with the magnetizing current. This assumption turns out to be completely unsatisfactory for modelling and simulation of a number of steady-state and transient regimes of both induction and synchronous machines. From a historical point of view, the need to incorporate somehow the real shape of the magnetizing curve within the frame of the general theory first occurred in studies of salient-pole synchronous generators [1]. With regard to induction machines fed from the mains, the only operating regimes where a need for saturation modelling has been experienced in the past are switch-off and reclosing transients of an induction motor with reactive power compensation [2]. The influence of main flux saturation on other transients of induction machines fed from the mains supply is in general small, the exception being the reclosing transient of an uncompensated machine with a large phase difference between the mains voltage and the induced electromotive force [3]. For example, a detailed study of a start-up process [4] shows that the main flux saturation effect can be fully neglected. However, developments in converter fed induction motor drives and in applications of induction machines as generators in autonomous power systems have led to a significant increase in interest in the possibilities of including the main flux saturation effect in models of induction machines. The reason for such a trend is rather simple: these new technologies have introduced new applications of induction machines where the omission of the main flux saturation effect from the model leads to either inaccurate results, or, even worse, makes simulation and performance prediction impossible.

All the existing methods of saturation modelling utilize the assumption that the stator and rotor flux components along the d and q axes can be resolved into sums of leakage flux and main flux components. Saturation of leakage and main flux paths is treated independently further on. Main flux saturation is far more important than leakage flux saturation. Saturation of leakage paths is usually of interest in analyses associated with induction motors fed from the mains, where during start-up and reversing transients large currents lead to significant saturation of the leakage
paths. From the modelling point of view, leakage flux saturation is a simple task and the models that account for it are readily available and simple to use [5,6].

In general, two common approaches to main flux saturation modelling can be identified. The first utilizes d,q-axis flux components as state space variables, while the other relies on currents as state space variables. The latter approach is the subject of this paper. Although the flux state space model is slightly simpler to use, its main drawback is that it conceals the effect of cross-saturation which is, however, implicitly accounted for. On the other hand, the current state space model contains explicit terms that describe the cross-saturation effect and therefore a better physical insight into the overall behaviour of saturated induction machines is achieved. Still, it should be noted that the corresponding flux state space model of a saturated induction machine may be utilized instead, without any loss of accuracy in the simulation results.

The cross-saturation effect is the usual way of denoting the mutual influence which occurs between two perpendicular axes, even in a uniform-air-gap machine under saturated conditions, and is purely due to main flux saturation. More specifically, a change of saturation level in one axis causes a corresponding change of flux level in the other, perpendicular, axis, and this is usually named cross-saturation or cross-magnetization. The cross-saturation effect was recognized a long time ago [1], but it was not until recently that it became the subject of wider interest. Cross-saturation between two perpendicular axes in smooth-air-gap machines has been studied in detail in a number of papers [7–16]. The existence of this effect has been verified in different ways, applying a variety of theoretical and experimental approaches. The approach in Ref. [9] relies on magnetic co-energy, while that in Ref. [10] uses different starting premises but yields the same results. The finite-element method enabled a detailed study of the electromagnetic fields in the machine and proved the existence of cross-saturation as well [13]. Sophisticated experiments, specially designed in order to achieve an experimental proof of the cross-saturation effect, are analyzed in Ref. [8,12,15]. In simulation examples [17,18] which refer to an induction machine under saturated conditions, the influence of cross-saturation is separated from other effects and shown explicitly.

The idea of modelling a saturated induction machine with currents as state space variables stems from the work of Kovács [19–21], where a special common reference frame fixed to the magnetizing current (or main flux) space vector is selected. This current state space model is derived by means of space vector theory and is a special case of a more general model of a saturated induction machine in an arbitrary common reference frame [22]. An important feature of the current state space model of a saturated induction machine in a common frame fixed to the magnetizing current space vector is the absence of the terms which describe cross-saturation, this being a consequence of the specific choice of the common reference frame. The above-mentioned general current state space model in an arbitrary frame of reference [22,23] contains explicit terms which account for cross-saturation. The origin of the more general model can be found in an earlier paper by Kovács [24]. Unfortunately, due to an incorrect mathematical derivation, the resultant model of a saturated induction machine given in Ref. [24] is not correct. The original intention to develop a saturated induction machine model in Ref. [24] came from the need to model and simulate the behaviour of a self-excited induction generator. The behaviour of an induction generator which operates in an autonomous power system and is equipped with capacitors which provide reactive power has been brought into the scope of research due to the development of small autonomous electric energy plants. The self-excitation process of such an induction generator cannot be modelled and simulated unless the main flux saturation is encompassed by the model. The standard linear models of induction machines simply fail in this case.

The generalized analysis of main flux saturation effects in orthogonal-axis models of electric machines [25–28] presents as a final result a general model of saturated electric machines, with currents as state space variables, which accommodates induction, synchronous and DC machines. Therefore, it easily reduces to the saturated induction machine model of Refs. [22,23]. Consequently, the current state space model of a saturated induction machine can be regarded as just a special case of the general current state space model of a saturated electric machine.

The current state space model of saturated induction machines for small-signal analysis [29] is just a special case of the corresponding large-signal model discussed so far. This model is commonly applied to determine the stability limits of induction machines under different operating conditions [30–34].

The large-signal current state space model of saturated induction machines has recently been developed for double-cage [18,35] and deep-bar [36] induction machines as well. In these cases the principle of main flux saturation modelling remains the same as for single-cage machines but the overall complexity of the model increases significantly due to addition of new differential equations.

The influence of main flux saturation on the behaviour of an induction machine fed from the mains is, as has already been stated, very small and can
usually be neglected. A similar conclusion holds true if the induction machine is fed from a voltage-source inverter, provided that the voltage to frequency ratio is held constant and that the magnetizing (mutual) inductance value in the linear model corresponds to the steady-state saturated value of the magnetizing inductance at the operating point of the magnetizing curve. If the machine is fed from a current source, the main flux saturation effect tends to be much more pronounced [18] and influences the drive behaviour more significantly.

Summarizing the cases where main flux saturation has to be accounted for, it may be stated that the need for a saturated induction machine model arises in two distinct types of application. The first type encompasses all the situations where the induction machine is not fed from a power electronic converter, there is a capacitance or three-phase bank of capacitors connected to the machine terminals and mains voltages are usually absent for one reason or another. More specifically, self-excitation of a single-cage [18,37–39] or double-cage [40,41] induction generator with terminal capacitors, operation of a compensated induction generator in an autonomous power system [18,38], switch-off and reclosing transients of a compensated induction motor [2,42], and three-phase symmetrical or single-phase asymmetrical capacitor braking of induction motors [18,35] all belong to this type of application. One specific case where mains voltages are present and there is still a need for inclusion of main flux saturation in the model is subsynchronous resonance instability in an induction motor supplied through a capacitor compensated feeder [18]. Here the inclusion of the saturation effect is important only if the unstable operation does take place. The second type includes all the converter fed induction motor variable-speed drives where the source may be treated as a current source. The current-source inverter fed induction machine [43,44] and the resonant parallel inverter fed induction machine [45–47] are two typical cases with simple control algorithms, while a field oriented induction machine fed from a current controlled pulse-width modulator (PWM) inverter represents a further example [48–51], this time with a rather sophisticated control structure. It is worth noting here that in the latter case the current state space model of a saturated induction machine is most easily achieved by application of the flux state space model of a saturated induction machine [52,53].

Most of the applications listed above are elaborated in this paper. More specifically, self-excitation in single-cage and double-cage induction generators, switch-off and reclosing transients of a compensated induction motor, induction motor drives fed from current-source inverters and from resonant parallel inverters, and finally the current fed rotor flux oriented induction machine are discussed. The second section presents a brief overview of the current state space model of a saturated induction machine. The third section deals with self-excitation in induction generators and transients associated with the compensated induction motor. The fourth section is devoted to variable-speed drives and encompasses the three previously mentioned cases. Emphasis in the paper is placed on simulation and experimental results, rather than on detailed mathematical modelling. A more detailed theoretical treatment is available in the references listed above for all the applications discussed.

2. Review of a saturated induction machine model with currents as state space variables

It is assumed that the machine has $P$ pairs of poles, mechanical losses are neglected, power invariant transformation is applied and the model is given in an arbitrary frame of reference whose angular velocity is $\omega_a$, in matrix form as

$$v = L \frac{di}{dt} + Bi \quad (1)$$

$$T_e - T_l = (J/P) \frac{d\omega_a}{dt} \quad (2)$$

Eq. (1) can be expressed in state space form, with currents as state space variables, as

$$\frac{di}{dt} = L^{-1} v - L^{-1} Bi \quad (1a)$$

The matrices and vectors contained in Eq. (1), as well as the expression for the induction torque $T_e$ in Eq. (2), differ depending on whether the machine is single cage or double cage. For this reason these two cases are addressed separately.

2.1. Single-cage induction machine

The matrices and vectors of Eq. (1) and the torque are given for the single-cage machine by the following expressions:

$$v = [v_{ds}, v_{qs}, 0, 0]^T$$

$$i = [i_{ds}, i_{qs}, i_{dr}, i_{qr}]^T \quad (3)$$
Indices \( s \) and \( r \) denote stator and rotor parameters and variables, respectively, index \( \gamma \) stands for leakage inductances, and index \( m \) defines parameters and variables associated with magnetizing flux and magnetizing current. The inductance terms in Eq. (5) are determined as

\[
L_{ddm} = L \cos^2 \mu + L_m \sin^2 \mu \quad (7a) \\
L_{qqm} = L \sin^2 \mu + L_m \cos^2 \mu \quad (7b) \\
L_{dq} = (L - L_m) \cos \mu \sin \mu \quad (7c)
\]

where \( L \) is the dynamic (tangent, slope) inductance defined as

\[
L = \frac{d\psi_m}{di_m} \quad \text{where} \quad L = L(i_m) \quad (8)
\]

\( L_m \) is the steady-state saturated magnetizing inductance

\[
L_m = \frac{\psi_m}{i_m} \quad \text{where} \quad L_m = L_m(i_m) \quad (9)
\]

and \( \mu \) is the angle between the \( d \) axis of the common reference frame and the magnetizing current (flux) space vector whose real and imaginary components are \( i_{dm} \) and \( i_{qm} \) (\( \psi_{dm} \) and \( \psi_{qm} \)), respectively. Due to the non-linearity of the magnetizing curve of the machine, both the steady-state magnetizing inductance and the dynamic inductance are variables and depend on the instantaneous value of the magnetizing current and the corresponding value of the magnetizing flux:

\[
\psi_m = (\psi_{dm}^2 + \psi_{qm}^2)^{1/2}, \quad i_m = (i_{dm}^2 + i_{qm}^2)^{1/2} \quad (10)
\]

The magnetizing current and magnetizing flux \( d, q \)-axis components are

\[
i_{dm} = i_{ds} + i_{dr}, \quad i_{qm} = i_{qs} + i_{qr} \quad (11a) \\
\psi_{dm} = L_{ddm}i_{dm}, \quad \psi_{qm} = L_{qdm}i_{qm} \quad (11b)
\]

The model given in Eqs. (1)–(11) is the current state space model which completely describes the single-cage saturated induction machine.

### 2.2. Double-cage induction machine

The matrices and vectors present in Eq. (1) take the following form for the case of the double-cage induction machine:

\[
v = [\psi_{ds}, \psi_{qs}, 0, 0, 0, 0]^T \\
i = [i_{ds}, i_{qs}, i_{dr1}, i_{qr1}, i_{dr2}, i_{qr2}]^T \quad (12)
\]

\[
B = 
\begin{bmatrix}
R_s & -\omega_a L_s & 0 & -\omega_a L_{rm} & 0 & -\omega_a L_{rm} \\
\omega_a L_s & R_s & \omega_a L_m & 0 & \omega_a L_{rm} & 0 \\
0 & -\omega_a L_{rm} & R_{r1} + R_c & -\omega_a (L_m + L_{r1} + L_{mr}) & R_{r1} + R_c & -\omega_a (L_m + L_{r1} + L_{mr}) \\
\omega_a L_{rm} & 0 & \omega_a (L_m + L_{r1} + L_{mr}) & R_{r1} + R_c & \omega_a (L_m + L_{r1} + L_{mr}) & R_{r1} + R_c \\
0 & -\omega_a L_m & R_c & -\omega_a (L_m + L_{mr}) & R_{r2} + R_c & -\omega_a (L_m + L_{r2} + L_{mr}) \\
\omega_a L_m & 0 & \omega_a (L_m + L_{mr}) & R_c & \omega_a (L_m + L_{r2} + L_{mr}) & R_{r2} + R_c
\end{bmatrix}
\quad (13)
\]

\[
L = 
\begin{bmatrix}
L_{ddm} + L_{qdm} & L_{dq} & L_{ddm} & L_{dq} & L_{ddm} & L_{dq} \\
L_{dq} & L_{ddm} + L_{qdm} & L_{dq} & L_{dq} & L_{dq} & L_{dq} \\
L_{qdm} & L_{dq} & L_{r1} + L_{mr} + L_{ddm} & L_{dq} & L_{r1} + L_{mr} + L_{dq} & L_{dq} \\
L_{ddm} & L_{dq} & L_{dq} & L_{dq} & L_{dq} & L_{dq} \\
L_{qdm} & L_{dq} & L_{dq} & L_{dq} & L_{dq} & L_{dq} \\
L_{dq} & L_{dq} & L_{dq} & L_{dq} & L_{dq} & L_{dq}
\end{bmatrix}
\quad (14)
\]
In Eqs. (12)–(15) index 1 denotes the upper rotor cage, while index 2 stands for the lower rotor cage. The slip angular frequency is introduced in Eq. (13) as \( \omega_s = \omega_a - \omega_t \), and the inductance term \( L_s = L_{s1} + L_m \). Eqs. (7)–(10) remain the same as for the single-cage machine. The magnetizing current components given in Eq. (11a) for the single-cage machine now become

\[
\begin{align*}
    i_{dm} &= i_{ds} + i_{dr1} + i_{dr2}, \\
    i_{qm} &= i_{qs} + i_{qr1} + i_{qr2}
\end{align*}
\]

(16)

The resistance \( R_c \) in Eq. (13) is the common end-ring resistance between the two cages and may or may not be present depending on the construction of the machine. Inductance \( L_{mr} \) denotes the mutual leakage inductance between the two rotor cages.

The data for all the induction machines employed in the simulation and experiments are given in the Appendix. The magnetizing curve of one of the machines (Machine A) is shown here, together with variations of the steady-state magnetizing inductance and dynamic inductance (Fig. 1). By inspecting Fig. 1 one easily concludes that the difference between the steady-state magnetizing inductance and the dynamic inductance is considerable over a major portion of the magnetizing curve; consequently, inductance \( L_{qsh} \), defined in Eq. (7), which describes the cross-saturation phenomenon, will be non-zero as well and every attempt to ignore cross-saturation will lead to incorrect simulation results [17,18].

3. Induction machine with reactive power compensation

Three different cases are discussed in this section, the first and the second being self-excitation in single- and double-cage induction generators, respectively. The third case encompasses switch-off and reclosing transients of an induction motor with a parallel reactive power compensator consisting of a three-phase capacitor bank.

3.1. Self-excitation in single-cage induction generators

It is assumed that the induction generator operates at constant speed and that there is no load connected to the output stator terminals. A three-phase capacitor bank is connected in parallel to the machine. Initiation of self-excitation is based on remanent magnetism and therefore an initial non-zero value of the magnetizing current is needed in the model in order to start self-excitation. This value is easily obtainable by running the machine at constant speed and measuring the remanent voltage. The additional equations that describe the capacitor bank are

\[
\begin{align*}
    \frac{dv_{ds}}{dt} &= \frac{1}{C} (L_s i_{qs} - i_{ds}) \\
    \frac{dv_{qs}}{dt} &= \frac{1}{C} (L_s i_{ds} - i_{qs})
\end{align*}
\]

(17a)

(17b)

As constant-speed operation is analysed here, the equation of mechanical motion (2) may be omitted. Hence, Eqs. (1a), (3)–(11) and (17) constitute the required mathematical model.

The star connection of the capacitor bank and induction generator stator windings was analysed experimentally and by simulation for Machine A with a per-phase capacitance of 25 \( \mu \)F. The value of the capacitance was chosen to provide steady-state operation at the rated point on the magnetizing curve. The speed was held constant at a synchronous value of 50 Hz (1500 rpm). Fig. 2 shows the experimentally recorded phase voltage and current build-up (note that in Fig. 2(a) the steady state has not yet been reached; the steady-state voltage peak value equals 307.2 V). The corresponding simulation results are displayed in Fig. 3. Comparison of the experimental and simulation results shows pretty good agreement.

3.2. Self-excitation in double-cage induction generators

The model needed in this case encompasses Eqs. (1a), (7)–(10), (11b) and (12)–(17). Once more self-excitation at constant speed was analysed under no-load conditions and Machine B was utilized in simulations and experiments. The machine stator windings and capacitor bank were delta connected this time. The capacitor value was chosen in the same way as for the single-cage machine. The experimental and simulation results of the voltage build-up at a speed of 3180 rpm with capacitors of 24.5 \( \mu \)F capacitance are summarized in Fig. 4. Once more the results are in satisfactory agreement, although the voltage build-up is slightly
Fig. 2. Experimentally recorded phase voltage and current build-up during single-cage induction generator self-excitation (current recorded as voltage drop on a resistor of 0.02 $\Omega$ resistance).

3.3. Switch-off and reclosing transients of an induction motor with parallel reactive power compensation

In this example a single-cage induction motor operates on the mains and its reactive power consumption is compensated by a three-phase capacitor bank connected in parallel to the stator terminals. During normal operation the capacitor voltages are dictated by the mains; however, if the machine is switched off from the supply, capacitor equations are again needed. Therefore the model of the machine consists of Eqs. (1a), (2) – (11) and (17) for the analysis of the switch-off transients (a single-cage machine is discussed here), while during start-up and for analysis of the reclosing transients Eq. (17) is omitted from the model. It is worth noting that the switch-off transient will fully correspond to the so-called symmetrical capacitor braking [18] if the machine is not reconnected to the supply, provided that the value of the capacitors is appropriately selected and zero initial conditions are used for capacitor voltages.

If the machine is overcompensated by the capacitor bank, switch-off may lead to overvoltages at the machine terminals. The amount of overvoltage and its duration will depend on the capacitance of the capacitors, the drive inertia and the load torque. The inclusion of saturation ensures correct prediction of the overvoltages, as an overestimate of the voltage peaks is obtained if the saturation is neglected. The induction machine used in this analysis is again Machine A, it runs unloaded and its stator winding and capacitor bank are star connected. The value of the capacitor which provides compensation of the induction motor's reactive power consumption is 25 $\mu$F, the same as was used in Section 3.1. With this capacitor value there will be no overvoltages at the machine terminals during the switch-off transient, as the experimentally recorded waveform shown in Fig. 5 clearly demonstrates.

However, if the machine is overcompensated, overvoltage will occur. Both simulations and experiments
were performed using a capacitance of 50 µF per phase (double the value that provides full compensation). Switch-off was performed first and the mains voltages were reconnected later on. The constant-parameter model with saturation neglected predicts an overvoltage of almost 30% (Fig. 6(a)) during the switch-off transient. If the saturated machine model is applied, the predicted overvoltage is much smaller (Fig. 6(b)) and corresponds to that obtained experimentally (Fig. 6(c)). The instant of switch-off is indicated in Fig. 6(a) and 6(b) by the broken line (the second broken line in Fig. 6(b) corresponds to the instant of mains reconnection).

Current waveforms for the same value of capacitance (50 µF) are given in Fig. 7, where the experimentally recorded currents during the switch-off (Fig. 7(a)) and

Fig. 4. Experimental and simulation results of voltage build-up in a self-excited double-cage induction generator operating at a speed of 3180 rpm, with capacitors of 24.5 µF: (a) experimental, initial portion of the transient (400 V/div., 100 ms/div.); (b) experimental, voltage in steady-state (400 V/div., 5 ms/div.); (c) simulation, complete voltage build-up.

Fig. 5. Experimentally recorded induction motor voltage during switch-off with capacitors of 25 µF per phase.

Fig. 6. Voltage waveforms during the switch-off transient with capacitors of 50 µF per phase: (a) simulation with main flux saturation neglected; (b) simulation with main flux saturation accounted for (reclosing transient included); (c) experimental.
4. Variable-speed converter fed induction motor drives

Main flux saturation plays an important role in cases when the variable-speed induction motor drive is fed from a current source. Induction motor drives fed from a current-source inverter and a resonant parallel inverter are discussed in the first two subsections. The third subsection deals with the field oriented control of an induction motor fed from an ideal current controlled PWM inverter.

4.1. Current-source inverter fed induction motor drive

The configuration of the current-source inverter (CSI) fed induction machine is shown in Fig. 8. A three-phase thyristor bridge rectifier supplies, through a DC link inductance, a standard autosequentially commutated three-phase CSI which consists of six thyristors, six diodes and six commutating capacitors. It is assumed that the CSI operates with independent output frequency control (i.e. without slip frequency control). Consequently, steady-state operation takes place on the stable part of the torque–slip curve and the machine operates in the highly saturated region of the magnetizing curve. Steady-state time domain analysis is performed here with the ultimate goal of predicting the steady-state voltage and torque of the machine. Ripple in the DC current and the commutation interval may or may not be disregarded in the analysis, depending on the desired accuracy. The approach here assumes that the DC link current may be treated as constant. However, commutation overlap is accounted for in such a way that the machine phase currents have a trapezoidal waveform. As the machine is current fed, the stator phase and consequently the d,q-axis currents are known. Thus the stator d,q-axis currents and their derivatives act as inputs to the model, while voltages become outputs. The machine model (Eqs. (1)–(11)) has to be modified accordingly.

The machine used in this analysis is Machine C. Fig. 9 summarizes experimental and simulation results for steady-state operation at a frequency equal to 40 Hz with a DC link current of 12.2 A and a load torque of
12 N m. The experimentally recorded waveform of the steady-state voltage is shown in Fig. 9(a). Values of the commutation overvoltages and steady-state peak values are given for better comparison with simulation results. The voltage waveform obtained by simulation is displayed in Fig. 9(b), together with the phase a input current, while the simulated induction motor torque and speed waveforms are depicted in Fig. 9(c). Comparison of the experimental and simulated voltage waveforms shows very good agreement between the steady-state voltage values (the peak value in the simulation is \( \pm 279 \) V, while the experimental value varies slightly between \( \pm 273 \) V and \( \pm 279 \) V); this was achieved by utilizing the saturated machine model. As far as the commutation overvoltages are concerned, the difference between the experimental and simulation values is quite significant in some commutation intervals. However, the predicted values of the overvoltages are dependent on the current approximation during the commutation interval only; considering the very simple current approximation utilized in the simulation, it can be stated that the agreement between the experimental and simulation commutation overvoltages is fairly good.

4.2. Parallel resonant inverter fed induction motor drive

The drive under consideration consists of a low-voltage three-phase voltage supply (an autotransformer operating with constant-voltage output), a three-phase diode bridge, a DC link inductance and capacitance, a three-phase thyristor inverter and a three-phase capacitor bank connected in parallel to the terminals of the induction motor (Fig. 10). A suitable choice of the DC link inductance and capacitors at the AC side of the inverter leads to resonant operation of the circuit; the current in the DC link is discontinuous and the inverter operates as a zero-switching-current resonant inverter. Two thyristors in the inverter conduct during the conduction intervals. If the maximum required output frequency is 50 Hz, the resonant frequency of the circuit has to be set above 150 Hz. It should be noted that the drive operation is possible over the entire designed frequency range with a constant DC link voltage. The motor’s voltage/frequency ratio is reasonably constant. However, due to the absence of feedback, instability results if a step increase in the load torque occurs; some feedback has therefore to be introduced in order to stabilize the drive operation.

Steady-state operation of the drive takes place on the saturated part of the magnetizing curve and simulation is not possible unless a model which accounts for main flux saturation is utilized. As the single-cage machine (Machine A) is used, the model that describes the machine is once more given by Eqs. (1a) and (2)-(11). The input into the model is the DC link voltage \( V_d \) across capacitance \( C_d \), which is assumed to be constant. The equation of the DC link circuit is

\[
\frac{dV_d}{dt} = \frac{(V_d - v_i)}{L_d}
\]

where \( v_i \) is the inverter input voltage and is a linear combination of the products of switching functions and induction motor terminal voltages. As there is a capacitor bank connected to the machine terminals, Eq. (17) is needed once more. However, it is more convenient in this case to use the capacitor bank equations in the original phase domain rather than Eq. (17).

The capacitor bank and stator windings were delta connected in the experiments and simulations, the load torque had a fan characteristic and steady-state operation at 40 Hz was investigated (the relevant data of the rig are given in the Appendix). The resonant frequency was approximately equal to 200 Hz. The simulation
results are shown in Fig. 11 and include motor line-to-line voltage $v$, motor line current $i_a$, and phase current $i_x$, motor torque $T_e$, and DC link current $i_d$.

The corresponding experimental results are shown in Fig. 12. The agreement between the experimental and simulation results is remarkably good. However, it should be noted that the DC voltage in the experiment

Fig. 10. Resonant parallel inverter fed induction motor drive.

Fig. 11. Simulation results of the resonant parallel inverter fed induction motor drive operation at 40 Hz: (a) line-to-line voltage; (b) motor line and phase currents; (c) torque; (d) DC link current.

Fig. 12. Experimentally recorded waveforms in a resonant parallel inverter fed induction motor drive at $f = 40$ Hz and $V_d = 64$ V: (a) voltage; (b) motor line current; (c) motor phase current.
was equal to 64 V, while that employed in the simulation was only 44 V. These voltages produced identical DC link current waveforms with identical peak values and consequently the motor voltage and currents obtained by experiment and in the simulation have the same waveforms with the same values. The difference in the DC voltage values required to produce the same waveforms in the simulation as in the experiment is attributed to the fact that all the losses in the DC link, inverter and AC part of the circuit prior to the motor are neglected in the simulation.

4.3. Vector controlled induction motor drives

Among the three possible choices of vector control (orientation along the stator, air-gap or rotor flux space vector), the one chosen for discussion here is the rotor flux oriented control, which is still the predominant type in actual realizations. Rotor flux oriented control may be realized in a number of different ways. Either a current-fed or a voltage-fed machine, with either an indirect type of orientation or with one of the direct schemes, where estimation of the rotor flux space vector is performed, may be selected depending on the application. The scheme discussed here is the simplest one, the so-called indirect rotor flux oriented control (Fig. 13). The machine is once more represented by model equations (1a) and (2)–(11) with the same change already mentioned in Section 4.1. As the machine is treated as current fed, the stator-axis currents and their derivatives are known and act as inputs to the model. A PWM inverter with local current feedback is taken as ideal, so that reference currents equal actual phase currents. There are two distinct operating regimes where main flux saturation will play an important role. The first is operation in the field-weakening region, where a change in the saturation level in the machine happens simply because of a change in the desired flux level. The other situation is operation in the constant-flux region, with a constant rotor flux reference value. In this case, a change of saturation level in the machine will occur purely due to the cross-saturation effect during dynamic regimes with high values of the stator q-axis reference current. The consequence of cross-saturation will be a reduction in the actual rotor flux and motor torque with respect to the reference values and an unwanted transient in the response.

Two operating regimes were simulated in order to illustrate the influence of saturation on the operation of the drive. In both cases the value of the magnetizing
inductance in the controller was set to rated (i.e., \( L_m = L_{mn} \)) and the speed loop was left open, so that the independent control variable is the reference torque and the drive operates in the torque mode. Machine A was used once more. The first transient applied was a step torque command equal to the rated torque while the machine operated in the field-weakening region with a rotor flux command equal to 75% of the rated. The commanded and actual torques and the rotor flux waveforms are shown in Fig. 14(a) (an asterisk denotes the commanded quantities). As can be seen from the figure, prior to the application of the step torque command the drive operates under no-load conditions and there is a large discrepancy between the actual and commanded rotor flux. This is a consequence of the value of the magnetizing inductance used in the controller being inadequate for the operating flux level. Once the torque command is applied, there are unwanted transients in both the torque and flux responses. Experimental results regarding the influence of saturation on operation in the field-weakening region are available in Ref. [49].

The second transient is concerned with operation in the constant rated rotor flux region. A large torque command, equal to seven times the rated torque, was first applied and then removed. The responses are illustrated in Fig. 14(b). Due to the cross-saturation effect there is a significant drop in the actual rotor flux and the torque produced by the machine is smaller than the commanded one. It is worth noting that, to the best of the author’s knowledge, experimental proof of the cross-saturation effect in rotor flux oriented induction machines has not been provided yet.

5. Conclusion

Main flux saturation in induction machines is of paramount importance in a number of everyday applications and, if successful analysis is to be performed, it has to be included in the model of induction machines. This paper has concentrated on the current state space model of saturated single-cage and double-cage (deep-bar) induction machines and its applications in the simulation of induction machines and the variable-speed induction motor drive. The majority of practical situations where a need for a saturated machine model exists are addressed. The applications studied include self-excitation of single-cage and double-cage induction generators, switch-off and reclosing transients of induction motors with reactive power compensation, variable-speed drives fed from current-source and parallel resonant inverters and finally vector controlled high-performance induction motor drives. A large number of simulation and experimental results are included and these are found to be in very good agreement.

The majority of cases studied here involve a single-cage induction machine. However, if the actual machine in any of the applications is double cage or deep bar, the corresponding model of a saturated double-cage machine should be used. The same degree of accuracy in the simulation results should be achieved with regard to the influence of the main flux saturation on the machine’s behaviour.

Appendix

Induction machine and drive data

**Machine A**

\[ P_n = 0.75 \, \text{kW}, \quad V_n = 380 \, \text{V}/220 \, \text{V}, \quad I_n = 2.1 \, \text{A}/3.6 \, \text{A} \]

\[ \cos \varphi = 0.72, \quad f = 50 \, \text{Hz}, \quad n_n = 1390 \, \text{rpm} \]

Index \( n \) denotes rated values

Parameters:

\[ R_s = 10 \, \Omega, \quad R_r = 6.3 \, \Omega \]

\[ L_{mn} = 0.42119 \, \text{H}, \quad L_s = L_y + L_{mn} = 0.46257 \, \text{H} \]

\[ L_r = L_y + L_{mn} = 0.46226 \, \text{H}, \quad J = 0.00442 \, \text{kg m}^2 \]

Magnetizing curve approximation in terms of phase r.m.s. values:

\[ \Psi_m = 0.86427 \times 0.59976 L_m^{1.211} \]

\[ L = d \Psi_m/d I_m \]

Additional data for Section 4.2

\[ C = 100 \, \mu\text{F}, \quad L_d = 4.2 \, \text{mH}, \quad T_{	ext{i}} = 0.025 \times 10^{-3} \omega_r^2 \]

**Machine B** (double-cage induction machine)

\[ P_n = 7.5 \, \text{kW}, \quad V_n = 380 \, \text{V}, \quad I_n = 14.7 \, \text{A} \]

\[ \cos \varphi = 0.9, \quad f = 50 \, \text{Hz}, \quad n_n = 2905 \, \text{rpm} \]

Delta connected stator winding

Parameters:

\[ R_s = 1.9685 \, \Omega, \quad R_{r1} = 2.82 \, \Omega, \quad R_{r2} = 1.36 \, \Omega \]

\[ R_r = 0.649 \, \Omega, \quad L_{mr} = 2.79 \, \text{mH} \]

\[ L_{mn} = 0.44977 \, \text{H}, \quad L_s = L_{yr} + L_{mn} = 0.46 \, \text{H} \]

\[ L_{r1} = L_{yr} + L_{mn} = 0.45256 \, \text{H} \]

\[ L_{r2} = L_{yr} + L_{mn} + L_{mr} = 0.46057 \, \text{H} \]

Magnetizing curve approximation in terms of phase r.m.s. values:

\[ \Psi_m = \begin{cases} L_{m0} I_m & I_m < 1.25 \, \text{A} \\ (a + b + c I_m^2)^{-1} & I_m > 1.25 \, \text{A} \end{cases} \]

\[ a = 0.67905, \quad b = 0.067911, \quad c = 0.94346, \quad L_{m0} = 0.59 \, \text{H} \]
\[ L = \begin{cases} L_m & I_m < 1.25 \text{ A} \\ L_m \left( b/|I_m|^2 + 2c/|I_m|^3 \right) & I_m > 1.25 \text{ A} \end{cases} \]

**Machine C**

\[ P_n = 3 \text{ kW}, \quad V_n = 220 \text{ V}, \quad I_n = 6.3 \text{ A} \]

f = 50 Hz, \quad n_n = 1400 \text{ rpm}

Delta connected stator winding

Parameters:

\[ R_s = 1.54 \text{ Ω}, \quad L_{ys} = 6.6526 \text{ mH} \]

\[ R_s = 2.22 \text{ Ω}, \quad L_{ys} = 6.6526 \text{ mH}, \quad J = 0.21 \text{ kg m}^2 \]

DC link inductance \( L_d = 28.5 \text{ mH} \)

Commutating capacitance \( C = 8 \mu\text{F} \)

Magnetizing curve approximation in terms of phase r.m.s. values:

<table>
<thead>
<tr>
<th>Magnetizing current ( I_m )</th>
<th>Magnetizing flux ( \psi_m )</th>
<th>Dynamic inductance ( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00–1.10</td>
<td>0.318 ( I_m )</td>
<td>0.318</td>
</tr>
<tr>
<td>1.10–1.85</td>
<td>0.227 ( I_m + 0.10 )</td>
<td>-0.184 ( I_m + 0.52 )</td>
</tr>
<tr>
<td>1.85–3.60</td>
<td>0.105 ( I_m + 0.33 )</td>
<td>-0.054 ( I_m + 0.28 )</td>
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<tr>
<td>3.60–6.10</td>
<td>0.068 ( I_m + 0.45 )</td>
<td>-0.016 ( I_m + 0.14 )</td>
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<tr>
<td>6.10–50.00</td>
<td>0.044 ( I_m + 0.60 )</td>
<td>0.044</td>
</tr>
</tbody>
</table>

**References**


