PROBLEMS

4.1. Given the time-harmonic representation of $f(t)$ in (4.23), show that

$$\frac{df}{dt} = \mathrm{Re}(i\omega F e^{i\omega t})$$

4.2. Verify the four relations for the real-part operator, given in (4.27)–(4.30). *Hint*: To prove the two relations for derivatives and integrals, begin with the basic definition of a derivative and a Riemann integral.

4.3. One of the important theorems of electromagnetic theory is the *principle of duality* [12],[13]. Using duality, make the necessary changes in (4.80)–(4.83) to obtain the fields produced by the magnetic sheet source

$$\mathbf{M}(z) = \hat{x} M_{s0} \delta(z)$$

where M_{s0} is a constant magnetic surface current density in volts/m.

4.4. From (4.72), the wavenumber with loss is given by

$$k = k_d \sqrt{1 - iS}$$

Show that the requirement $\mathrm{Im}(k) < 0$ implies that $\mathrm{Re}(k) > 0$. *Hint*: Write ik in terms of its real and imaginary parts, viz.

$$ik = \alpha + i\beta = ik_d \sqrt{1 - iS}$$

Solve for α and β by squaring both sides and discarding the extraneous root. Note that $\mathrm{Im}(k) < 0$ implies $\mathrm{Re}(\alpha) > 0$. From the sign of α, it is then possible to infer the sign of β.

4.5. In (4.98), we obtained

$$\int_{-\infty}^{\infty} \frac{e^{i\beta z}}{\beta^2 - k^2} d\beta = \frac{\pi}{ik} e^{-ikz}, \qquad z > 0$$

By contour integration and the calculus of residues, obtain the result for $z < 0$. *Hint*: In Example 4.1, we closed the contour on a semi-circle through the upper half of the β-plane. For $z < 0$, close the contour through the lower half of the β-plane.

4.6. An interesting variation [14] on the line source problem examined in Section 4.3 is the line source located at (x', y') parallel to the z-axis and polarized in the ρ-direction (Fig. 4-10). Such a source can be represented by

$$\mathbf{J} = \hat{\rho} I_0 \frac{\delta(\rho - \rho')}{\rho} \delta(\phi - \phi')$$

Show that the magnetic field radiated by this source is given by

$$H_z = -\frac{I_0}{4\rho'}\sum_{-\infty}^{\infty} n e^{in(\phi-\phi')}\begin{cases} H_n^{(2)}(k\rho')J_n(k\rho), & \rho < \rho' \\ H_n^{(2)}(k\rho)J_n(k\rho'), & \rho > \rho' \end{cases}$$

Show that, despite the presence in the sum of the multiplicative factor n, the series converges as $n \to \pm\infty$.

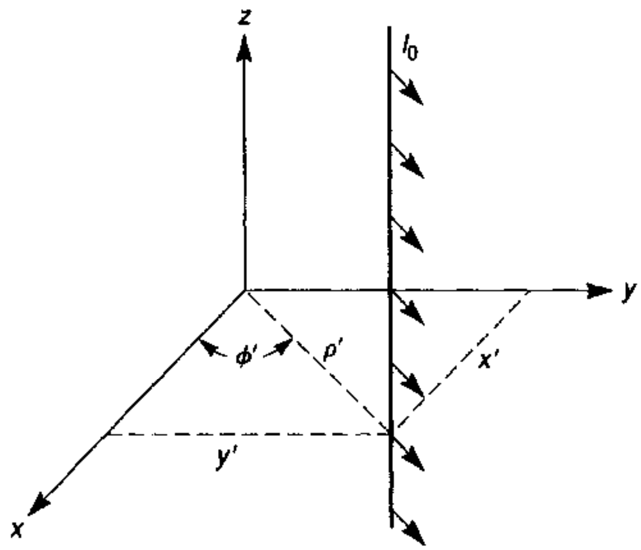

Fig. 4-10 Line source parallel to z-axis and ρ-polarized.

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