Modeling electricity markets with hidden Markov model

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Abstract

This paper proposes to model the movements of electricity markets as partially observable Markov processes driven by underlying economic forces. An electricity market is modeled as a dynamic system evolving over time according to Markov processes. At any time interval, the electricity market can be in one state and transition to another state in the next time interval. This paper models the states of an electricity market as partially observable, while each state has incomplete observations such as market-clearing price and quantity. The true market states are hidden from a market participant behind the incomplete observation. The hidden Markov model (HMM) is of a more fundamental approach and focuses on capturing the interaction of supply and demand forces on electricity markets. Such an approach is appropriate because the simultaneous production and consumption of electricity eliminates the storage sector, while limited transmission networks segment electricity markets. This model is shown to be able to link the fundamental drivers to the price behaviors; therefore, it provides forecast power for mid-term and long-term price movements.

This work applies HMM to historical data from New York independent system operator (NYISO), and examples are given to illustrate the forecast power of HMM.

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Keywords: Deregulation; Electricity markets; Hidden Markov model; Markov process

1. Introduction

The deregulation of electricity markets around the world has raised many new challenges for all stakeholders. The modeling of electricity markets is a basic yet critical problem for all stakeholders and provides the foundation for many decision-making problems within electricity markets. The movements of price on electricity are often modeled as stochastic processes [1–3]. The geometric Brownian motions (GBM) as shown in Eq. (1) are often assumed. Features of electricity price movement have been modeled as mean-reversion, jumps, regime-switching, and time-varying volatilities beyond GBM. The electricity markets are implicitly assumed to be weak form efficient, and prices are forecasted by observing the historical price movements only. Econometric approaches are employed to estimate the parameters of such stochastic processes using historical data. Although an econometric approach captures some features of markets and provides insights on short-term forecasting, it is incomplete because the market structure and architecture of electricity markets and the underlying economic forces are ignored.

\[
\frac{dY(t)}{Y(t)} = \mu \ dt + \sigma \ dW(t)
\]  

where \( Y(t) \) is the price on electricity at time \( t \), \( \mu \) the drift rate of price on electricity, \( \sigma \) the volatility of price on electricity, and \( dW(t) \) is the increment of a Brownian motion.

This paper analyzes the electricity markets’ structure and architecture, and proposes a fundamental economic approach to model electricity markets movements in mid-term and long-term. The electricity markets are modeled with hidden Markov models (HMM), which are only partially observed by market-clearing prices and quantities. The prices on electricity are determined by market states, which are in turn defined by the states of demand, supply on electricity and transmission network. An HMM is estimated with historical data from NYISO.

2. Electricity market structure and architecture

Market structure is defined as the properties closely tied to the ownership and technology of a market, while market archi-
Deregulation of the electric power industry breaks the vertically integrated utilities into horizontally independent entities, such as generation company (GENCO), transmission company (TRANSCO), distribution company (DISCO), electricity service company (ESCO), electricity management company (EMCO), and market/system coordinator such as Regional Transmission Organization (RTO)/Independent System Operator (ISO), shown in Fig. 1. The independent entities are the buyers, sellers, and coordinators of electricity markets. The market participants own and operate different parts of the electric power systems, which defines the electricity markets structure.

Electricity markets trade electric energy, ancillary services, and other services, and discover their time- and space-varying values. Locational marginal price (LMP) addresses the space-varying nature of prices on electricity, illustrating a nonlinear dependency between the prices of electricity and locations, supported by a limited capacity transmission network. The non-storable nature of electricity inhibits temporal arbitrage and contributes to the time-varying nature of prices on electricity. However, the physical unit constraints do link the generation production level of one hour to a previous hour, as does the short-term demand on electricity. The aggregated online generation capacity at the next time interval depends solely on the current level of online generation capacity and the unit commitment and dispatch decisions of GENCOs. Due to the limited transmission capability and non-storable nature, electricity is heterogeneous between different locations and time intervals. This leads to segments of electricity markets depending on time horizon and electrical distances. A regional electricity market can be segmented into base-load, intermediate-load and peak-load sub-markets shown in Fig. 2 and the sub-markets have different market players.

The technology of generation, transmission, storage, and consumption of electricity determines the time-varying and space-varying prices of electricity in all segments of electricity markets. For base-load electricity markets, the key suppliers are base-load generators that utilize similar technology to achieve efficiency of producing electricity. The transmission network is less utilized during base-load periods, thus the base-load electricity market is more easily congested during peak-load periods. Electricity demand could be categorized into industry, commercial and residential customers based on the objectives and technologies of electricity consumption. Base demand includes invariant parts of all three-load groups, and depends mainly on population size and macroeconomic trend. Peak electric demand is due to commercial and residential customers, and is highly correlated with weather. The difference in the supply and demand forces in base-load and peak-load electricity markets suggests different market participants, structures, behaviors, and models. This, in turn, leads to different market prices movements.

Market structure defines market players and their competition positions. Market architecture defines how market players interact with each other and how electricity and information are exchanged and shared on electricity markets. The market architecture must be consistent with the market structure in a way that all the functionalities are properly assigned and aligned within each market architecture design. Sheble defines the electricity market set to include forward and spot markets in short-term, and future and planning markets in mid-term and long-term [5]. Forward contracts are normally traded for physical delivery and allow the scheduling of both generation facilities, and transmission networks. Typically, a forward contact is traded at least 1 day prior to the operating day, which includes 24 h markets. Spot
contracts and real-time markets are used to re-schedule and rectify forecast errors. Future contracts are more often employed as hedging instruments, while physical delivery is also possible but rare. There are also derivatives, such as options trading on all of the various contracts. A swap market is also included to facilitate the exchange of different contracts and risk sharing among all market players. Fig. 3 illustrates the market architecture of electricity markets categorized by contracts.

Besides contracts traded, the sub-markets could also be categorized according to other criteria, such as commodities traded, trading mechanisms and the authorities of the central ISOs in such markets. The main commodities traded include electric energy and ancillary services. Fig. 4 illustrates the market architecture of short-term electricity markets categorized by commodities traded.

Contracts and real-time markets are used to reschedule and rectify forecast errors. Future contracts are more often employed as hedging instruments, while physical delivery is also possible but rare. There are also derivatives, such as options trading on all of the various contracts. A swap market is also included to facilitate the exchange of different contracts and risk sharing among all market players. Fig. 3 illustrates the market architecture of electricity markets categorized by contracts.

State transition probability matrix \( A \)
\[
A : a_{ij} = P[q_{t+1} = S_j | q_t = S_i] \\
1 \leq i, j \leq N \text{ (number of distinct observation symbols per state)} \\
\text{(number of states)}
\]

Observation symbol distribution given a state \( B \)
\[
B : b_{j}(k) = P[v_k \text{ at } t| q_t = S_j] \\
1 \leq j \leq N, \quad 1 \leq k \leq M \text{ (number of distinct observation)} \\
\text{(symbols per state)}
\]

Initial state distribution \( \pi \)
\[
\pi : \pi_i = P[q_1 = S_i] \\
1 \leq i \leq N
\]

HMM falls in a subclass of Bayesian networks (BN) known as dynamic Bayesian networks (DBN), which are simply Bayesian networks for modeling time series data. A Bayesian network is a graphical model for representing conditional interdependencies between a set of random variables. The graphical modularity of BN provides both an intuitively modeling interface and a data structure for algorithm development. Probability theory is employed to ensure that the BN as a whole is consistent, and to interface models to data. The nature of time series simplifies the design of the DBN with directed arcs as shown in Fig. 5.

Several features of HMM make it a good candidate for theoretical analysis. First, HMM has a very rich mathematical

![Fig. 5. Dynamic Bayesian network, X, Y, Z and W are random variables.](image)

**3. Hidden Markov model**

A pair of stochastic processes \((S, Y)\) is a hidden Markov model (HMM) if \(S\) (the state process) is a Markov process and \(Y\) (the observable process) is an incomplete observation of \(S\) [8]. The observation can be deterministic or probabilistic and the observable can be a state or a state transition. Mathematically, HMM is a doubly embedded stochastic process with an underlying stochastic process that is not observable, but can only be observed through another set of stochastic processes that produce the sequence of observations. Eq. (2) defines a HMM with discrete observable variables:

\[
\begin{align*}
A & : a_{ij} = P(q_{t+1} = S_j | q_t = S_i) \\
& 1 \leq i, j \leq N \text{ (number of distinct observation symbols per state)} \text{(number of states)} \\
B & : b_{j}(k) = P(v_k \text{ at } t| q_t = S_j) \\
& 1 \leq j \leq N, \quad 1 \leq k \leq M \text{ (number of distinct observation)} \text{(symbols per state)} \\
\pi & : \pi_i = P(q_1 = S_i) \\
& 1 \leq i \leq N
\end{align*}
\]
structure such that a sequence of HMMs of increasing size can approximate any ergodic stochastic process in the weak and cross entropic sense [9]. The second merit of HMM is its strength to explain extreme variations in the observed process based on a postulated hidden process. In particular, an HMM attributes this over-dispersion to the key model feature that observations come from one of several different marginal distributions, each associated with a different latent state. An HMM is capable to capture the over-dispersion in the observed electricity market prices by showing that the markets are transitioning among different states. The third merit is that there are very efficient algorithms such as the forward–backward method to solve HMM. The closed state space of HMM reduces the complexity of model estimation and solution. Markov lattice has been shown to be easier to construct, and it converges faster than multiple-period multinomial trees when applied to option pricing [10]. Finally, HMM can be extended to many special cases, which have potential application in modeling electricity markets. More details on HMM can be found in references [8,11].

4. Modeling of electricity markets with hidden Markov models

Inspired by the significant volatilities of price on electricity has been observed, different approaches have been proposed to interpret the observed phenomenon. One approach is regime switching [12,13]. Regime switching states that there are unobserved market regimes following Markov processes underlying the observed price movements. Often, electricity markets are categorized into stable regime with less volatility and unstable regimes with extreme volatility. For each regime, an individual econometric model is proposed and fitted with historical data. This study realizes that the regime switching approach is a simplified case of HMM, probably the simplest form of HMM, shown in Fig. 6. Notations to be used in Figs. 6–9 are also given in Fig. 6.

The simplest HMM shown in Fig. 6 encodes information of a time series with the value of a single multinomial variable, the hidden system state \( S \). This multinomial approach severely limits the representation capacity of an HMM. For example, to represent 30 bits of information about the history of a time series, an HMM would need \( 2^{30} \) distinct states while an HMM with a distributed state representation could achieve the same task with 30 binary variables. The distributed state representation incorporates flexibility into HMM and increases the modeling power in two ways. First, such representation decomposes the state space into drivers that naturally decouple the dynamics of the process generating the time series. In this study, supply and demand on electricity determine the market-clearing price and quantity together. Although, the relationship between supply and demand can be modeled as system states such as under-supply, equilibrium and over-supply, a more natural approach is to model the supply and demand sides as distributed system states. Second, distributed state representations simplify the task of modeling multiple time series generated by the interaction of multiple independent processes. For example, the demand for ancillary services could be modeled as dependent on the demand for electric energy. This feature allows incorporating market architecture into the market model by explicitly defining how the observations depend on incomplete observable states. An HMM with distributed states is defined as a factorial hidden Markov model (FHMM), shown in Fig. 7 [14].

In FHMM, the state variables at a time interval are assumed to be dependent only on the corresponding state variables at the previous time interval. This assumption can be relaxed by
introducing coupling between the state variables in different time intervals. Tree-structured HMM (TSHMM) allows modeling of the interdependence between state variables of different time intervals. The TSHMM, shown in Fig. 8, models the supplied generation capacities at time $T+1$ to be dependent on the demand on electricity at time $T$. This feature captures behaviors of generation companies to turn on more generators when higher prices are expected.

The switching HMM (SHMM) allows multiple dependencies to be dynamically chosen by a switch variable. The SHMM as shown in Fig. 9 incorporates the impact of transmission networks. Whether the transmission networks are congested or not defines how the supply and demand forces determine the electricity prices. Other extensions of HMM are also possible and see potential applications in modeling electricity markets. In addition, electric markets can be modeled with multiple HMMs. Each HMM approximates a time-segment of the electricity market such as base-load, intermediate-load and peak-load markets. Multiple HMMs are linked together by the system states. The ending states of the last HMM are the initial states of the next HMM in the chain.

HMM can also be used to model electricity markets in mid-term and long-term. Mid-term market movements are assumed to be limited to changes caused by the different competition strategies employed by market participants within the same market structure. The same HMM with different sets of parameters is used to model the mid-term market movement as shown in Fig. 10. An electricity market is assumed to settle with different equilibrium within the same market structure until a threshold is reached, which in turn leads to changes of market structure in the long-term. Long-term market movements are assumed to be market structure changes, such as long-term demand trends, fuel costs, transmission networks, and technology advancements. HMMs with different structures are employed to model the long-term market movements as shown in Fig. 10.

HMM combines electricity market structure, architecture, and competition strategies of market players into one integrated mathematical framework. The electricity market structure and architecture determine the structure of an HMM, such as how many market states exist and the definitions of a market state. The economic position of a market player determines its competition strategies, which in turn jointly determines how an electricity market evolves from one state to another, namely the transition matrix. The randomness caused by unintended forecast errors is defined with the distribution of prices for each specific market state. All market movements are triggered by the arrival of information, such as observation on market prices, and, modeled as state transition in short-term, the HMM parameter changes in mid-term and the HMM structure changes in long-term.

5. Numerical examples

The part demonstrates how to estimate HMM from simulated or historical data. The first example models an hourly electric energy market with the simplest HMM, having three market states where each has a discrete distribution for observable prices. The markets states are defined based on the relationship between supply and demand for electricity. For a given level of demand on electricity, there are three possible levels of online

<table>
<thead>
<tr>
<th>Market states definition</th>
<th>Punch-in</th>
<th>Harvesting</th>
<th>Ripping-off</th>
</tr>
</thead>
<tbody>
<tr>
<td>State-1 (S-1)</td>
<td>S-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>True parameters</td>
<td>0.58</td>
<td>0.32</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial parameters</td>
<td>0.2</td>
<td>0.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Estimated values</td>
<td>0.5624</td>
<td>0.3254</td>
<td>0.1122</td>
</tr>
<tr>
<td>State-2 (S-2)</td>
<td>S-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>True parameters</td>
<td>0.275</td>
<td>0.45</td>
<td>0.275</td>
</tr>
<tr>
<td>Initial parameters</td>
<td>0.05</td>
<td>0.9</td>
<td>0.05</td>
</tr>
<tr>
<td>Estimated values</td>
<td>0.299</td>
<td>0.4228</td>
<td>0.2773</td>
</tr>
<tr>
<td>State-3 (S-3)</td>
<td>S-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>True parameters</td>
<td>0.1</td>
<td>0.32</td>
<td>0.58</td>
</tr>
<tr>
<td>Initial parameters</td>
<td>0.0</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Estimated values</td>
<td>0.1143</td>
<td>0.3071</td>
<td>0.5786</td>
</tr>
</tbody>
</table>

![Fig. 10: HMM for mid/long-term electricity markets modeling.](image-url)
Table 2
True and estimated parameters for price distribution for all electricity market states

<table>
<thead>
<tr>
<th>Market State</th>
<th>True Parameters</th>
<th>Initial Parameters</th>
<th>Estimated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1</td>
<td>0.18 0.64 0.18</td>
<td>0.7 0.2 0.1</td>
<td>0.1767 0.6386 0.1847</td>
</tr>
<tr>
<td>S-2</td>
<td>0.18 0.64 0.18</td>
<td>0.1 0.8 0.1</td>
<td>0.1866 0.6323 0.1811</td>
</tr>
<tr>
<td>S-3</td>
<td>0.18 0.64 0.18</td>
<td>0.1 0.2 0.7</td>
<td>0.1587 0.6484 0.1929</td>
</tr>
</tbody>
</table>

The second example estimates FHMM from historical data on an electricity market within the west control area of NYISO from 7th April to 27th July 2003, shown in Fig. 12 [15]. The FHMM is shown in Fig. 8, including prices on electric energy and ancillary services (30 min spinning reserve). The model estimation employs the Baum–Welch algorithm, and the estimated parameters are shown in Table 4.

The FHMM provides more insight into electricity markets such as the interaction of demand on electric energy and ancillary services, the interdependency between demand on electricity and supplied online generation capacity. Although the FHMM is much richer than example 1, it is still a very simplified model. More work on modeling such as incorporating the impacts of transmission networks requires more information.

Fig. 11. Simulated price paths for example 1.

Fig. 12. Prices on electric energy and ancillary services of west control area in NYISO, April–July 2003.
The rich mathematical structure of HMM provides full potential for its applications in modeling electricity markets. The intuitive graphical features enable HMM to be easily built and understood. The application of probability theory enables HMM to be estimated and solved efficiently. HMM provides modelers the capability to model the electricity market structure, architecture, and market participants’ competition strategies within an integrated framework.

6. Conclusion

The rich mathematical structure of HMM provides full potential for its applications in modeling electricity markets. The intuitive graphical features enable HMM to be easily built and understood. The application of probability theory enables HMM to be estimated and solved efficiently. HMM provides modelers the capability to model the electricity market structure, architecture, and market participants’ competition strategies within an integrated framework.

Acknowledgments

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References


Table 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low Mean Normal High</th>
<th>Low Normal High</th>
<th>Normal Mean High</th>
<th>Normal Mean Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand on electric energy states transition matrix</td>
<td>Demand at T=low 0.9229 0.0771 0 0.0000 Demand at T=normal 0.9334 0.0666 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td>Demand at T=low 0.9229 0.0771 0 0.0000 Demand at T=normal 0.9334 0.0666 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td>Demand at T=low 0.9229 0.0771 0 0.0000 Demand at T=normal 0.9334 0.0666 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td></td>
</tr>
<tr>
<td>Online generation capacity states transition matrix</td>
<td>Demand at T=low 0.9007 0.0993 0.0000 Demand at T=normal 0.8469 0.1531 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td>Demand at T=low 0.9007 0.0993 0.0000 Demand at T=normal 0.8469 0.1531 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td>Demand at T=low 0.9007 0.0993 0.0000 Demand at T=normal 0.8469 0.1531 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td></td>
</tr>
<tr>
<td>Price on electric energy (normal distributions, mean, normalized)</td>
<td>Demand at T=low 0.9900 0.0100 0.0000 Demand at T=normal 0.9892 0.0100 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td>Demand at T=low 0.9900 0.0100 0.0000 Demand at T=normal 0.9892 0.0100 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td>Demand at T=low 0.9900 0.0100 0.0000 Demand at T=normal 0.9892 0.0100 0.0000 Demand at T=high 0.0000 1.0000 0.0000</td>
<td></td>
</tr>
<tr>
<td>Price on electric energy (variance, normalized)</td>
<td>Supply Demand at T=low 0.0341 0.0616 0.0000 Demand at T=normal 0.0342 0.0615 0.0000 Demand at T=high 0.0680 0.0863 0.0275 Demand at T=normal 0.0341 0.0616 0.0000 Demand at T=normal 0.0342 0.0615 0.0000 Demand at T=high 0.0680 0.0863 0.0275 Demand at T=high 0.0680 0.0863 0.0275</td>
<td>Demand at T=low 0.0341 0.0616 0.0000 Demand at T=normal 0.0342 0.0615 0.0000 Demand at T=high 0.0680 0.0863 0.0275 Demand at T=normal 0.0341 0.0616 0.0000 Demand at T=normal 0.0342 0.0615 0.0000 Demand at T=high 0.0680 0.0863 0.0275 Demand at T=high 0.0680 0.0863 0.0275</td>
<td>Demand at T=low 0.0341 0.0616 0.0000 Demand at T=normal 0.0342 0.0615 0.0000 Demand at T=high 0.0680 0.0863 0.0275 Demand at T=normal 0.0341 0.0616 0.0000 Demand at T=normal 0.0342 0.0615 0.0000 Demand at T=high 0.0680 0.0863 0.0275 Demand at T=high 0.0680 0.0863 0.0275</td>
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