Fuzzy Decision Making based on Relationship Analysis between Criteria

Abdoos Monireh
Department of Computer Engineering
Iran University of Science and Technology
Tehran, Iran
abdoos@mail.iust.ac.ir

Mozayani Nasser
Department of Computer Engineering
Iran University of Science and Technology
Tehran, Iran
mozayani@iust.ac.ir

***Abstract*- In this paper, we propose a solution for multi criteria group decision making problems, where the preferences are stated by fuzzy numbers. A fuzzy model has been applied to rank the possible alternatives of the problem. The criteria of decision making problems are usually dependent on each others. By considering dependency of the criteria, three relations between them have been used: conflicting, cooperative and irrelevant relations. In this implementation, the conflicting degree and the cooperative degree between any two criteria are first formulated. Relationships between criteria are identified based upon their conflicting and cooperative degree and a new fuzzy method for multi criteria decision making problems is proposed based on relationship analysis between criteria. Herein, the ordered list of alternatives is provided in four steps: calculating conflicting and cooperative degree for each pair of criteria, dividing criteria into two classes using an algorithm, calculating criteria satisfaction of each class for each alternative and finally, the ordered list of alternatives is provided by using a ranking method.**

I. INTRODUCTION

This paper presents a new methodology for Decision Making problems with dependent criteria. In particular, we have studied the problems that consider more than one criterion, which is known as Multiple Criteria Decision Making (MCDM). MCDM problems are widespread in real life decision situations and have been one of the fastest growing problem areas in many disciplines [1]. MCDM refers to making decision in the presence of multiple, usually conflicting criteria [2]. A MCDM problem is to find a best compromise solution from all feasible alternatives assessed on multi criteria, both quantitative and qualitative. There are two types of MCDM methods. One is compensatory and the other is non-compensatory [Hwang & Yoon, 1981]. The proposed method is compensatory-based method. Compensatory methods permit tradeoffs between criteria, meaning that a slight decline in one criterion is acceptable if it is compensated by some enhancement in one or more other criteria. Empirical studies have indicated that human decision making is better described by operators which allow trade-off between criteria [3]. MCDM problems may not always have a conclusive or unique solution. Fuzzy DM is an uncertain decision making mechanism applied to fuzzy environment. Under many conditions, however, crisp data are inadequate or insufficient to model real-life decision problems [4, 5]. Indeed, human judgments are vague or fuzzy in nature and as such it may not be appropriate to represent them by accurate numerical values. A more realistic approach could be to use linguistic variables to model human judgments [6]. Suppose the decision makers have to choose one of or rank m alternatives: $A_1, A_2, \ldots, A_m$ based on n criteria: $c_1, c_2, \ldots, c_n$. Denote an alternative set by and a $A = \{A_1, A_2, \ldots, A_m\}$ and a criteria set by $C = \{c_1, c_2, \ldots, c_n\}$; Let $\mu_i^j(x)$ be the satisfaction of criterion c_j $(j = 1,2,\ldots,n)$ by alternative A_i $(i = 1,2,\ldots,m)$, where stated by fuzzy numbers, and suppose w_j is the relative weight of criterion c_j, where $w_j \geq 0$ $(j = 1,2,...,n)$ and $\sum_{j=1}^{n} w_j = 1$. Denote a weight vector by $W = \langle w_1, w_2, \ldots, w_n \rangle$. A MCDM problem can then be expressed as the following decision matrix:

$$\tilde{D} = (\mu_i^j(x))_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} \mu_1^1(x) & \mu_1^2(x) & \cdots & \mu_1^n(x) \\ \mu_2^1(x) & \mu_2^2(x) & \cdots & \mu_2^n(x) \\ \vdots & \vdots & \vdots & \vdots \\ \mu_m^1(x) & \mu_m^2(x) & \cdots & \mu_m^n(x) \end{bmatrix} \end{array}$$

Where, the elements $\mu_i^j(x)$ are the membership functions which represent the satisfaction of criterion c_j by alternative A_i.

In this paper, we present a new methodology for selecting the appropriate alternative among feasible alternatives based on relationship analysis between criteria. In this methodology, triangular fuzzy numbers are used to capture fuzziness in decision information and group decision making processes by means of a fuzzy decision matrix. One of the most basic concepts of fuzzy set theory that can be used to generalize crisp mathematical concepts to fuzzy sets is the extension principle [7]. In this paper, the extension principle have been used for fuzzy mathematical.

This paper is organized as follows. In next section, the basic definitions and relations are defined and a new method for MCDM problems is proposed. In section 3, a model for multi criteria group decision making problems is presented.

The proposed method is also illustrated by a numerical example in section 4. The paper is concluded in section5.

II. FUZZY MULTI CRITERIA DECISION MAKING METHOD

Our proposed method uses a decision matrix that the preferences are stated by fuzzy numbers. For the sake of simplicity and without loss of generality assume that all fuzzy numbers are positive triangular fuzzy numbers. It is easy to see that a triangular fuzzy number $N = (\alpha, \beta, \gamma)$ is reduced to a real number N if $\alpha = \beta = \gamma$. Conversely, a real number of quantitative criteria can be written as a triangular fuzzy number $N = (\beta, \beta, \beta)$. Consequently, assume that all of the preferences are triangular fuzzy numbers.

A. Definitions of relations

$\mu_c(A)$ indicates the degree of satisfactory of the criterion c of alternative A. considering a pair of alternatives two criteria are named conflicting if an increase in the degree to which one criterion is satisfied often decreases the degree to which another criterion is satisfied, that is, the $\mu_c(A)$ decreases between the two alternatives (called a conflicting decision alternative pair). On the other hand, two criteria are said to cooperate with each other if an increase (or a decrease) in the degree to which one criterion is satisfied often increases (or decreases) the degree to which another criterion is satisfied, that is, the $\mu_c(A)$ increases (or decreases) between the two alternatives (called a cooperative decision alternative pair). Note that the third possibility is that the $\mu_c(A)$ remains unchanged between the two alternatives, which is called an irrelevant decision alternative pair [8]. We formally define conflicting, cooperative and irrelevant pairs below, followed by the formal definitions of conflicting and cooperative degrees.

Definition 1: (Conflicting, cooperative and irrelevant pairs)

A_p denotes a set of alternatives pairs:

$$A_P = \{(a_i, a_j) \mid \ \forall a_i, a_j \in A \ , i \neq j\}$$

We define a triangular fuzzy number $\mu_{ij}^k(x)$ by:

$$\mu_{ij}^k(x) = \mu_i^k(x) \Theta \mu_j^k(x) \qquad (1)$$

Such that Θ denote to extended subtraction to fuzzy numbers.

Assume that c_k, c_l be two criteria, a set of conflicting decision alternative pairs (for short, conflicting pairs) is defined as:

$$CF = \left\{ (a_i, a_j) \middle| \left[\int_{x>0} \mu_{ij}^k(x) - \int_{x<0} \mu_{ij}^k(x) \right] \times \left[\int_{x>0} \mu_{ij}^l(x) - \int_{x<0} \mu_{ij}^l(x) \right] < 0 \right\} \quad (2)$$

A set of cooperative decision alternative pairs (for short, cooperative pairs) is defined as:

$$CP = \left\{ (a_i, a_j) \middle| \left[\int_{x>0} \mu_{ij}^k(x) - \int_{x<0} \mu_{ij}^k(x) \right] \times \left[\int_{x>0} \mu_{ij}^l(x) - \int_{x<0} \mu_{ij}^l(x) \right] > 0 \right\} \quad (3)$$

A set of irrelevant decision alternative pairs (for short, irrelevant pairs) is defined as:

$$IR = \left\{ (a_i, a_j) \middle| \left[\int_{x>0} \mu_{ij}^k(x) - \int_{x<0} \mu_{ij}^k(x) \right] \times \left[\int_{x>0} \mu_{ij}^l(x) - \int_{x<0} \mu_{ij}^l(x) \right] = 0 \right\} \quad (4)$$

Hence, A_p can be divided into three classes: conflicting, cooperative and irrelevant, in such a way that:

$$A_P = CF \cup CP \cup IR$$
$$CF \cap CP \cap IR = \Phi$$

Definition 2: (Conflicting and cooperative degrees)

Let c_k, c_l be two criteria and CP and CF denote conflicting and cooperative pairs of alternatives, respectively. If we present $\mu_{ij}^k(x)$ by triangular fuzzy number, we can get:

$$\mu_{ij}^k(x) = (\alpha_{ij}^k, \beta_{ij}^k, \gamma_{ij}^k)$$

As we know, the degree of membership β_{ij}^k is 1.

The conflicting degree between two criteria, c_k and c_l is defined as:

$$cf(c_k, c_l) = \frac{\sum_{(a_i, a_j) \in CF} (|\beta_{ij}^k| + |\beta_{ij}^l|)}{\sum_{(a_i, a_j) \in A_P} (|\beta_{ij}^k| + |\beta_{ij}^l|)} \quad (5)$$

The cooperative degree between two criteria, c_k and c_l , is defined as:

$$cp(c_k, c_l) = \frac{\sum_{(a_i, a_j) \in CP} (|\beta_{ij}^k| + |\beta_{ij}^l|)}{\sum_{(a_i, a_j) \in A_P} (|\beta_{ij}^k| + |\beta_{ij}^l|)} \quad (6)$$

The relationships among criteria are crucial for adequate treatment of fuzzy decision making, because they reflect the structure of interaction among the criteria and represent user's preferences of the criteria [8]. Together with information about the criticality of criteria, the relationships among criteria can serve as a guideline for ranking alternatives.

B. The algorithm for dividing criteria based on their relations

The next step, after calculation of the conflicting and cooperative degree between any two criteria, is dividing criteria into two classes. We need an algorithm to divide the criteria into two discrete classes based on the defined relations, such that criteria belonging to each class have maximum cooperative degree with each other. For this reason, differences of cooperative degree and conflicting degree for each pair of criteria can be used. We propose the following algorithm for dividing criteria into two classes with the foregoing property. If A_C refers to a set of pairs of criteria and C a set of criteria, we have:

$$A_C = \{(c_i, c_j) \mid \ \forall c_i, c_j \in C \ , \ i \neq j\}$$

The output of this algorithm is two classes; S_1 , S_2 which any of them is a set of criteria such that: $S_1 \cap S_2 = \Phi$, $S_1 \cup S_2 = C$.

ALGORITHM 1

FOR DIVIDING CRITERIA INTO TWO CLASSES

$S_1 = \{c_1\}$
$S_2 = \{c_2, c_3, ..., c_n\}$
while true *do*
{
  for all $c_j \in S_2$ *do*
    $P_j = \sum_{c_i \in S_1} cp(c_i, c_j) - cf(c_i, c_j)$
  $P_k = \max_j P_j$ // k : *index of* max*imum value among* $P_j s$
  if $P_k \succ 0$ *then*
    {
      $S_1 = S_1 + \{c_k\}$
      $S_2 = S_2 - \{c_k\}$
    }
  else
    exit
}

This algorithm, first assigns the criterion c_1 to S_1 and then a criterion which belongs to S_2 and has maximum positive value among the other criteria of S_2 , is removed and assigned to S_1 .

To obtain the ranking of the alternatives, μ_{S_1}, μ_{S_2} for each alternative is defined as:

$$\mu_{S_1}(A_k) = \sum_{c_i \in S_1} w_i \sum_{c_i \in S_1} \mu_k^i(x) \quad (7)$$

$$\mu_{S_2}(A_k) = \sum_{c_j \in S_2} w_j \sum_{c_j \in S_2} \mu_k^j(x) \quad (8)$$

Where, the second $\sum$ is extended sums of fuzzy numbers. Ordered list of the alternatives is provided according to $\max(\mu_{S_1}(A_i), \mu_{S_2}(A_i))$ for each alternative.

III. FUZZY MULTI CRITERIA GROUP DECISION MAKING

There are two possible approaches for solving multi person MCDM problems. In the first, before applying this algorithm, aggregated criteria importance and satisfaction coefficient are obtained. In the Second, a MCDM problem with multi decision makers is divided into multi MCDM problems with single decision maker and then the appropriate alternative is specified according to all decision makers' opinions. In spite of less calculation, the first has less accuracy because of applying the aggregation operator for twice, in computing the matrix and importance of criteria. Hence, we apply the second. (See Fig. 1)

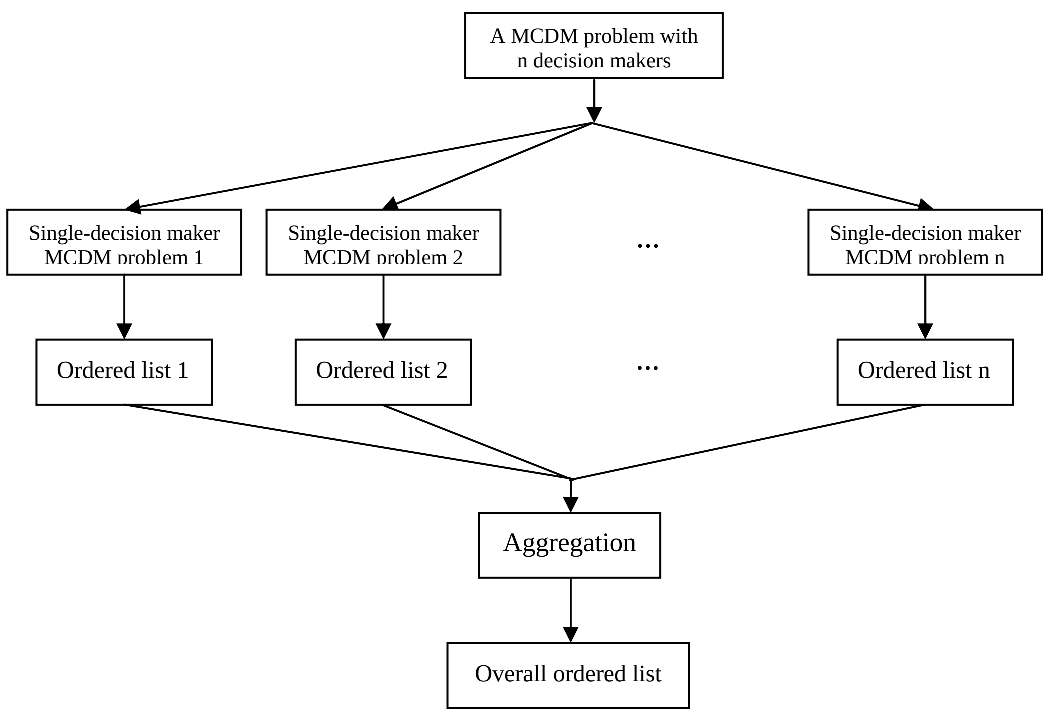

Fig.1 A solution for Multi Criteria Group Decision Making.

MCDM problems with n decision makers are subdivided to n problems with single decision maker and then the aggregation operator is applied. Average operator as an aggregation operator has been used in this method. According to aggregated values of alternatives, by using a ranking method, ordered list is provided.

IV. A Numerical Example

Let $C = \{c_1, c_2, c_3, c_4\}$ a set of criteria, $A = \{A_1, A_2, A_3, A_4\}$ a set of possible alternatives. Suppose there are three decision makers J_1, J_2 and J_3. The decision matrices are given by the three decision makers J_1, J_2 and J_3 as in Table I-II-III, respectively.

TABLE I
Decision Information given by the decision maker J_1

Alternatives	Criteria			
	c_1	c_2	c_3	c_4
A_1	(0,0.1,0.2)	(0.4,0.5,0.6)	(0.1,0.2,0.3)	(0.8,0.9,1)
A_2	(0.7,0.8,0.9)	(0.1,0.2,0.3)	(0.2,0.4,0.5)	(0,0.1,0.2)
A_3	(0.5,0.6,0.8)	(0.7,0.8,0.9)	(0.6,0.7,1)	(0,0.1,0.2)
A_4	(0.2,0.4,0.5)	(0.4,0.5,0.6)	(0.6,0.7,1)	(0.4,0.5,0.6)
weight	0.2	0.4	0.3	0.1

TABLE II
Decision Information given by the decision maker J_2

Alternatives	Criteria			
	c_1	c_2	c_3	c_4
A_1	(0.2,0.4,0.5)	(0.5,0.6,0.8)	(0,0.1,0.2)	(0.1,0.2,0.3)
A_2	(0.4,0.5,0.6)	(0.7,0.8,0.9)	(0.6,0.7,1)	(0.4,0.5,0.6)
A_3	(0,0.1,0.2)	(0.8,0.9,1)	(0.2,0.4,0.5)	(0.1,0.2,0.3)
A_4	(0.2,0.4,0.5)	(0.7,0.8,0.9)	(0,0.1,0.2)	(0.2,0.4,0.5)
weight	0.1	0.3	0.2	0.4

TABLE III
Decision Information given by the decision maker J_3

Alternatives	Criteria			
	c_1	c_2	c_3	c_4
A_1	(0,0.1,0.2)	(0.4,0.5,0.6)	(0.5,0.6,0.8)	(0.6,0.7,1)
A_2	(0,0.1,0.2)	(0.1,0.2,0.3)	(0.2,0.4,0.5)	(0.7,0.8,0.9)
A_3	(0.2,0.4,0.5)	(0.5,0.6,0.8)	(0.6,0.7,1)	(0.8,0.9,1)
A_4	(0.4,0.5,0.6)	(0.1,0.2,0.3)	(0.2,0.4,0.5)	(0.6,0.7,1)
weight	0.3	0.3	0.2	0.2

Cooperative degree and conflicting degree between any two criteria were calculated using (5), (6) as follow:

TABLE IV
The cooperative and conflicting degree (J_1)

	cp	cf
c_1, c_2	0.32	0.61
c_1, c_3	0.66	0.29
c_1, c_4	0	0.96
c_2, c_3	0.64	0.14
c_2, c_4	0.39	0.39
c_3, c_4	0.15	0.69

TABLE V
The cooperative and conflicting degree (J_2)

J_2	cp	cf
c_1, c_2	0.14	0.71
c_1, c_3	0.63	0.36
c_1, c_4	0.78	0
c_2, c_3	0.6	0.13
c_2, c_4	0.45	0.35
c_3, c_4	0.68	0.15

TABLE VI
The cooperative and conflicting degree (J_3)

J_3	cp	cf
c_1, c_2	0.38	0.41
c_1, c_3	0.4	0.4
c_1, c_4	0.43	0.33
c_2, c_3	1	0
c_2, c_4	0.84	0.18
c_3, c_4	0.66	0.16

S_1, S_2 two classes of criteria are obtained by applying the algorithm:

TABLE VII
The classes of criteria

	J_1	J_2	J_3
S_1	$\{c_1, c_2, c_3\}$	$\{c_1, c_3, c_4\}$	$\{c_1, c_2, c_3, c_4\}$
S_2	$\{c_4\}$	$\{c_2\}$	Φ

The $\max(\mu_{S_1}(A_i), \mu_{S_2}(A_i))$ values for all alternatives and by the three decision makers were calculated using (7), (8). These values are given in Table VIII.

TABLE VII

THE VALUES OF $\max(\mu_{S_1}(A_i), \mu_{S_2}(A_i))$

	A_1	A_2	A_3	A_4
J_1	(0.45,0.72,0.99)	(0.9,1.26,1.5)	(1.62,1.98,2.43)	(1.08,1.44,1.89)
J_2	(0.21,0.49,0.7)	(0.98,1.19,1.5)	(0.21,0.49,0.7)	(0.28,0.63,0.84)
J_3	(1.5,1.9,2.6)	(1,1.5,1.9)	(2.1,2.6,3.3)	(1.3,1.8,2.4)

According to Table VIII, aggregated values are as follow:

$A_1 = (0.71, 1.03, 1.42)$

$A_2 = (0.78, 1.31, 1.62)$

$A_3 = (1.21, 1.59, 2.04)$

$A_4 = (0.88, 1.29, 1.71)$

Fig. 2 shows these fuzzy numbers.

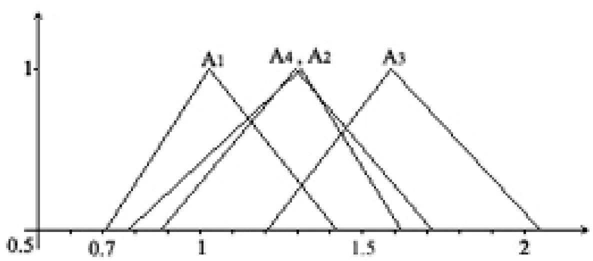

Fig. 2 Aggregated values of alternatives.

By using Adamo ranking method [9] with $\alpha \geq 0.85$ [1], the overall ordered list is provided:

$A_3 \succ A_2 \succ\approx A_4 \succ A_1$

V. CONCLUSION

Most MCDM problems include both quantitative and qualitative criteria which are often assessed using imprecise data and human judgment. Fuzzy set theory is well suited to dealing with such decision problems. In this paper, we proposed a method for MCDM problems using fuzzy set theory. The method can be applied to solving many practical decision problems. Our method has the following advantages:

- In real world, there are many MCDM problems that their criteria are conflicted to each other, and this method gives a good solution for these problems.
- Triangular fuzzy numbers have been used in this paper to assess alternatives with respect to both quantitative and qualitative criteria. Hence, this method contains a broad scope of problems.
- In this method, triangular fuzzy numbers can be extended to any LR fuzzy numbers.
- This method can be used in both crisp and fuzzy environment.

REFERENCES

[1] Evangelos Triantaphyllou, *Multi-Criteria Decision Making Methods: A Comparative Study - Applied Optimization*, Vol. 44, kluwer Academic, 2000.

[2] Ling Xu, Jain –Bo Yang, "Introduction to Multi-Criteria Decision Making and the Evidential Reasoning Approach", http://www.sm.umist.ac.uk , 2001.

[3] Uzay Keymak, Hans R. Van Nauta Lemke, "A sensitivity approach to introducing weight factors into decision functions in fuzzy multicriteria decision making", Fuzzy Set and Systems 97 (1998) 169-182.

[4] C Carlsson, R. Fuller, " Multiobjective linguistic optimization", Fuzzy Sets and Systems 115 (2000) 5-10.

[5] C.T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment", Fuzzy Sets and Systems 114 (2000) 1-9.

[6] Deng-Feng Li, Jian-Bo Yang, "Fuzzy linear programming technique for multiattribute group decision making in fuzzy environments ", Information Sciences 158 (2004) 263-275.

[7] H.-J. Zimmermann, *Fuzzy Set Theory and its Applications*, Kluwer Academic Publisher, 1996.

[8] Jonathan Lee, Jong-Yih Kuo, "Fuzzy decision making through trade-off analysis between criteria", Journal of Information Sciences 107 (1998) 107-126.

[9] M.J. Asgharpour, *Multiple Criteria Decision Making*, Tehran University publications, 2003.

1 - In this method, α is an acceptable threshold given by decision maker.