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Surface Matching Degree

[1]Hosein Alizadeh, [2]Hamid Parvin, [3]Hamid Alinejad-Rokny, [4]Naser Mozayani

[1]Computer Engineering Department, Islamic Azad University,
Science and Research Branch, Tehran, Iran.
[2]Computer Engineering Department, Islamic Azad University,
Science and Research Branch, Tehran, Iran.
[3]Computer Engineering Department, Islamic Azad University,
Science and Research Branch, Tehran, Iran.
[4]Department of Computer Engineering, Iran University of Science and Technology,
Tehran, Iran.

Abstract: Generally, the input of a fuzzy system can be a crisp value or a fuzzy linguistic value. The input of most fuzzy systems is a crisp value while a fuzzy linguistic input is employed in a few of them. The Max-Min algorithm is used to identify matching degree, when the input is a fuzzy linguistic input. This method suffers from some drawbacks which are discussed in this paper. An alternative approach which is called Surface Matching Degree (SMD) is proposed. It can manage all mentioned disadvantages of the common approach. This new method which applies an adaptive and parametric equation to determine matching degree can be adapted to any kind of applications.

Key words: Surface Matching Degree; Fuzzy Inference; Adaptive Matching; Max-Min Method.

INTRODUCTION

Fuzzy logic inference is used as inference engine of a fuzzy expert system (Zadeh, A., 1973; Zadeh, 1975; Verikas, 2010; Borroto, 2010). Suppose A'⇒B' to be an *if-then* rule in the rule base of a fuzzy expert system. Inference engine makes a mapping between input fuzzy variable, A′, and output fuzzy variable, B′ (Lee, 1990). A fuzzy *if-then* rule is interpreted as a fuzzy relationship between the input and output of product space, U×V (Wang, Li-Xin. 1997). If the fuzzy rule base only consists of a single rule, the mapping from the fuzzy variable A′ defined in U, to the fuzzy variable B′ defined in V, will be specified by generalized modus ponens (Wang, Li-Xin. 1997; Duboisa, 1984; Gupta, 1985). If the fuzzy rule base consists of more than one rule, then the fuzzy inference engine will infer from a set of rules (Fukami, 1980; Baldwin, 1980; Baldwin, 1980).

There are two ways to infer from a set of rules: (a) composition-based inference and (b) individual-rule-based inference (Mamdani, 1974; Sugeno, 1985; Lee, Chuen, 1990). Generally, the operations are divided into three main parts in an individual-rule-based inference:

(i) To determine the membership degree between the input and the rule-antecedent.
(ii) To compute the rule consequences.
(iii) To aggregate the rule consequences using the fuzzy control-action set (Zimmermann, Hans-Jürgen. 1996; Mamdani, 1977).

At the first step, usually *Max-Min* approach is used (Mamdani, 1974; Mamdani, 1977; Zadeh, 1979).

This approach is discussed in more details in section 2. We address the problems of the *Max-Min* method in section 3. The problems which we discuss in detail in section 3 are our motivation to offer a new alternative method. We propose a new adaptive approach in section 4. This new idea which is originally the results of our works (Alizadeh, 2007; Alizadeh, 2008) is an adaptive surface-based approach. It can be applied to all kinds of inference engines. It improves the performance and the convergence speed, in fuzzy systems, especially, for oscillatory systems. This new method is Surface Matching Degree (SMD). Finally, section 5 concludes this paper.

2. Related Works:

Suppose that A(x) is the antecedent part of a rule in a fuzzy system, and the crisp value u_0 is the input of the system, then the matching degree, α, has been derived from equation 1:

Corresponding Author: Hamid Alinejad-Rokny, Computer Engineering Department, Islamic Azad University, Science and Research Branch, Tehran, Iran
E-mail: alinejad@ualberta.ca

$$\alpha = \mu_{A(u_0)} \tag{1}$$

When the input fuzzy term is A′(x), the matching degree can be obtained by equation 2:

$$\alpha = \max_x \{\mu_{A(x)} \wedge \mu_{A'(x)}\} \tag{2}$$

If we use the *Min* operator for the intersection, the equation 2 will be reformulated into equation 3 (Tsukamoto, 1979; Sugeno, 1983; Wangming, Wu. 1990).

$$\alpha = \max_x \min_x \{\mu_{A(x)}, \mu_{A'(x)}\} \tag{3}$$

3. Problem and Motivations:

Again suppose the fuzzy input term which is the antecedent part of the rule illustrated in Fig1, is denoted by *A*. The inputs of the system u_0 include a crisp zero and three fuzzy triangular term values 1, 2, 3 and 4. The drawbacks of *Max-Min* approach are as follows:

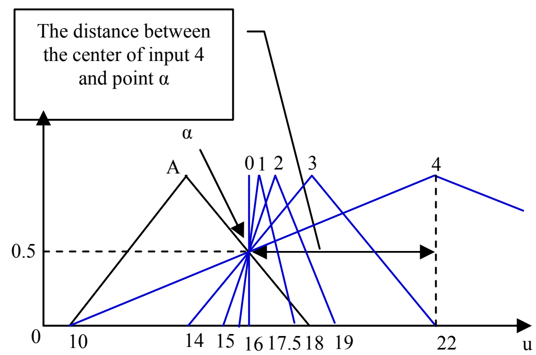

Fig. 1: The draw backs of Max-Min approach.

First, the matching degree is fixed on changes of input membership values. As it is shown in Fig 1, the point α, is the matching degree obtained by *Max-Min* method. Inspire from the fact that the input membership values are different from each other, this point is expected not to be fixed. The membership value for each of the inputs, including the crisp input 0 and fuzzy input terms, 1, 2, 3 and 4, is equal to 0.5. As you can observe the matching degree is fixed by changing the input values.

Suppose that the fuzzy input term is an isosceles. The center of this term is its center of gravity. As it is shown in Fig 1, the inputs 3 and 4 cut the fuzzy input term *A*, in the fixed point α. The input 4 is wider than 3 and therefore is fuzzier than it. Also, the gravity center of the input value 4 is more distant from antecedent term *A* than the gravity center of the input value 4. In the same situation, the fuzziness of the input increases, when the gravity center of input gets away. It should be logical that, the value of matching degree decreases, whatever the input term is more distant from than point α. However, the input 4 is wider and fuzzier than 3, the matching degrees are the same. It is equal to 0.5 for all of those inputs. In reality, in this approach, the distance between center of input and point α, does not influence in determining the matching degree. It is the second challenge of common approach to determine the matching degree.

The highlighted area in Fig 2, denoted by A_1, is the communal area between input value *A′* and fuzzy term *A*. This area is a section of input that can be considered as true part of the rule antecedent *A*. The ratio of communal area to total input area is denoted by *R*. *R* value shows that how much of input ratio is true in the rule. In the same situation, it can be meaningful that the greater value of *R* the greater matching degree. There are some cases between fuzzy input term and the rule antecedent that yield R to be negligible; whereas the matching degree obtained by Max-Min method is significant (like fuzzy input number 4 in Fig 1). It does not take into account the value of input that is true of rule antecedent. It was the third shortcoming of above mentioned approach. This ratio is 0.17 for fuzzy input value 4. It is also 0.58 for fuzzy input value 1. However, while the ratio for each of them is considerably different, the matching degrees for both of them by *Max-Min* method are equal values (here 0.5). Thus, the *Max-Min* approach does not consider *R*.

To sum up, the problem is as follow:

1. Unchanging the matching degree facing with changes of input membership values.
2. Uncareness of the gravity of the fuzzy input.
3. Disgracing the intersection ratio between the fuzzy input and the rule antecedent.

We propose a new adaptive approach to determine matching degree. This method inspires from the approach that we have already proposed it on [25, 26]. The paper contribution is three folded:

1. Illustration of the drawbacks of the *Max-Min* method, in determining matching degree.
2. Proposing a new matching degree method which manages the motioned drawbacks of the *Max-Min* method
3. Illustration of a straightforward way to compute it in a very fast time, i.e. O(1).

It can be applied for all kinds of inference engines. It improves the performance and the convergence speed, in fuzzy systems, especially, for oscillatory systems.

In this section, three drawbacks of the common approach to determine the matching degree have been investigated. It has been shown that the above mentioned method does not operate logically, on the discussed cases.

4. Surface Matching Degree: SMD

The main contribution of the paper is to use the communal area between the fuzzy input term value and the rule antecedent. The aim is to find a method that compensates the drawbacks of *Max-Min* method as well. The R value is used as a parameter in determining the matching degree. Considering its drawbacks explained in section 3, it depends on input shape, amount of fuzziness of input term and the value of R.

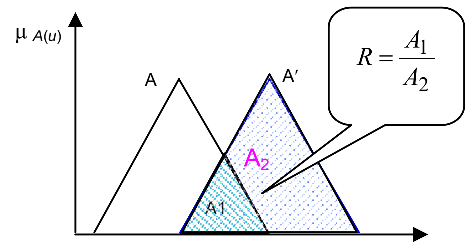

Fig. 2: The ratio of areas (R).

Depend on A_1 and A_2 that are shown in Fig 2, we define the ratio of areas (R) as equation 4:

$$R = \frac{A_1}{A_2} \tag{4}$$

Where A_1 is the communal area and A_2 is the total input area. R will be faded, if fuzzy inputs are wider and fuzzier. Therefore, it can be considered as a good parameter to model the fuzziness of input and the distance from point α. This ratio, R, for fuzzy input term values 4, 3, 2 and 1 and in boundary state for crisp input value 0 are respectively equal to 0.17, 0.25, 0.37, 0.58, and 0.75. It can be observed that this parameter makes different values for different fuzzy inputs intersecting the antecedent term in the same fixed point α.

In other words, by getting the center of fuzzy input away from α, the changes of R, tend to fade. In fact, this is what the equation 3 is lacked. A very well-suited aspect of this parameter is its adaptation. It is able to adapt itself to input transformation. By using this parameter with its adaptability feature in definition of the matching degree, we will have an adaptive matching degree. Also, we can use R as another parameter combined with the parameter α (derived from equation 3). This combination can yield a dynamic and adaptive way to determine the matching degree. It will have the advantages of both *Max-Min* and the proposed approaches, simultaneously.

Our proposition to define SMD is to combine the parameters, normalized R and α, as it is in equation 5:

$$SMD(R_n, \alpha) = \frac{w_1 f(R_n) + w_2 g(\alpha)}{w_1 + w_2} \tag{5}$$

Where R_n is the normalized ratio of surfaces, α is the value that comes from equation 3. f and g are two functions in range [0,1] that can be chosen depending on the problem. Also, w_1 and w_2 are two weights in [0,1]. By defining the equation 5, we can profit from both approaches by weights w_1 and w_2.

4.1. Computation of R:

In this section, the ratio R is computed in three states between isosceles fuzzy terms. These three possible states between fuzzy input terms and rule antecedent are shown in Fig 3.

State 1 shows that the input A′ and rule antecedent A cut each other at just one point. The junction point in this state is called μ. State 2 shows that the input A′ and rule antecedent A cut each other at two points, when A′ is smaller than A. State 3 shows that the input A′ and rule antecedent (A) cut each other at two points, when A′ be larger than A. In two recent states, the larger and smaller points are called μ and μ_2, respectively.

As shown in Fig 4, suppose that the half bases of terms A′ and A are respectively I and r. Then to compute the ratio of areas in state 1, R_1, we have:

$$\cot\theta 1 = \frac{I}{1} = \frac{S1}{\mu} \Rightarrow S1 = I\mu$$
$$\cot\theta 2 = \frac{r}{1} = \frac{S2}{\mu} \Rightarrow S2 = r\mu \tag{6}$$

Then:

$$S1 + S2 = (I + r)\mu \tag{7}$$

By substituting equation 7 in equation 4, we have:

$$R_1 = \frac{A_1}{A_2} = \frac{0.5*(S1+S2)\mu}{0.5*2*I} = \frac{(r+I)\mu*\mu}{2*I} \tag{8}$$

The equation 9 will be obtained by summarizing of equation 8:

$$R_1 = \frac{(r+I)\mu^2}{2I} \tag{9}$$

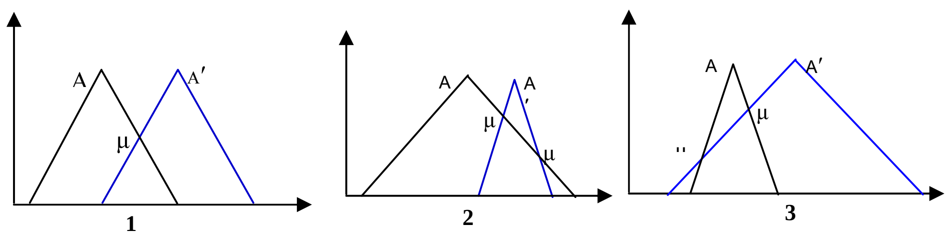

Fig. 3: three possible states between fuzzy input term and rule antecedent.

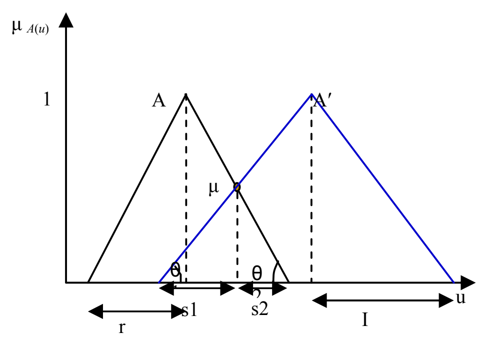

Fig. 4: computation of R_n .

Similarly, it has been proved that R_2 and R_3 for states 2 and 3 obtained from the equations 10 and 11(Alizadeh, H. 2007):

$$R_2 = \mu + \frac{(r-I)(\mu-\mu_2)\mu_2}{2I} \tag{10}$$

$$R_3 = \frac{r}{I}\mu - \frac{(r-I)(\mu-\mu_2)\mu_2}{2I} \tag{11}$$

4.2. Normalization Of R By Fixed μ:

It is apparent from equation 1, that the matching degree for the crisp input is the membership value in the antecedent part of rule at junction point. Since this value is determined by human expert (Mizumoto, 1982; Mizumoto, 1987; Mamdani, 1976), the value of *R* must be normalized such it is μ for crisp input. Therefore, when a fuzzy input tends to crisp input (crispness), the value of R has to tend to μ.

As shown in Fig 5, if we go from input 2 toward narrower input 1, the R_1 will be replaced with R_2. With a fixed point α (here, as μ), whatever we go from input 1 toward 0 ($I \to 0$), the ratio of R goes from R_2 to its. In the other hand, whatever the fuzziness of input decreases, R goes to its maximum limit value. It has been shown that this value occurs in state 2 (R_2) and its value trends to $2\mu-\mu^2$ (Alizadeh, 2007) So we have:

$$\lim_{I\to 0} R = 2\mu - \mu^2 \tag{12}$$

Since this value is obtained from limit state of fuzzy input, it must be such normalized that it becomes equal to μ.

$$R = 2\mu - \mu^2 \Rightarrow \mu = \frac{R+\mu^2}{2} \tag{13}$$

So, the normalized ratio of areas (R_n) has been defined as:

$$R_n = \frac{R+\mu^2}{2} \tag{14}$$

This equation is a normalized equation that returns the true value for crisp input. By applying this equation to equation 5, we can reach to a parametric and adaptive relation. We can select the functions f and g, depending on the problem.

4.3. Choice Of Parameters:

Equation 5 is the main contribution which has been introduced in this paper. Until here, the term R_n from the equation 5 is computed. Now we can replace it in the general equation.

$$SMD(R_n,\alpha) = \frac{w_1 f(R_n) + w_2 g(\alpha)}{w_1 + w_2} \tag{15}$$

The functions *f* and *g,* and the weights w_1 and w_2 can be chosen depends to the problem. We can tune the effect of R_n with respect to α by changing the weight values w_1 and w_2. By choosing the functions *f* and *g* as identical functions and $w_1=w_2=0.5$ from equation 5, we will have:

$$SMD(R_n,\alpha) = \frac{R_n + \alpha}{2} \tag{16}$$

Replacing R_n as equation 14 in equation 15 SMD will be:

$$SMD(R,\alpha) = \frac{R+\mu^2+2\mu}{4} \tag{17}$$

Equation 16 performs an averaging between *Max-Min* and *SMD* methods. It means that, we can average over both common and proposed methods to simultaneously benefit from both of them.

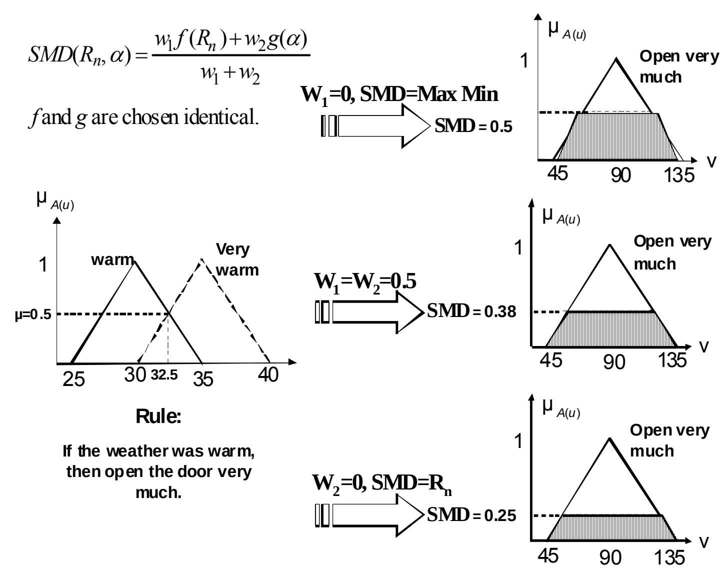

Fig. 5: comparison between α and SMD for fuzzy input “very warm” by R=0.25, α=μ=0.5.

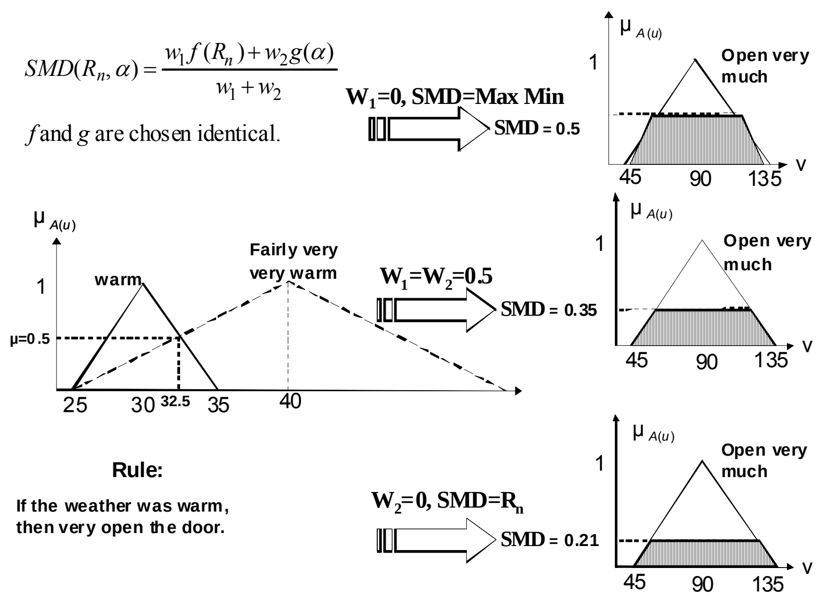

Fig. 6: comparison between α and SMD for fuzzy input "fairly very very warm" by R=0.17, α=μ=0.5.

4.4. Example And Comparison:

By illustrating two examples in this section, we try to make a comparison between the *Max-Min* method (which chooses α) and *SMD*.

The reader can infer from Fig 5 and Fig 6 that "the wider the input fuzzy term, the smaller SMD" and also "the more distant the center of gravity of the input fuzzy term, the smaller SMD".

Table 1: A comparison between α and SMD for fuzzy input terms in figure 1.

	Input 4	Input 3	Input 2	Input 1	Crisp input 0
W1=0,SMD=Max Min=α	0.5	0.5	0.5	0.5	0.5
W1=W2=0.5, SMD	0.35	0.38	0.41	0.46	0.5
W2=0, SMD=R_n	0.21	0.25	0.31	0.42	0.5

In Table 1, SMD is computed for input terms of Fig 1. Table 1 shows that, while α is 0.5 for all of those inputs, however SMD is adaptively different, depending on shape of inputs.

Table 2: A comparison between α and SMD in limit states by a fixed μ.

	The limit value for crisp input	The limit value for fuzzy input by infinite amount of fuzziness
W1=0, SMD=Max Min=α	$\alpha = \mu$	$\alpha = \mu$
W1=W2=0.5, SMD	$SMD = \mu$	$SMD = \frac{2\mu + \mu^2}{4}$
W2=0, SMD=Rn	$SMD = \mu$	$SMD = \frac{\mu^2}{2}$

Table 2, shows the matching degree values obtained using common and SMD method, in limit states.

Two points in Tables 1 and 2 are worthy to note: (a) while, the values obtained from common approach are fixed for mentioned states, different values are obtained from SMD for them. Second, the SMD values with any parameter choices are trended to μ', in limit state.

5. Conclusion:

In this paper, three shortcomings of common *Max-Min* approach to determine the matching degree is illustrated. Then, an adaptive method called "Surface Matching Degree (SMD)" is proposed to handle them. We can change and adapt the parameters of SMD depends on the kind of inference engine and its application. It can help the inference engine in different applications. However, the values obtained from *Max-Min* method are fixed for the states of inputs that have a same μ; different values are obtained from SMD, adaptively. It depends on input shape and the amount of input term that is true of the rule antecedent.

In addition, SMD has the following benefits: first it seems that using SMD can be more logical than common approach. Second, emphasizing to the ratio of input which is satisfied in the rule affect the stability of the fuzzy system. It can cause to faster convergence by decreasing fuzziness in feedback systems.

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