Exercises – Solutions

Note, that we have not formulated the answers for all the review questions. You will find the answers for many questions by reading and reflecting about the text in the book.

Chapter 2 Manufacturing and process systems

2.5

Hint: think about how a transient disturbance is evaluating in a continuous system. Then compare with a batch operation. What happens at the beginning and the end of the batch? Usually the batch looks like a transient.

Chapter 5 Discrete manufacturing problems

5.4

a) The state of a dynamical system is defined by the number of independent differential equations. The future can be completely defined if the state is known at time t and the inputs for all future times are given.
b) The state of a discrete system is defined completely differently. A discrete system is located in one and only one state at a time. The number of states can be finite or infinite. To move from one state to another some condition has to be satisfied, which is called an event. The time that it takes to move from one state to another is considered infinitely small.

5.7

a) Assume that machine k is finished and machine $k+1$ is not. If the robot picks up the product from machine k first, then it has nowhere to deliver it. Machine $k+1$ cannot be released from its product.
b) The robot has to pick up the finished product in the last machine and then work its way backward. (To introduce a buffer is of course another solution, but that is not part of the assumptions).

<u>Comment</u>: It is not sufficient to say that we need to introduce a scheduler.

5.12

Hint: give some examples how machines are not efficiently used.

5.19

If you are not familiar with semaphores, read Chapter 15.6.

Chapter 6 Stochastic Modelling of Manufacturing Systems

For computational help for Markov chains and Markov processes you may use the Matlab m-files `markovchain` and `markovprocess` respectively. They are available for downloading on the homepage www.iea.lth.se/aut.

6.2

Assume that the customers of Alfa are described as state 1 and the others are state 2. Then the transition probability matrix can be written

$$\mathbf{P} = \begin{pmatrix} 0,88 & 0,12 \\ 0,15 & 0,85 \end{pmatrix}$$

Initially the probability distribution is $\mathbf{p}(0) = \begin{pmatrix} 0,60 & 0,40 \end{pmatrix}$. We calculate

a) $\mathbf{p}(3) = \begin{pmatrix} 0,57 & 0,43 \end{pmatrix}$; b) $\mathbf{p}(\infty) = \begin{pmatrix} 5/9 & 4/9 \end{pmatrix}$ c) The same as b)

The market share will decrease! Independent of the initial market share for Alfa it will end up in 5/9, i.e. 56%.

6.3

a) (i) yes, (ii) ergodic, $\overline{p} = (1\ 0)$ b) (i) no (ii) not relevant

c) (i) no (ii) not relevant
d) (i) yes, (ii) not ergodic. It is certainly possible to find a unique stationary solution but the system is not positive recurrent. The time for recurrence to state 2 is infinity. The state is periodically changing between states 1 and 3. This is manifested by one of the eigenvalues. They are 1.0, 0.3 and –1.0. The eigenvalue 1.0 makes sure that the **p** vector is normalized, i.e. that the sum of its components is 1 at all times. The eigenvalue –1.0 is also on the unit circle and is a sign that the system is periodic and does not converge to a stationary value.

e) (i) yes (ii) not ergodic. This is apparent from the state graph.

f) (i) yes, (ii) ergodic $\overline{p} = \frac{1}{45}\begin{pmatrix} 19 & 17 & 9 \end{pmatrix}$ = (0.42, 0.38, 0.2). The eigenvalues of **P** are 1.0, 0.5 and –0.8. The negative real eigenvalue (-0.8) will cause two of the states to oscillate back and fourth but will finally converge towards the stationary value.

g) (i) yes, (ii) ergodic, $\overline{p} = (1\ 0\ 0)$

6.4

Yes, **P** is ergodic. We have $\mathbf{p}(\infty) = (2/7 \quad 5/7)$. The prob. to stay in state 1 is 2/7.

6.5

(a) Not ergodic. If we start in one state we will remain there.
(b) Not ergodic. The system is periodic.

(c) This is ergodic and the stationary probabilities are $p_1 = a/(a+b); \quad p_2 = b/(a+b)$

6.6

Define the states 1 = *A*; 2=*B*; 3=*C*; 4=out; 5=*Ra*; 6=*Rb*; 7=reject.

b) The transition probability matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 0,97 & 0 & 0 & 0,03 & 0 & 0 \\ 0 & 0 & 0,9 & 0 & 0 & 0,1 & 0 \\ 0 & 0 & 0 & 0,93 & 0 & 0 & 0,07 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0,5 & 0 & 0 & 0 & 0 & 0,5 \\ 0 & 0 & 0,4 & 0 & 0 & 0 & 0,6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

c) The prob. to reach the output buffer from machine *A* is

$$(0,97 + 0,03 \cdot 0,5) \cdot (0,9 + 0,1 \cdot 0,4) \cdot 0,93 =$$
$$= 0,985 \cdot 0,94 \cdot 0,93 = 0,86$$

Thus 117 units must be fed into machine *A* in order to get 100 approved jobs.

d) The prob. to reach the output from machine *A* is $0,97 \cdot 0,9 \cdot 0,93 = 0,811$

Thus 124 units must be fed into machine *A* in order to get 100 approved jobs.

e) See Chapter 6.3.2.

6.7

The process can be modeled as a Markov chain with three states, the number of unfinished jobs at the operator, *just before* the courier arrives. The states 1, 2 and 3 represent that there are 0, 1 or 2 unfinished jobs waiting for the operator. Every 30 minutes there is a state transition. This means that at the end of the period the number of unfinished jobs has decreased with 1 (if 0 delivery), stayed the same (if 1 job delivered) or increased with 1 (if 2 jobs delivered).

Assume that the operator has 1 job waiting in the beginning of a time interval (just before the courier arrives)

- If the courier delivers 1 job (with prob. 0,5), then there will be 1 unfinished job at the end of the time period, since the operator will finish one job (surely). Therefore we have $p_{22} = 0,5$.
- If the courier delivers 0 job, then there will be 0 jobs waiting at the next time interval. Thus $p_{21} = 0,3$

- If the courier delivers 2 jobs, then there will be 2 jobs waiting at the next time interval, i.e. $p_{23} = 0,2$

Now assume that the operator has 2 unfinished jobs waiting in the beginning of the period.

- If the courier delivers 1 job (with prob. 0,5) there will be 3 jobs waiting. However, if he delivers 2 jobs (with prob. 0,2) one of them will be rejected, and there are still 3 jobs waiting. So the probability to get 3 jobs in the beginning of the period is 0,5+0,2=0,7. At the end of the period there will be 2 unfinished jobs. In other words $p_{33} = 0,7$

- If the courier delivers 0 job (with $p_{33} = 0,7$ prob. 0,3) there will be 1 unfinished job at the end of the period, so we get $p_{32} = 0,3$

In a similar manner we can find the rest of the transition probabilities and obtain:

$$\mathbf{P} = \begin{pmatrix} 0,8 & 0,2 & 0 \\ 0,3 & 0,5 & 0,2 \\ 0 & 0,3 & 0,7 \end{pmatrix}$$

The stationary probability distribution is calculated:

$$\mathbf{p}(\infty) = \left(\frac{9}{19}, \quad \frac{6}{19}, \quad \frac{4}{19} \right) = \begin{pmatrix} 0,47 & 0,32 & 0,21 \end{pmatrix}$$

In the long run the operator will be in state 1 (there are no unfinished jobs) during 9/19 of the time. The courier arrives, and with 30% probability he does not deliver any new job. This means that the probability that the operator will remain without any job is (9/19)*0.3 = 0,14, i.e. 14% of the time.

6.8

a) We calculate

$$f_1^1 = 0,75 \qquad f_1^2 = 0,25 \cdot 0,25$$

$$f_1^3 = 0,25 \cdot 0,75 \cdot 0,25 \qquad f_1^n = 0,25 \cdot (0,75)^{n-2} \cdot 0,25 \qquad n \geq 2$$

Summarizing all the probabilities we obtain

$$\sum_{n=1}^{\infty} f_j^n = f_j = 0,75 + 0,25 \cdot (1 + 0,75 + 0,75^2 + \ldots) \cdot 0,25 =$$

$$= 0,75 + 0,25 \cdot \frac{1}{1 - 0,75} \cdot 0,25 = 0,75 + 0,25 \cdot 4 \cdot 0,25 = 1$$

i.e. the prob. to ever return to state 1 is 1.

b) The average time to return to state 1 is calculated as:

$$\mu_1 = \sum_{n=1}^{\infty} n \cdot f_1^n = 1 \cdot 0,75 + 0,25^2 \cdot (2 + 3 \cdot 0,75 + 4 \cdot 0,75^2 + 5 \cdot 0,75^3 \cdot \cdot\cdot)$$

Replace 0,75 with x and calculate the sum

$$2 + 3 \cdot x + 4 \cdot x^2 + 5 \cdot x^3 + \cdots \qquad (1)$$

It can be rewritten as a sum of two series:

$$1 + x + x^2 + x^3 + \cdots \qquad (2)$$

and

$$1 + 2 \cdot x + 3 \cdot x^2 + 4 \cdot x^3 + \cdots \qquad (3)$$

The latter can be written as

$$\frac{d}{dx}\left(1 + x + x^2 + x^3 + \cdots\right) \qquad (4)$$

The sum (2) is calculated to $\frac{1}{1-x}$ while the sum (4) is $\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$

Inserting $x = 0,75$:

$$\mu_1 = 0,75 + 0,25^2 \cdot \left(\frac{1}{1-0,75} + \left[\frac{1}{1-0,75}\right]^2\right) = 0,75 + 1,25 = 2$$

c) In Example 6.9 of the book we find $\overline{p}_1 = 0,5$ which verifies the statement. Note, that this is true only if we start in state 1 or 2. The complete Markov chain is *not* ergodic.

d) Calculate the probability to ever return to state 4:

$$\sum_{n=1}^{\infty} f_4^n = f_4 = 0,25 + 0,25 \cdot (1 + 0,25 + 0,25^2 + ...) \cdot 0,25 =$$

$$= 0,25 + 0,25 \cdot \frac{1}{1-0,25} \cdot 0,25 = \frac{1}{4} + \frac{1}{12} < 1$$

Thus, state 4 is transient. The same can be shown for state 3.

6.9
Use Eq. (6.14). The time to return to the same state is (at an average) for p_1: $1/0.19 \approx 5$, for p_2: $1/0.58 \approx 1,7$ and for p_3: $1/0,23 \approx 4,3$, respectively.

6.10

Denote the states according to:

1=In; 2=*M1*; 3=*M2*; 4=*M3*; 5=Out; 6=*M1s*; 7=*M2s*; 8=*M3s*; 9=reject

a) The probability to pass from In to Out is:

$$1{,}0 \cdot (0{,}95 + 0{,}05 \cdot 0{,}8) \cdot (0{,}9 + 0{,}1 \cdot 0{,}8) \cdot (0{,}9 + 0{,}1 \cdot 0{,}8) =$$

$$= 0{,}99 \cdot 0{,}98 \cdot 0{,}98 = 0{,}951$$

So, one has to start with at least 106 articles in order to get 100 approved.

b) The probability to arrive at M2 is $(0{,}95 + 0{,}05 \cdot 0{,}8) = 0{,}99$.

The probability to pass from M2 to M3 is $(0{,}9 + 0{,}1 \cdot 0{,}8) = 0{,}98$

The probability to be rejected after that the product has arrived at M3 is $0{,}1 \cdot 0{,}2 = 0{,}02$

Conclusion: The probability to reach M3 and then be rejected is $0{,}99 \cdot 0.98 \cdot 0{,}02 = 0{,}019$

c) The transition probability matrix is

$$\mathbf{P} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0{,}95 & 0 & 0 & 0{,}05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}9 & 0 & 0 & 0{,}1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}9 & 0 & 0 & 0{,}1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0{,}8 & 0 & 0 & 0 & 0 & 0 & 0{,}2 \\ 0 & 0 & 0 & 0{,}8 & 0 & 0 & 0 & 0 & 0{,}2 \\ 0 & 0 & 0 & 0 & 0{,}8 & 0 & 0 & 0 & 0{,}2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

6.11

a) The state are "under repair" =1, "operating" =2. The transition probability matrix is

$$P = \begin{pmatrix} 0{,}6 & 0{,}4 \\ 0{,}2 & 0{,}8 \end{pmatrix}$$

b) According to (6.8): $\mathbf{p}(0) = (0 \quad 1)$; $\mathbf{p}(2) = \mathbf{p}(0) \cdot \mathbf{P}^2 = (0 \quad 1) \cdot \begin{pmatrix} 0{,}44 & 0{,}56 \\ 0{,}28 & 0{,}72 \end{pmatrix} = (0{,}28 \quad 0{,}72)$

c) Stationary solution: $\mathbf{p}(\infty) = \bar{\mathbf{p}} = \begin{pmatrix} 1/3 & 2/3 \end{pmatrix}$

Hint: Use the Matlab code `markovchain` to verify your results!

6.12

b) The transition probability matrix is

$$P = \begin{pmatrix} 0.7 & 0.3 \\ 0.1 & 0.9 \end{pmatrix}$$

The stationary probabilities are $\bar{\mathbf{p}} = \begin{pmatrix} 0{,}25 & 0{,}75 \end{pmatrix}$.

c) According to (6.14) the average time to return to the state *operating* is 1/0,75=1,33 time units. It can also be calculated using (6.13).

d) The availability is defined as $\bar{p}_2$ i.e. 0,75.

6.13

The Markov chain is ergodic. The stationary solution is $\bar{\mathbf{p}} = \begin{pmatrix} 0{,}372 & 0{,}149 & 0{,}479 \end{pmatrix}$

Hint: Use the Matlab code `markovchain` to verify your results!

6.14

The transition probability matrix is

$$\mathbf{P} = \begin{pmatrix} q_0 & q_1 & q_2 & \cdots & q_m \\ 1 & 0 & 0 & \ldots & 0 \\ 1 & 0 & 0 & \ldots & 0 \\ .. & .. & .. & .. & .. \\ 1 & 0 & 0 & .. & 0 \end{pmatrix}$$

6.15

See Sections 6.5.3 and 6.5.4. The *superposition* principle is valid, which means that the sum of many Poisson processes is still a Poisson process. A Poisson process can also be decomposed into many processes, where each one of the outputs is a Poisson process. Furthermore, the propagation of the probabilities is described by a linear differential equation.

<u>Comment</u>: It is true that a Markov process has no memory. This, however, is not sufficient to guarantee than the system is linear. Note, that a system described by the non-linear differential equation $\frac{dx}{dt} = f(x,u)$ has no memory. It stores all its history in the state *x*.

6.16

This is a birth process, that can be described by the Figure 6.16. Define p_k(t) as the probability that there are *k* jobs in the system at time *t*. Then the Markov process can be described by

$$\frac{d}{dt} p_0 = -\lambda p_0$$

$$\frac{d}{dt} p_k(t) = \lambda p_{k-1} - \lambda p_k$$

In analogy with Example 6.16 in the book we find that the general solution can be found analytically:

$$p_k(1) = \frac{(\lambda)^k}{k!}e^{-\lambda} = \frac{(2)^k}{k!}e^{-2} \text{ which implies } p_0(1) = e^{-2} = 0{,}135$$

$$p_1(1) = \frac{2^1}{1!}e^{-2} = 0{,}27\,,\ p_2(1) = \frac{2^2}{2!}e^{-2} = 0{,}27\,,\ p_3(1) = \frac{2^3}{3!}e^{-2} = 0{,}18$$

The probability that more than 3 articles have arrived after 1 minute is equal to

$$p_4 + p_5 + p_6 + \ldots = 1 - p_0 - p_1 - p_2 - p_3$$

$$= 1 - 0{,}14 - 0{,}27 - 0{,}27 - 0{,}18 = 0{,}14$$

The expected number of articles is $E\{N(t)\} = \lambda t = 2$ with the variance $\mathrm{var}\{N(t)\} = \lambda t = 2$, i.e. the standard deviation $\sqrt{2} = 1{,}4$.

6.17

This is a pure birth process, where the probability $p_n(t) = (\lambda t)^n e^{-\lambda t} / n!$ is defined as the probability that the system contains *n* pieces at time *t*. In our case $\lambda = 1$ and we calculate
$p_o(2) = 0{,}135;\quad p_1(2) = 0{,}271;\ p_2(2) = 0{,}271;\quad p_3(2) = 0{,}180$
The prob. to have *at least* 2 articles is $1 - p_0 - p_1 = 0{,}59$
The prob. to have *at least* 4 articles is $1 - p_0 - p_1 - p_2 - p_3 = 0{,}14$

6.18

This is a birth process. We define $p_k(t)$ as the probability that there are *k* jobs in the system at time *t*. Here $\lambda = 1$. The probability that there are more than 2 jobs in the system at time $t = 1$ is

$$p_3(1) + p_4(1) + p_5(1) + \ldots = 1 - p_0(1) - p_1(1) - p_2(1) = 1 - e^{-1} - e^{-1} - 0{,}5 * e^{-1} = 0.080$$

6.19

This is a continuous Markov process with the generator

$$A = \begin{pmatrix} -r & r \\ f & -f \end{pmatrix}$$

where $f = 0{,}1$. The stationary solutions are $p_1 = 0.1/(0.1+r)$; $p_2 = r/(0.1+r)$; .
Given that $p_1 < 0{,}05$ we get the required repair rate $r > 1{,}9$. This means that the machine has to be repaired in about half a day at an average.

6.20

Define: p_1 = the prob. that the machine is under repair, p_2 = the prob. that the machine is operating. The failure probability for the machine is $0{,}5dt$ which is the same as to say that the time between errors is an exponentially distributed stochastic variable with the average 2, while the time for repair has the average 3. The continuous Markov process can be modeled as:

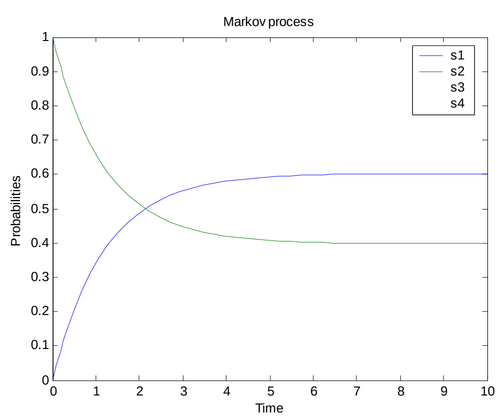
$$\frac{dp_1}{dt} = -0{,}33 p_1 + 0{,}5 p_2$$
$$\frac{dp_2}{dt} = 0{,}33 p_1 - 0{,}5 p_2$$

$$\frac{d\mathbf{p}}{dt} = \mathbf{p}(t)\cdot \mathbf{A} = \mathbf{p}(t)\cdot \begin{pmatrix} -0{,}33 & 0{,}33 \\ 0{,}5 & -0{,}5 \end{pmatrix}$$

The initial condition is $\mathbf{p}(0) = (0\ 1)$. The general solution is $\mathbf{p}(t) = \mathbf{p}(0)e^{\mathbf{A}t}$:

$\mathbf{p}(5) = (0.59 \quad 0.41)$ --> the prob. for operation is 41%
$\mathbf{p}(\infty) = \overline{\mathbf{p}} = (0.60 \quad 0.40)$ --> the prob. for operation is 40%

The Matlab code `markovprocess` generates the following figure, showing how the state probabilities propagate as function of time.



6.21

a) 7·2·2 = 28 states
b) *M1* can produce when it is not blocked or failed, i.e. 12 states.
c) See Eq. (6.60) in the book.

6.22

(a) 20 states
(b) Both the machines must be operational. The buffer most not be empty or full, i.e. 3·1·1 = 3 states.
(c) $$\frac{dp_{311}}{dt} = -(\mu_1 + \mu_2 + f_1 + f_2)\cdot p_{311} + \mu_1 p_{211} + \mu_2 p_{411} + r_1 p_{301} + r_2 p_{310}$$
No machine is blocked or starved.
(d) M_2 is allowed to be both operating and under repair. The buffer may not be full.

$$E_1 = \sum_{n=0}^{3} \sum_{\alpha_2=0}^{1} p_{n,1,\alpha_2}$$

For a very large buffer we have according to (6.55):

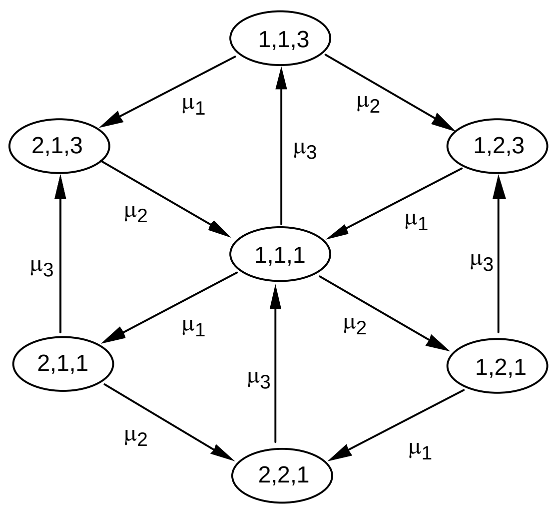
$$e_1 = \frac{r_1}{r_1 + f_1} = \frac{0,2}{0,21} = 95\%$$

It is true that $E_1 \le e_1$ since the machine cannot be blocked or starved if the buffer is very large.

(e) See Eq. (6.60). The dimension is $time^{-1}$

6.23

Machines 1 and 2 can be in the states 1 or 2. Machine 3 can be in the states 1 or 3. This makes 8 states, called s(m_1, m_2, m_3) or s(111), s(113), s(121), s(123), s(211), s(213), s(221), and s(223). However, we can never reach s(223). Consequently the system has 7 states that can be represented in the diagram below.



c) The efficiencies are

E(m₁) = p(113) + p(123) + p(111) + p(121)

E(m₂) = p(113) + p(213) + p(111) + p(211)

E(m₃) = p(111) + p(211) + p(121) + p(221)

d) The production rate is $\mu_3 E(m_3)$

6.24

a) 32 states

b) Consider:

- a machine under repair cannot produce;
- a starved or blocked machine can not produce;
- the number of articles in each buffer can be 0 or 1.

Neighboring states to s(0,1,1,1,0) are the following:

s(0,1,0,1,0) – *M1* and *M3* under repair
s(0,1,1,0,0) - *M2* and *M3* under repair
s(0,1,1,1,1) – all machines operating
s(0,0,1,1,0) - both *M1* and *M2* are operating but are starved.
s(1,1,1,1,0) - both M1 and M2 are blocked, M3 is under repair

$$\frac{d}{dt}p(01110) = -(f_1 + f_2 + r_3 + \mu_1)p(01110) + r_1 p(01010) +$$
$$+ r_2 p(01100) + f_3 p(01111)$$

c) $\mu_1 \cdot p(0,*,1,*,*)$, i.e. the sum of all arguments 2, 4 and 5.

6.25

(a) The system has *n*+1 states
(b) Call the states $P(k_1, k_2) = P_{k_1 k_2}$. For n=3 we denote the states

$p_{30}, p_{21}, p_{12}, p_{03}$. The state equations are:

$$\frac{dp_{30}}{dt} = -\mu_1 p_{30} + \mu_2 p_{21}$$
$$\frac{dp_{21}}{dt} = \mu_1 p_{30} - (\mu_1 + \mu_2)p_{21} + \mu_2 p_{12}$$
$$\frac{dp_{12}}{dt} = \mu_1 p_{21} - (\mu_1 + \mu_2)p_{12} + \mu_2 p_{03}$$
$$\frac{dp_{03}}{dt} = \mu_1 p_{12} - \mu_2 p_{03}$$

6.26

NOTE: There is a misprint in the problem formulation:
It reads: Calculate the average lead time …
It should read: Calculate the average number of operating machines if $q = 0$ and for $q = 0.2$. This number is defined as: $L = \sum_{n=0}^{M} n * p_n$

As a preparation for this example, study Example 6.17. Consider this problem a birth-death process. We refer to Figure 6.20 as an illustration of the process.

Assume that state s_k is defined as k machines are in operation. Thus s_0 means that all the M machines are failed. State s_M means that all the machines are in operation. Referring to Figure 6.20 we can see that the state increases from s_{k-1} to s_k with the rate r. All the λ in Figure 6.20 are here called r.

If only 1 machine is operational, then the failure rate is *f*. Is *k* machines are operational, then the failure rate is *k* times larger, *k*f*. The quality control means that a fraction *q* of 1 machine getting repaired is returned. Another way to express this is that the failure rate for each state will increase with *q*r*. In other words,
$\mu_1 = f+q*r$; $\mu_2 = 2f+q*r$; …. $\mu_M = Mf+q*r$

According to (6.57) we have:

$p_{k+1} = r*p_k/(k*f+q*r)$

Then

$p_1 = r/(f+qr)p_0$; $p_2 = r*p_1/(2f+qr)$; …. $p_M = r*p_{M-1}/(Mf+qr)$

Now we insert the values $M = 5$, $r = 10$, $f = 2$, $q = 0$:

We get:

$p_1 = 5*p_0$; $p_2=10*p_1/4$; $p_3 =10*p_2/6$; $p_4 =10*p_3/8$; $p_5 =10*p_4/10$;

Knowing that the sum $p_0 + … + p_5 = 1$ we can calculate p_0, which gives:

$p_0 = 0.01$; $p_1 = 0.05$; $p_2 = 0.14$; $p_3 = 0.23$; $p_4 = 0.285$ $p_5 = 0.285$

The average number of operating machines can be calculated as the sum

$$L = \sum_{n=0}^{M} n * p_n = 3.41$$

Comment:
You can also calculate the probabilities by using the Matlab code `markovprocess`. Verify that the generator **A** (for $q = 0$) is:

-10	10	0	0	0	0
2	-12	10	0	0	0
0	4	-14	10	0	0
0	0	6	-16	10	0
0	0	0	8	-18	10
0	0	0	0	10	-10

For $q = 0.2$ we the generator A is:

-10	10	0	0	0	0
4	-14	10	0	0	0
0	6	-16	10	0	0
0	0	8	-18	10	0
0	0	0	10	-20	10
0	0	0	0	12	-12

The steady state solution then becomes:

$p_0 = 0.045$; $p_1 = 0.11$; $p_2 = 0.186$; $p_3 = 0.23$; $p_4 = 0.23$ $p_5 = 0.19$

$L = 3.08$

Comment:
Verify that (6.57) is still valid.

Chapter 7 Queuing Systems

For computational help for queuing systems you may use the Matlab m-files `mmmqueue` and `mmmkqueue` respectively. The first one solves key parameters for M/M/1 and M/M/m queuing systems while the second one solves the M/M/1/K and M/M/m/K problems. They are available for downloading on the homepage www.iea.lth.se/aut.

7.1
Queuing systems are special cases of birth-death processes. They in turn are special cases of Markov processes, that are governed by $d\mathbf{p}/dt = \mathbf{pA}$. For the queuing systems we consider only the steady state condition of the Markov process.

7.3
The system can be described by an *M/M/*1-queue. We have $\lambda = 4$ (per hour), $\mu = 6$ (per hour).

a) The utilization rate $\rho = \lambda / \mu = 2/3$. The number of articles waiting in the queue: $\bar{L}_q = \dfrac{\rho^2}{1-\rho} = 1,33$. The waiting time in the queue: $W_q = \dfrac{\rho}{\mu - \lambda} = \dfrac{2}{3(6-4)} = \dfrac{1}{3}$ (h) = 20 minutes

b) From $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)} = \dfrac{4}{\mu(\mu-4)} = \dfrac{1}{12}$ we get $\mu = 2 + \sqrt{52} \approx 9,2$ (per hour). The new operation time thus has to be 1/9,2 h = 6,5 minutes.

7.4

This is a D/D/1 system, where the time between arrivals is exactly 3 minutes and the operation time exactly 4 minutes. Then we can write in a table what happens during the first 30 minutes. Only the times when something happens are marked.

Time minutes	Article number being served	Article number waiting in line
0	..	..
3	1	..
6	1	#2
7	2	..
9	2	#3
11	3	..
12	3	#4
15	4	#5
18	4	#5, #6
19	5	#6
21	5	#6, #7
23	6	#7
24	6	#7,#8
27	7	#8, #9
30	7	#8, #9, #10

We can see that there is no queue during the time intervals 0-6, 7-9, 11-12, i.e. during 9 minutes. There is 1 article in the queue during the time intervals 6-7, 9-11, 12-18, 19-21, 23-24, i.e. during 12 minutes. In the same way we see that there are 2 articles waiting during 9 minutes and 3 waiting during 0 minutes (during the period 0-30 minutes). The average length then is calculated to $\frac{0 \cdot 9 + 1 \cdot 12 + 2 \cdot 9 + 3 \cdot 0}{30} = 1$ article.

Note that the arrival rate is larger than the service rate. This means that the queue will grow indefinitely.

7.5

It is an M/M/1 queue. a) $L_q = 2{,}25$; b) $W_q = 4{,}5$ minutes; c) $p_0 = 0{,}25$.

7.6

ρ will not be changed. Therefore L and L_q will be the same, while the waiting time is halved.

7.7

We consider the M/M/1/4 problem.

a) $\lambda = 20$. The utilization is $\rho = 2/3$. $\overline{L} = 2 - \frac{5 \cdot 32}{211} = 1{,}24$

$p_4 = \frac{16}{211} = 0{,}076$. $\bar{\lambda} = 20 \cdot (1 - 0.076) = 18{,}5$ (i.e. at an average one must turn away 20 - 18,5 = 1,5 articles per hour).

$W = \frac{1{,}24}{18{,}5} = 0{,}067$ (h) = 4 minutes

$W_q = 0{,}067 - \frac{1}{30} = 0{,}034$ (h) = 2 min

b) $\lambda = 30$ implies $\rho = 1$. The average number of articles in the system is $\bar{L} = 2$

$p_4 = \frac{1}{5} = 0{,}2$ $\bar{\lambda} = 30 \cdot (1 - 0.2) = 24$.
At an average one must turn away 30 - 24 = 6 articles per hour .

$W = \frac{2}{24} = 0{,}083$ h = 5 minutes

$W_q = 0{,}083 - 0{,}033 = 0{,}05$ h = 3 minutes

The reason that the service time is still quite reasonable when $\rho = 1$ is of course, that the queue is not allowed to be larger than 4 articles.

For an M/M/1 queue we know that the queue length will grow to infinity when ρ=1.

7.8

We simply set up a table for the job arrivals and the completion time of the operations. Here we have assumed, that operator #1 will always stay with the job, so operator #2 cannot take over his job. Therefore operator #2 cannot leave if operator #1 happens to be idle, while operator #2 is busy.

time	arrival	op 1 finished	op 2 finished	# jobs	# jobs in the q				
5	1			1	0				
10	1			2	1				
13		1		1	0				
15	1			2	1				
20	1			3	2				
21		1		2	1				
25	1			3	2				
29		1		2	1				
30	1			3	2				
35	1			4	2	op 2 arrives			
37		1		3	1				
40	1			4	2				
43			1	3	1				
45	1	1		3	1				
50	1			4	2				
51			1	3	1				
53		1		2	0	op 2 still busy			
55	1			3	1				
59			1	2	0	op 2 still busy			
60	1			3	1				
61		1		2	0				
65	1			3	1				
67			1	2	0				
69		1		1	0	op 2 still busy, op 1 idle (not allowed to leave)			
70	1			2	0				
75	1		1	2	0				
77		1		1	0	op 2 still busy, op 1 idle (not allowed to leave)			
80	1			2	0				
83			1	1	0	op 2 can leave			
85	1	1		1					

a) The second operator arrives at 35 minutes and leaves at 83 minutes. In other words, he works for 48 minutes.

In order to calculate the average number of jobs in the system we simply set up the number of jobs on a minute-by-minute basis:

minutes	#jobs	# queue	
1	0	0	
2	0	0	
3	0	0	
4	0	0	
5	1	0	
6	1	0	
7	1	0	
8	1	0	
9	1	0	
10	2	1	
11	2	1	
12	2	1	
13	1	0	
14	1	0	
15	2	1	
16	2	1	
17	2	1	
18	2	1	
19	2	1	
20	3	2	
21	2	1	
22	2	1	
23	2	1	
24	2	1	
25	3	2	
26	3	2	
27	3	2	
28	3	2	
29	2	1	
30	3	2	
31	3	2	
32	3	2	
33	3	2	
34	3	2	
35	4	2	
36	4	2	
37	3	1	
38	3	1	
39	3	1	
40	4	2	
41	4	2	
42	4	2	
43	3	1	
44	3	1	
45	3	1	

46	3	1	
47	3	1	
48	3	1	
49	3	1	
50	4	2	
51	3	1	
52	3	1	
53	2	0	
54	2	0	
55	3	1	
56	3	1	
57	3	1	
58	3	1	
59	2	0	
60	3	1	
61	2	0	
62	2	0	
63	2	0	
64	2	0	
65	3	1	
	2,53	1,03	average from t=5 including t=64

From this we calculate the average number of jobs in the system (work in progress) to be 2,53 and the number of jobs in the queue to 1,03. Note that more than 1 job is in operation at an average.

7.9

This is an *M/M/*1 system. $\lambda = 24\ h^{-1}$, $\mu = 30\ h^{-1}$ which implies $\rho = 24/30 = 0,8$.

(a) According to (7.18) we get $\overline{L} = \dfrac{\rho}{1-\rho} = \dfrac{0,8}{1-0,8} = 4$ jobs at the station.

(b) According to (7.24): $W = \dfrac{1}{\mu-\lambda} = \dfrac{1}{30-24} = \dfrac{1}{6}\,(\text{h}) = 10\,(\text{minutes})$

(c) The operator is idle if there is no product in the system. The probability for this is, according to (7.17) : $p_0 = 1-\rho = 0,2.$ He is idle at an average 20 % of the time.

(d) Eq. (7.23) gives the average waiting time in the queue:

$$W_q = \frac{\rho}{\mu-\lambda} = \frac{0,8}{30-24} = 0,133\,(\text{h}) = 8\,(\text{min})$$

(e) According to Eq. (7.25) we can calculate for t = 10 min = 1/6 hour:

$$W_q\left(\frac{1}{6}\right) = 0,8 \cdot e^{-10/10} = 0,8 \cdot e^{-1} = 0,29$$

7.10

This is an *M/M/*2 system with $\lambda = 100$ and $\mu = 60$ which implies $\rho = \frac{100}{2 \cdot 60} = \frac{5}{6} < 1$.

Thus, it is possible to reach stationarity. Use (7.38) to calculate

$$\frac{1}{p_0} = \frac{2^2 (5/6)^3}{2!\left[1-(5/6)\right]} + \sum_{n=0}^{2} \frac{(5/3)^n}{n!} = \frac{125}{18} + 1 + \frac{5/3}{1} + \frac{(5/3)^2}{2!} = 11$$

i.e. $p_0 = 1/11 = 0,0909$.

Similarly we use (7.39) to calculate the remaining probabilities:

$$p_1 = \frac{(5/3)}{1} \cdot \frac{1}{11} = 0,152 \qquad p_2 = \frac{(5/3)^2}{2!} \cdot \frac{1}{11} = 0,126$$

$$p_3 = \frac{2^2 (5/6)^3}{2!} \cdot \frac{1}{11} = 0,105$$

(a) The probability for more than 3 jobs in the system is

$$1 - (p_0 + p_1 + p_2 + p_3) = 1 - (0,091 + 0,151 + 0,126 + 0,105) = 0,526$$

(b) A given machine is idle, if there are no products to service in the cell, or if there is one product in the system and this is serviced by the other machine. The probability for this is

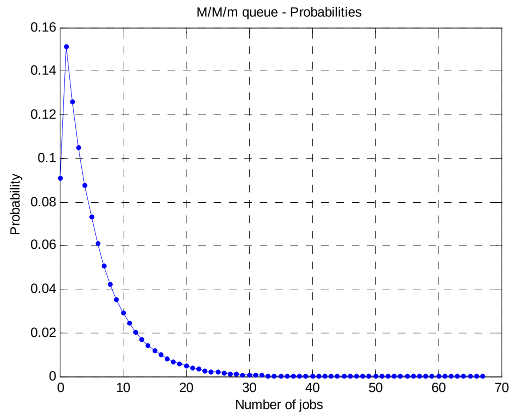
$$p_0 + \frac{1}{2} p_1 = 0,167$$

Comment:

We execute the Matlab program `mmmqueue` and find:

L = 5.45; Lq = 3.79; Wq = 0.038; W = 0.055;

(as a check, calculate W – Wq and compare with 1/μ). The probabilities are plotted in the figure below.



7.11

This is an M/M/1/6 problem with $\rho = 1$. We have that $p_n = 1/(K+1) = 1/7$

(a) We find $\overline{L} = K/2 = 3$. The average arrival rate, taking the refusals into consideration is $\overline{\lambda} = \lambda \cdot (1 - p_6) = 60/7$

Little's theorem gives that $W = \overline{L}/\overline{\lambda} = 3 \cdot 7/60 = 0{,}35$ hours = 21 minutes

b) $W_q = W - 1/\mu = 0{,}35 - 0{,}1 = 0{,}25$ hours or 21 – 6 minutes = 15 minutes.

c) W = 21 minutes

$\overline{L}_q = W_q * \overline{\lambda} = 0{,}25 \cdot 60/7 = 2{,}1$

b) The average service time is 21-15 minutes = 6 minutes, which is expected. 15 minutes

c) $p_0 = 1/7$

$p_6 = 1/7$, i.e. 14,3 % is refused

7.12

This is an M/M/1/K problem and we have to determine the size of *K*. We need to be able to service 90 % of the jobs. This means than p_K has to be less than 10%. From (7.52) we can calculate p_K. A system of *K* jobs requires *K*-1 m^2 storage space. We now calculate (for example in Matlab). We calculate $\rho = 25/30$.

K=2: $p_0 = 0{,}40$ $p_1=0{,}33$ $p_2=0{,}27$ (note that the sum is 1)
K=3: $p_3=0{,}186$
K=4: $p_4=0{,}134$
K=5: $p_5=0{,}1007$
K=6: $p_6=0{,}077$

Thus, we need *K*=6 to make the refusal rate less than 10%, and the required space is **5** m^2.
As an extra check we calculate all the probabilities for *K*=6:

$p_0=0{,}231$	$p_1=0{,}193$	$p_2=0{,}161$
$p_3=0{,}134$	$p_4=0{,}112$	$p_5=0{,}093$
$p_6=0{,}077$		

The sum $p_0 + ... + p_6 = 1$

7.13

This is an open queuing network. Note that each machine can be considered as an M/M/1 queuing system. The given parameters are $\lambda = 0,5$; $\mu_1 = 1$; $\mu_2 = 2$, which implies $\rho_1 = 0,5$ and $\rho_2 = 0,25$. Then we can calculate $\overline{L}_1 = \frac{\rho_1}{1-\rho_1} = 1$ and $\overline{L}_2 = \frac{\rho_2}{1-\rho_2} = \frac{1}{3}$. Little's theorem gives: $W_1 = 2$ minutes and $W_2 = \frac{2}{3}$ minutes. The total time in the system is: $W = W_1 + W_2 = \frac{8}{3}$ minutes. Note that $W = \frac{L_1 + L_2}{\lambda}$.

7.14

Alternative (i): This is an *M/M*/1 system with $\lambda = 6$ (per hour) and $\mu = 10$, i.e. $\rho = 0,6$.

$$L_q = \frac{0,6^2}{0,4} = 0,9$$

The average waiting time in the queue is $W_q = \frac{0,6}{10-6} = 0,15$ (h) = 9 minutes;

The lead-time: $W = 0,15 + 1/10 = 0,25$ h = 15 minutes.

Alternative (ii): This is an *M/M*/2 system with $\lambda = 6$ (per hour) and $\mu = 5$, i.e. $\rho = 0,6$. We calculate the average queue length. We have $p_0 = 0,25$. $L_q = 0,675$

The average waiting time in the queue: $W_q = \frac{L_q}{\lambda} = \frac{0,675}{6} = 0,113$ h = 6,8 minutes.

The lead time: $W = 0,113 + 1/5 = 0,313$ h = 19 minutes

To summarize :

a) the average queue length is 0,9 for 1 machine and 0,68 for 2 machines

b) the lead-time for 1 machine is 15 minutes, and 19 minutes for 2 machines. With **two** identical machines there is a shorter waiting time in the queue. When the job arrives to a machine the average service time is 12 minutes. With **one** fast machine the waiting time in the queue is longer than for two machines, while the operational time is shorter, which leads to a shorter lead-time.

c) Two separate queues implies $\rho = \frac{3}{5} = 0,6$. Each machine can be considered a separate *M/M*/1 system. The lead-time is $W = \frac{1}{5-3} = 0,5$ h = 30 minutes. This takes a longer time.

d) 2 or more jobs in the queue means 3 or more jobs in the system. From (7.17) we get $p_0 = 0,4$, $p_1 = 0,4 \cdot 0,6 = 0,24$, $p_2 = 0,144$. The prob. for 2 or more jobs in the queue is $1 - p_0 - p_1 - p_2 = 0,216$

7.15

The arrival rates are $\lambda_n = \lambda$ and the operation rate is $\mu_n = n^\alpha \mu$

We have a birth and death process with

$$p_n = \frac{(\lambda/\mu)^n}{(n!)^\alpha S} \text{ where } 1/p_0 = S$$

and $S = \sum_{n=0}^{\infty} \frac{(\lambda/\mu)^n}{(n!)^\alpha}$

Chapter 8 Simulators

8.1

Explanation: b) - Analytical modeling is not always faster than simulation but for any system with a practical use (relatively complex) simulation modeling becomes very computationally expensive. Because of this it is clearly much slower for large systems.

Chapter 10 Measurement processing

10.1

a) Study Section 10.2.2. We find that the alias frequency is $f_0 = f_s - f$. With $f_0 = 1$ and $f_s = 8$ we have the smallest signal frequency 7 Hz.
b) We have $f_s = 70$ and $f = 50$. Therefore the alias frequency is 20 Hz
c) The sampling frequency has to *at least* 2 times higher than the highest frequency component, i.e. 100 Hz

10.2

The Shannon sampling theorem assumes that there is an infinite measurement time.

10.3

a) A moving average: all $a_i = 0$. $b_0 = \ldots = b_4 = 1/5$. $b_k = 0$ for $k > 4$
b) Exponential filter: $a_1 = -\alpha$; $b_0 = 1 - \alpha$; All the other coefficients = 0.
c) Hint: see Section 10.4.3.
d) Hint: see Equation 10.15.

10.4

Hint: see Example 10.8 in the textbook

10.5

Hint: see Example 10.8 in the textbook

10.6

See Equation 10.12 in the textbook

10.7

Assume that y changes from 0 to 1 at time 0, which means y(0) = 0 and y(h) = 1. The subsequent y values are 1. The step response for the moving average filter will change 0.1 per time step. This means that it takes 6.3 times steps to reach 63% of the step size.

Chapter 13 Combinatorial and sequencing control

13.1

Remember that the state concept is of fundamental interest in sequencing control

13.3

Hint: study the concept of semaphores.

13.4

The combinatorial network has no memory, and the output is an algebraic (Boolean) expression of the input, $y = f(u)$. In a sequencing network there is a memory, and the system can appear in more than one state.

Chapter 14 Production control

14.2

Consider *n* jobs with the processing times t_i respectively. Then the average waiting time can be expressed as

$$\frac{1}{n}\left[(t_1) + (t_1 + t_2) + \ldots + (t_1 + t_2 + \ldots + t_n)\right]$$

It is obvious that the average time would be shortest if the shortest job is placed first, since the first job waiting time will have the largest weight.

14.5

Compare problem 14.2.

14.6

NOTE, misprint: j_1 has the delivery time 3.

a) The operational order will be j_3--j_2--j_5--j_4--j_6--j_1. The MFT is 75/6 = 12,5. The mean lateness is (here we only consider real delays) 26/6 = 4,3. Only j_1 is delayed.
b) Jobs j_1 and j_6 have the weight 2, while all the other jobs have the weight 1. The operational order will be j_3--j_2--j_6--j_5--j_1--j_4. The MFT is 84/6 = 14. The mean lateness (average of true delays) is 34/6 = 5,7. Jobs j_1 and j_4 are delivered too late.
c) The mean lateness is now 29/6 = 4,8

Comment: In the solution above we have defined, that a delivery is late only if there is a positive delay between delivery time and the true delivery. If the delivery is ahead of the required delivery time we have said that the delay is 0 instead of negative. In the textbook we have said that the delivery can be *both positive and negative*. This is the assumption behind the expression (14.10) in the textbook. Only in the case of both positive and negative delays the mean lateness *L* is minimized if the MFT is minimized.

14.7

a) If the jobs are processed in the order $j_1 - j_2$ -- -- j_6 then the total time is 45 minutes. Machine 1 is idle for 10 minutes and machine 2 is also idle for 10 minutes. The average waiting time is 161/6 = 26.8 minutes.
Using Johnson's procedure we find the order j_4—$j_5 - j_2 - j_6 - j_3 - j_1$. The total processing time is now 37 minutes. Machine 1 and 2 are both idle during 2 minutes. The average waiting time is 143/6 = 23,8 minutes

14.8

a) Using Johnson's algorithm we get the right job sequence j_6--j_3--j_4--j_1--j_5--j_2
b) 37 time units. Machine 1 has to wait 11 time units at the end.
c) 32 time units. Machine 1 has to wait 6 time units
Comment to b) and c): Make a Gantt diagram to find out the total time.

14.9

The jobs have to be ordered like $j_5 - j_1 - j_4 - j_3 - j_2$. It takes 47 minutes.

14.10

(a) The jobs have to be ordered in the sequence $j_2 - j_5 - j_4 - j_3 - j_1$. The last job will be finished after 36 minutes.
(b) Machine 2 is idle during 0-3 and 9-10 minutes, while machine 1 is idle during 33-36 minutes.
(c) The total time will be 46 minutes.

14.11

This is a 2/*m* problem and we use a graphical method like Figure 14.3 to solve the problem. The following sequence can be derived from the figure:

- Both P1 and P2 work for 3 time units
- At *t*=3 both P1 and P2 request A. Since A was requested first by P1 the job P2 has to wait for 2 time units, while P1 finishes on A
- P1 uses E for 2 time units, then P2 also requests E and has to wait for 2 time units
- While P1 uses machine F the machine E is used by P2 for 5 time units. P1 is finished on F just in time for P2 to request F
- P1 uses D for 5 time units when P2 is requesting D. Again P2 has to wait for 1 time unit.

To summarize:

- 3 machines are in conflicting situations, A, E, and D. Both P1 and P2 request them at the same time.
- P1 has not been forced to wait anything. It will be completed in 21 time units.
- P2 has been waiting for 2 (machine A) + 2 (machine E) + 1 (machine D) = 5 time units. It will be completed in 18+5 time units.

Chapter 15 Real time control of dynamical systems

15.1

a) At this stage M_1 is blocked because the AGV is not available. The AGV will not become available unless M_2 is free, but M_2 cannot be free until the AGV is available. This has become a deadlock situation.
b) Four necessary conditions have been identified for the occurrence of deadlocks
a. Mutual exclusion – a resource (here the AGV) cannot be used by two or more processes (steps in the manufacturing process) simultaneously;
b. No preemption – when a resource is being used, it is not released unless the process using it finishes with it;
c. Hold and wait – there must exist a process that is holding at least one resource and is waiting to acquire additional resources that are currently being held by other processes;
d. Circular wait – there must exist a set of waiting processes (p_1, p_2, …, p_n) such that p_1 is waiting for a resource held by p_2; p_2 is waiting for a resource held by p_3;… p_{n-1} is waiting for a resource held by p_n; and p_n is waiting for a resource held by p_1.

The two machines and the AGV are the resources. Mutual exclusion is satisfied since each one of these can only handle one job at a time. Since the resource cannot be released unless the parts holding it leave it, the no-preemption condition is satisfied. The hold-and-wait is satisfied since part p_1 is holding M_2 and waiting for the AGV. Part p_2 is holding the AGV and waiting for M_2, and part p_3 is holding M_1 and waiting for the AGV. The circular wait is established in the form of p_2 waiting for p_3, and p_3 waiting for p_2.

15.2

Pre-emption means that any execution can be interrupted. In real time execution this means that the current state has to be temporarily stored (context switching). In manufacturing such a context switching means that material has to be moved, and not only information. This will of course both take unnecessarily long time and will influence the product quality. For example, we let a drilling operation work until completion.

15.12

Think about non-interrupted sequences of instructions!