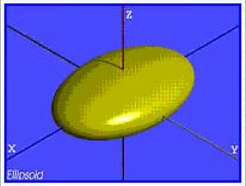
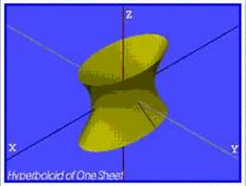
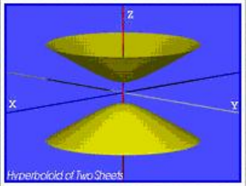
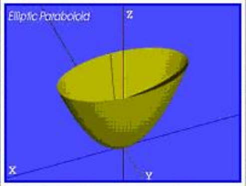
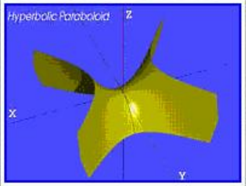
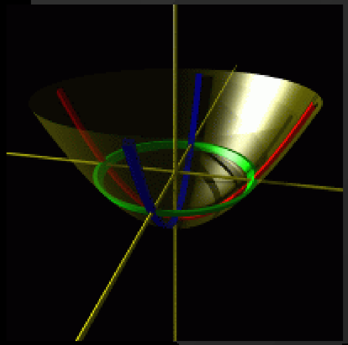
Quadric Surfaces

- Equation
- Types of surfaces
  - Ellipsoid
  - Hyperboloid of one sheet
  - Hyperboloid of two sheets
  - Elliptic paraboloid
  - Hyperbolic paraboloid
  - Elliptic cone (degenerate)

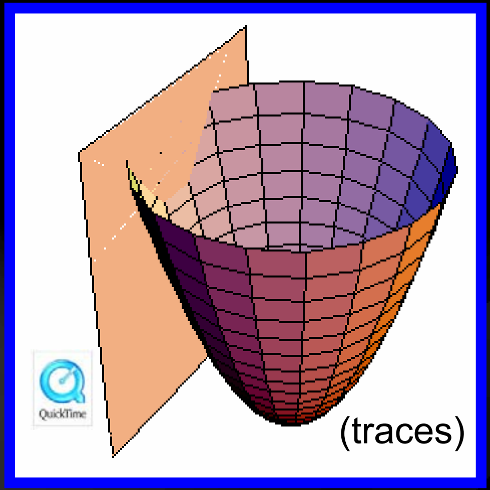
$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

Ellipsoid		$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperboloid of One Sheet		$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of Two Sheets		$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Elliptic Paraboloid		$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2cz$
Hyperbolic Paraboloid		$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2cz$



Quadric Surfaces - Traces

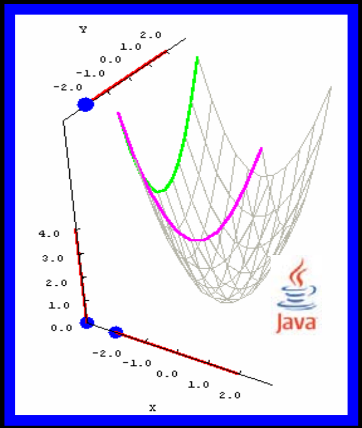
TJ Murphy, OU



Traces are cross sections parallel to a plane.

The *xy trace* is found by setting $z = 0$.
The *yz trace* is found by setting $x = 0$.
The *xz trace* is found by setting $y = 0$.

Traces applet
Jon Rogness,
Univ Minn.

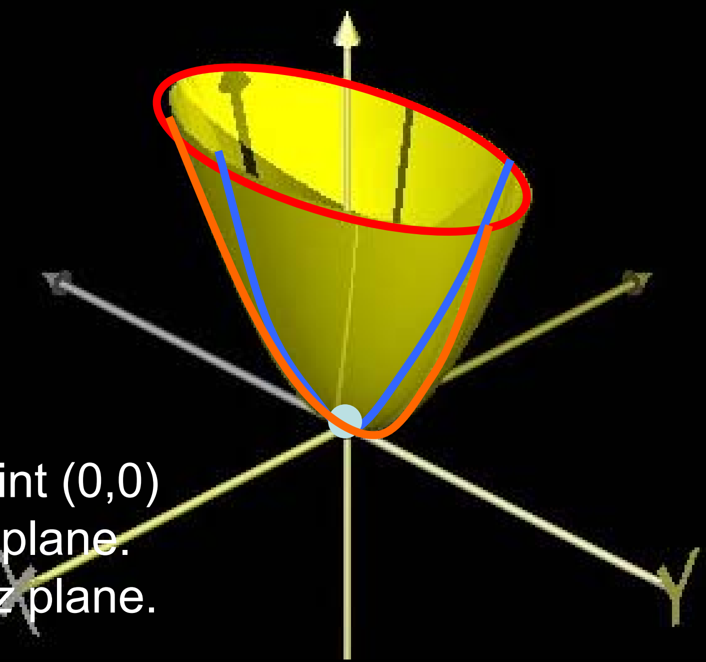


Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

Elliptic paraboloid

$z = 4x^2 + y^2$



Example:

For the elliptic paraboloid $z = 4x^2 + y^2$:

xy trace - set $z = 0 \rightarrow 0 = 4x^2 + y^2$ This is point (0,0)

yz trace - set $x = 0 \rightarrow z = y^2$ Parabola in *yz* plane.

xz trace - set $y = 0 \rightarrow y = 4x^2$ Parabola in *xz* plane.

Trace $z = 4$ parallel to *xy* plane:

Set $z = 4 \rightarrow 4 = 4x^2 + y^2$ or $x^2 + y^2/4 = 1$. This is an ellipse parallel to the *xy* plane.

Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

Hyperboloid of one sheet

$$-x^2 + y^2/9 + z^2/4 = 1$$

Example:

xy trace - set $z = 0 \rightarrow -x^2 + y^2/9 = 1$

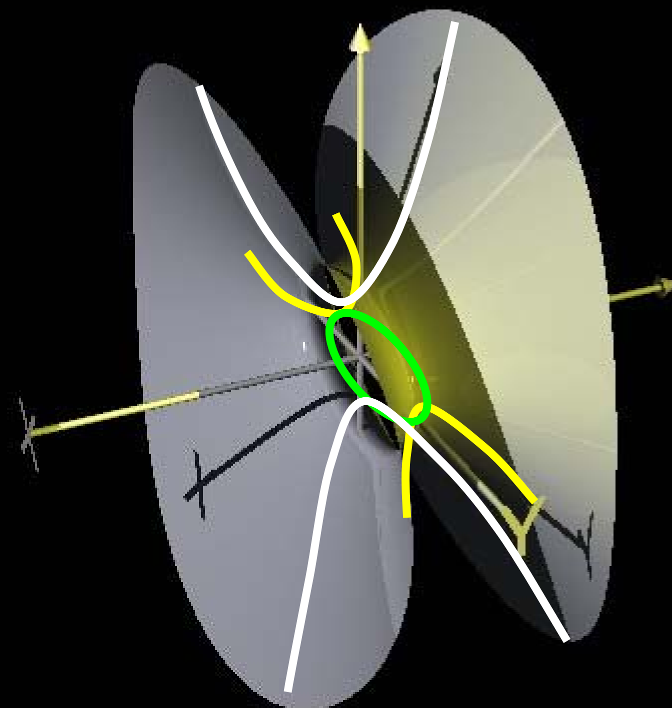
Hyperbola in the *xy* plane

yz trace - set $x = 0 \rightarrow y^2/9 + z^2/4 = 1$

Ellipse in *yz* plane.

xz trace - set $y = 0 \rightarrow -x^2 + z^2/9 = 1$

Hyperbola in *xz* plane.

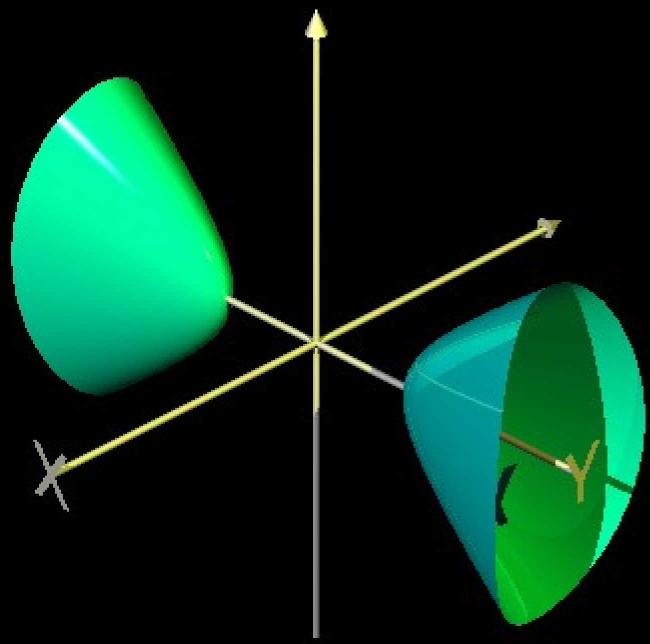


Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

Hyperboloid of two sheets

$$-x^2 + y^2/9 - z^2/4 = 1$$

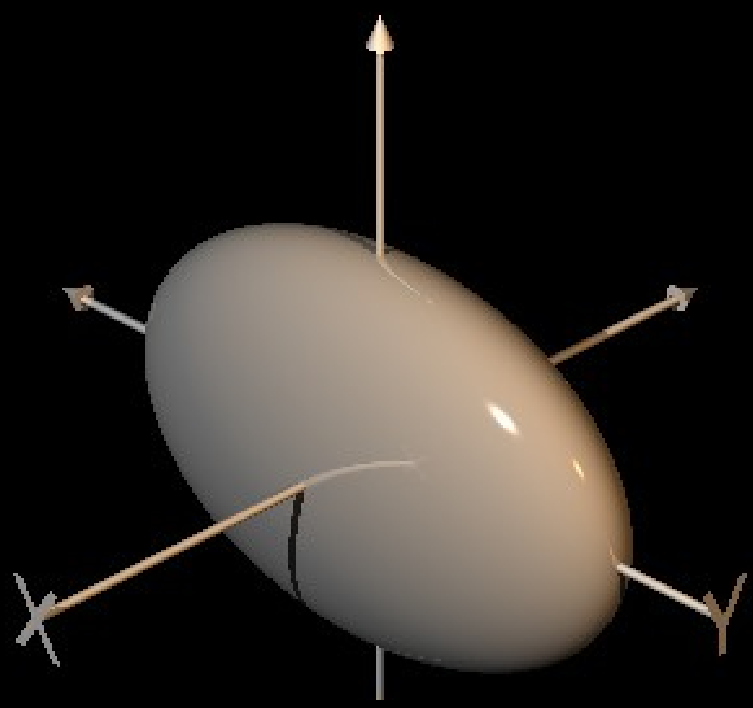


Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

Ellipsoid

$x^2 + y^2/9 + z^2/4 = 1$



Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

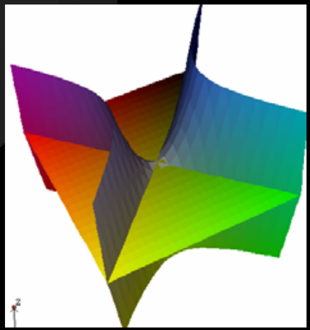
Hyperbolic Paraboloid

$z = x^2 - y^2$

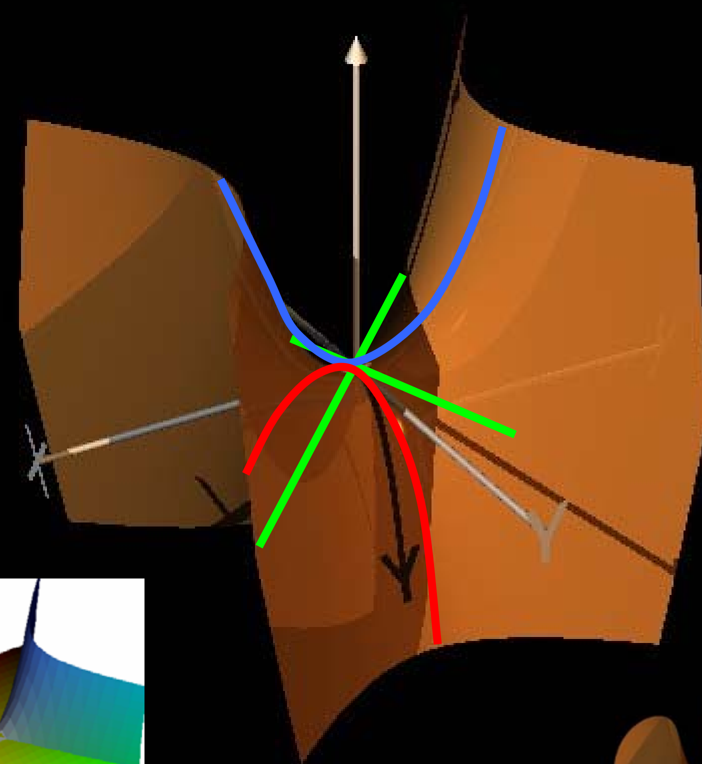
xy trace - set $z = 0 \rightarrow x^2 = \pm y^2$ This is two lines through (0,0)

yz trace - set $x = 0 \rightarrow z = -y^2$ Parabola in *yz* plane

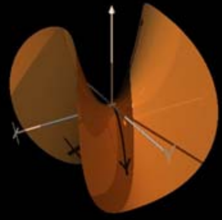
xz trace - set $y = 0 \rightarrow y = x^2$ Parabola in *xz* plane



Grapher



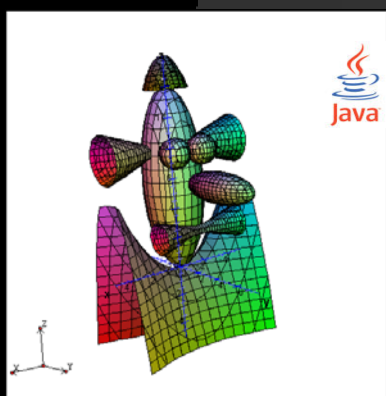
Polyray



Quadric Surfaces - Graphers

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + F = 0$$

Multitype grapher; does implicit and explicit functions

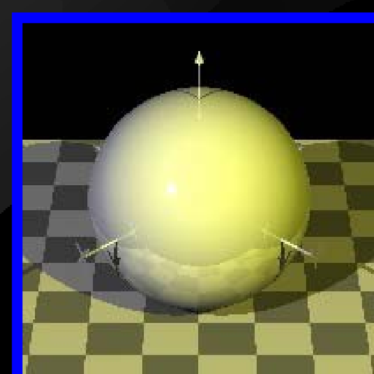


Can do implicit plots f(x,y,z)=0
Tips:
Shift X toggles axes on and off

For big image:
Right click on image: new display

Easiest to use; but z = *f(x,y)* form only

POLYRAY
does implicit polynomials
(goto calculators | polyray)
Xiao Gang, WIMS, France

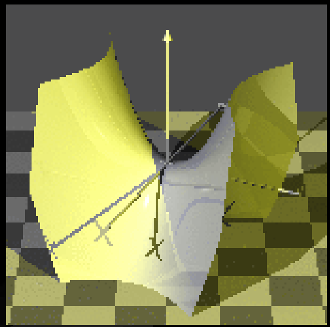


POLYRAY alternate WSU link

Hyperbolic Paraboloids

$x^2 - y^2 = cz$

$-1 \leq c \leq 1$



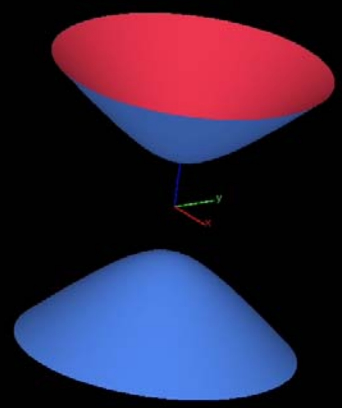
Paraboloids

“Parabola” by Maureen Bell, Scotland
Wax, silk, rivets, and washers

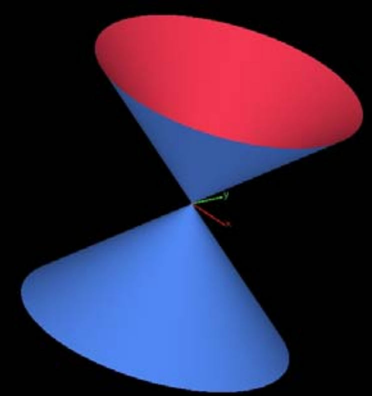
Ellipsoids

Winning entry in the 2003 Kansas Poultry Association Decorated Egg Contest

Hyperboloids Descending



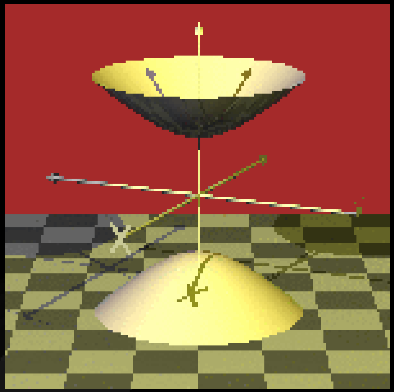
Zero locus of f(x,y,z) = x^2+y^2-z^2+1

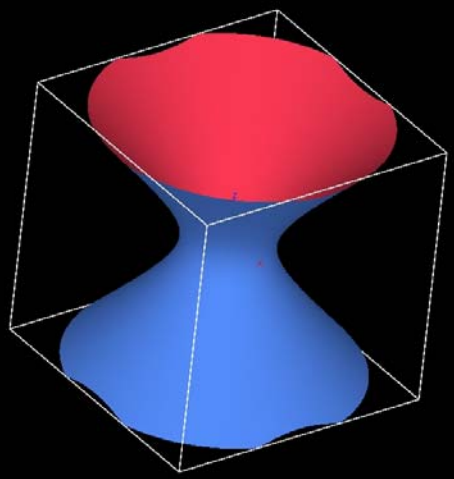


Zero locus of f(x,y,z) = x^2+y^2-z^2

$x^2+y^2-z^2 = c$

$-1 \leq c \leq 1$





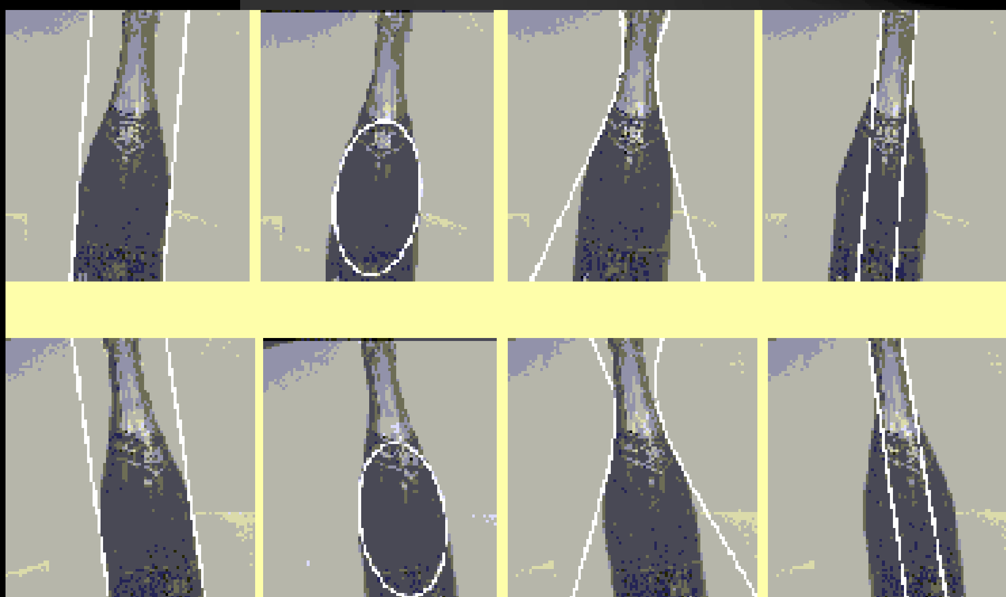
Zero locus of f(x,y,z) = x^2+y^2-z^2-1

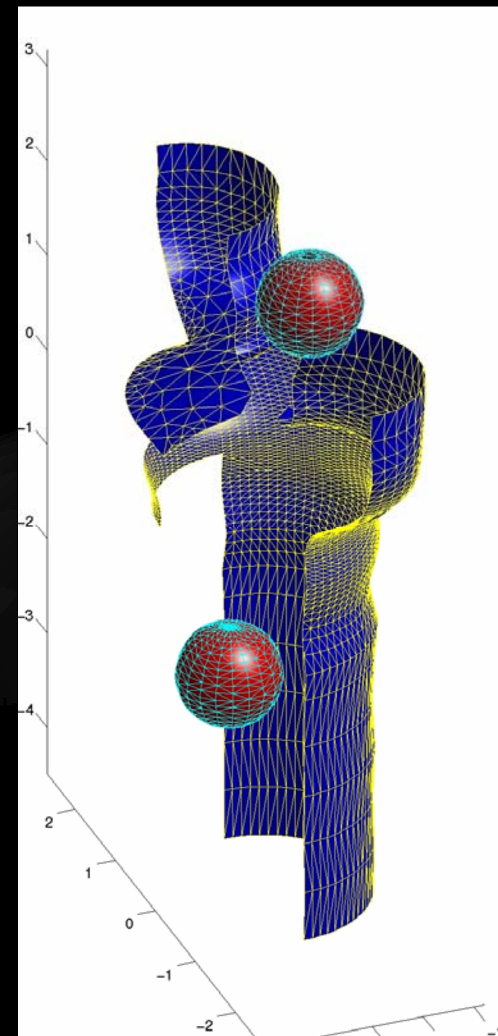
Hyperboloid Examples

Kobe, Japan

Modeling

Modeling software is based on pieces of quadric surfaces





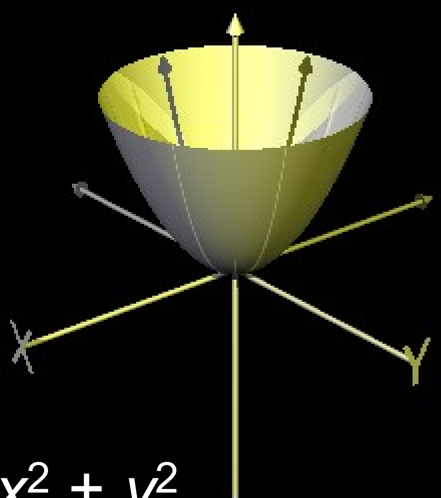
Quadric Surface Modeling

Quadric Surfaces - Transformations

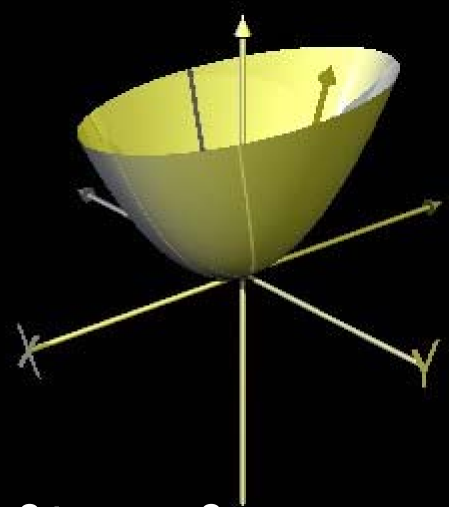
Quadric surfaces can be modified in several ways

Stretching

- Modifying *a, b,* or *c* causes the surface to stretch or shrink



$z = x^2 + y^2$

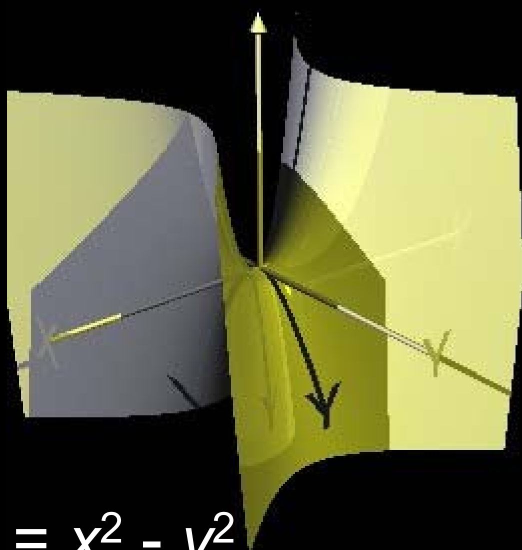


$z = x^2/4 + y^2$

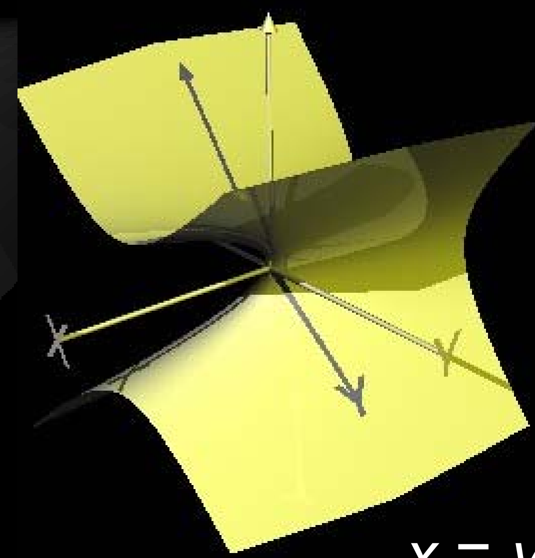
Quadric Surfaces - Transformations

Rotations

- Interchanging variables in the standard equation of a surface rotates the surface



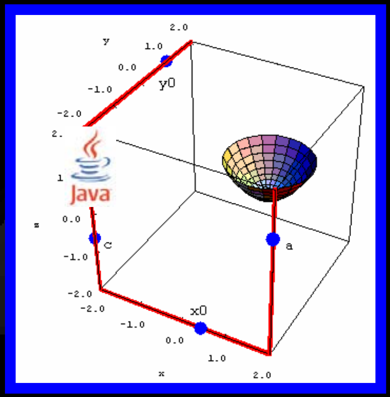
$z = x^2 - y^2$



$x = y^2 - z^2$

Quadric Surfaces - Transformations

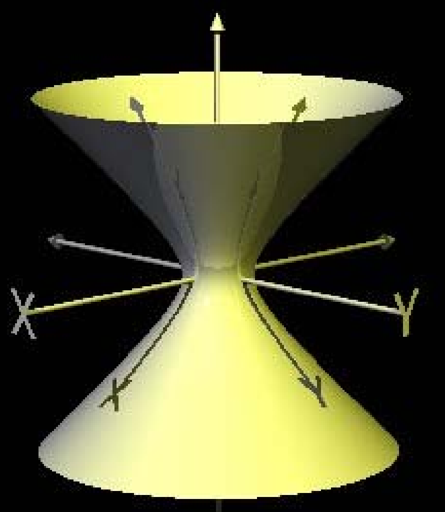
Dennis Nykamp, Univ. of Minn-Translations



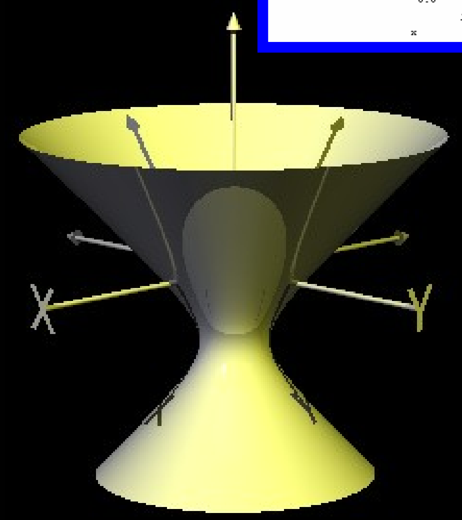
Translation

- You may shift a surface using the translations

$$x \to x - h$$
$$y \to y - k$$
$$z \to z - L$$



$x^2 + y^2 - z^2 = 1$



$x^2 + y^2 - (z+1)^2 = 1$