

Homework on Markov chains

Given a two-state DTMC with the transition probability matrix

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \quad 0 \leq a, b \leq 1 \quad |1-a-b| < 1$$

show that the n -step transition probability matrix $P(n) = P^n$ is given by:

$$P(n) = \begin{bmatrix} \frac{b + a(1-a-b)^n}{a+b} & \frac{a - a(1-a-b)^n}{a+b} \\ \frac{b - b(1-a-b)^n}{a+b} & \frac{a + b(1-a-b)^n}{a+b} \end{bmatrix}$$

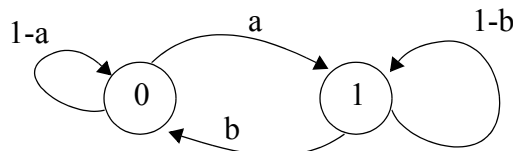


Figure 1: Model of a binary communication channel

If $a = \frac{1}{4}$ and $b = \frac{1}{2}$, choose any initial distribution and see what happens to $P(n)$ in the limit $n \rightarrow \infty$. Is the pmf of X_n dependent upon the initial distribution? Is it dependent on value of n ?