A new criterion to distinguish stochastic and deterministic time series with the Poincaré section and fractal dimension

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In this paper, we propose a new method for detecting regular behavior of time series: this method is based on the Poincaré section and the Higuchi fractal dimension. The new method aims to distinguish random signals from deterministic signals. In fact, our method provides a pattern for decision making about whether a signal is random or deterministic. We apply this method to different time series, such as chaotic signals, random signals, and periodic signals. We apply this method to examples from all types of route to chaotic signals. This method has also been applied to data about iris tissues. The results show that the new method can distinguish different types of signals. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3096413]

The main contribution of this paper is a new method to discriminate random and deterministic signals. This new method is based on two specific signal features. The test cases include random signals, deterministic signals, and combinations thereof. These test cases were sufficient to test the method's performance. This method was also applied to a real biological signal, yielding results that are consistent with medical experiments.

I. INTRODUCTION

Nonlinear signal processing is an important research area with many applications. Specifications and identifications of nonlinear signals can help us to detect nonlinear behavior of dynamical systems. One specification, the discrimination of stochastic and deterministic behaviors of nonlinear time series, is a basic topic in nonlinear dynamic fields.[1] This specification has attracted researchers for a long time.[2]

Some studies have shown that when fractal dimension is a function of the state-space dimension, deterministic properties of a system can be determined by low values of fractal dimension, and stochastic properties of a system can be determined by high values of fractal dimension.[3–5] Although this idea has seen wide use, some researchers have cast doubt on the usefulness of the fractal dimension for identifying the behavior of time series.[6–8] The fractal dimension indicates the amount of complexity and repeatability in different scales.[9] Among different versions of fractal dimension that are used for complexity estimation, the Higuchi fractal dimension is a precise and applicable mechanism to estimate self-similarity that also gives a stable value for the fractal dimension.[10–13] The Higuchi fractal dimension is a stable method to compute the fractal dimension of irregular time series of natural phenomena. In contrast with the conventional methods to estimate fractal dimension in the state space, the Higuchi fractal dimension can be calculated directly in the time domain and is therefore simple and fast. It has also been found to estimate samples as short as 150–500 data points reliably.[14] In the past few years, fractal analysis techniques have gained increasing attention in medical signal and image processing. For example, analyses of encephalographic data and other biosignals are among its applications.[15] Fractal complexity of the signal in the time domain (calculated using Higuchi's algorithm) seems to be the simplest method. The same method may also be used in other biomedical applications.[16–18]

Because of the similarity between chaotic and stochastic signals, distinguishing these two types is difficult. To overcome this problem, specification of chaotic signals can be used. Out of the different properties of chaotic signals, we focus on the Poincaré section: a method to detect periodic signals.

In this paper, we propose a new, efficient method to distinguish random signals from deterministic signals based on the Poincaré section and the modified version of the Higuchi fractal dimension: the P&H method. The P&H method has several steps.

The first step is to intersect the time series trajectory with the Poincaré section. This intersection induces a set of points that indicate dynamic flow. Now there is a series (S) specified by these points. Applying the Higuchi fractal dimension to the S series yields a vector $L(k)$. This vector is a basis for decision making about series specification: it suggests typical criteria to distinguish stochastic signals from deterministic signals. We also apply this method to examples from all types of route to chaotic signals. All chaotic systems can be mapped into their examples based on their route to chaos (period doubling, quasiperiodic, or intermittent), and that examples of each of these routes produce a peak in $L(k)$ which shows that these time series are deterministic.

This paper is organized as follows. In Sec. II, we explain Poincaré section. In Sec. III, we describe the Higuchi fractal

