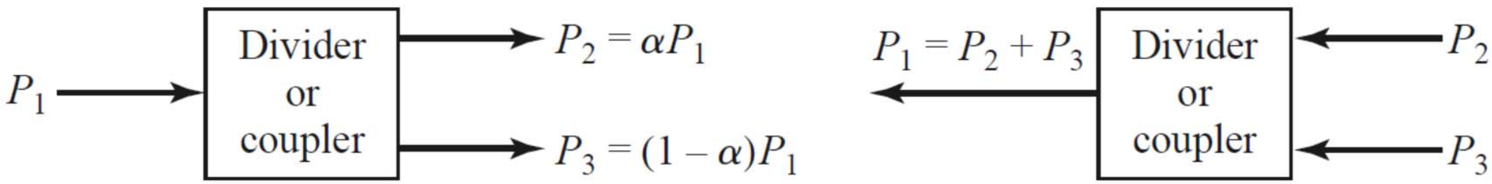
Power Dividers and Directional Couplers

- Passive components used for power division or power combining
- In the form of three-port (T-junction) networks and four-port (directional) networks



<u>Basic Properties of Dividers and Couplers</u>

Three-Port Networks (T-Junctions)

It would be useful to have a passive lossless network that divides input port power at any port between the other two ports while being matched at all three ports. This would require the network to be <u>matched</u>, <u>lossless</u> and <u>reciprocal</u>.

Network	S-matrix
(1) Matched	$S_{ii} = 0$
(2) Lossless	Unitary [S]
(3) Reciprocal	Symmetric [S]

$$\Longrightarrow [S] = \begin{matrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{matrix}$$

This matrix can satisfy (1) and (3), but not (2).

Therefore, using "normal" lossless components such as transmission lines, capacitors and inductors it is impossible to construct a 3-port network matched at all 3 ports.

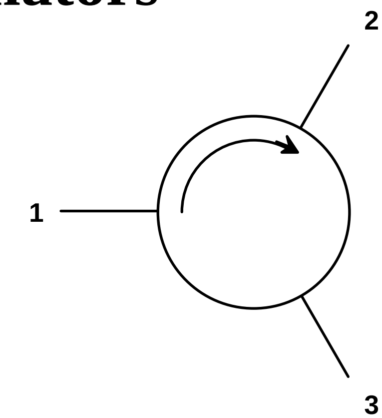
Need to relax one of the restrictions.

A. For a *nonreciprocal 3-port network* ($S_{ij} \neq S_{ji}$), using anisotropic materials (such as ferrite), all ports can be matched and a *circulator* is created.

Two Types of Circulators

(1) Clockwise circulation

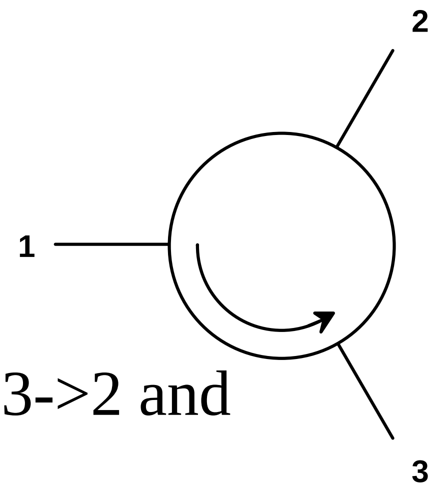
$$[S] = \begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$



it rotates power from port 1->2, port 2->3 and port 3->1.

(2) Clockwise circulation

$$[S] = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$$



it rotates power from port 1->3, port 3->2 and port 2->1.

Applications of Circulators:

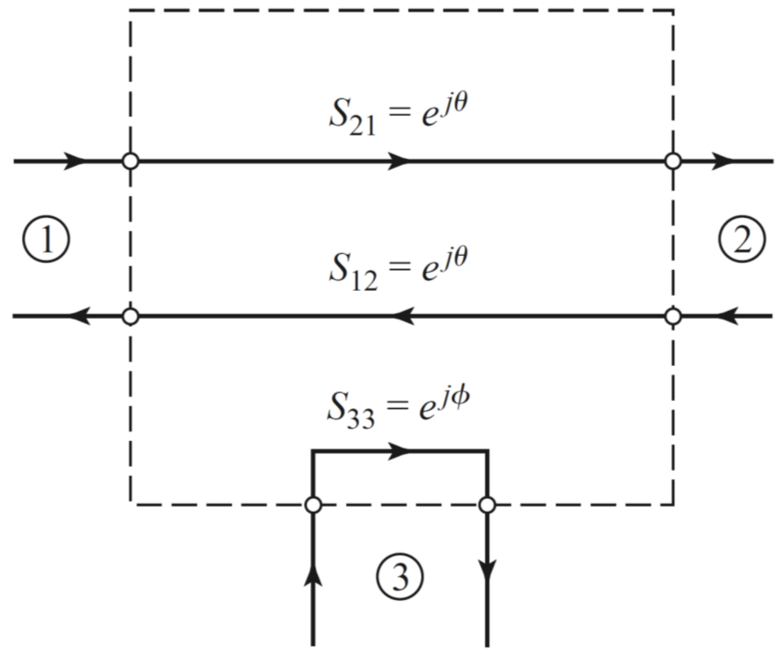
- Protects a power amplifier from output mismatch.
- Allows a transmitter and receiver to share an antenna.

B. For a *lossless and reciprocal 3-port network* ($S_{ij} = S_{ji}$), but with only two ports matched.

$$[S] = \begin{matrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{32} & S_{33} \end{matrix}$$

The requirement for unitary matrix leads to

$$[S] = \begin{matrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{matrix}$$



Totally mismatched

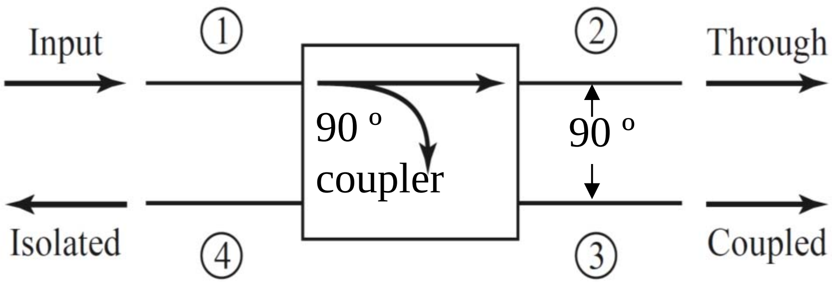
C. For a *reciprocal and all-matched 3-port network* ($S_{ij} = S_{ji}$), but with lossy components.

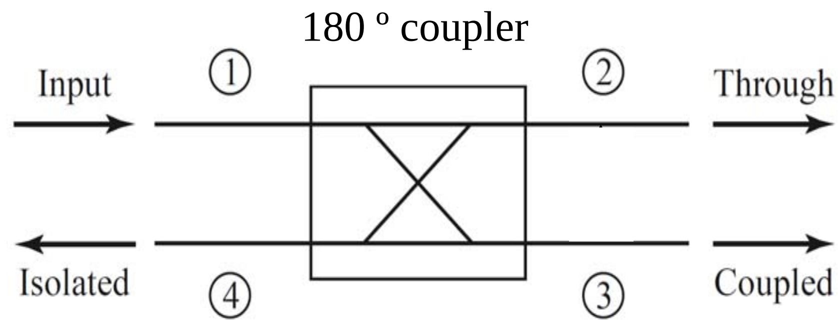
This is the case of the resistive divider. A lossy 3-port network can be made to have isolation between its output ports.

Four Port Networks (Directional Couplers)

Pozar shows that a matched, reciprocal, lossless four-port network is possible, and that it has directional coupling between pairs of ports.

There are two possible forms of [S] for a directional coupler, one with outputs differing by 90° in phase and the other with outputs differing by 180° in phase. Any 90° coupler can be made into a 180° coupler by adding a 90° transmission to one port, and *vice versa*.



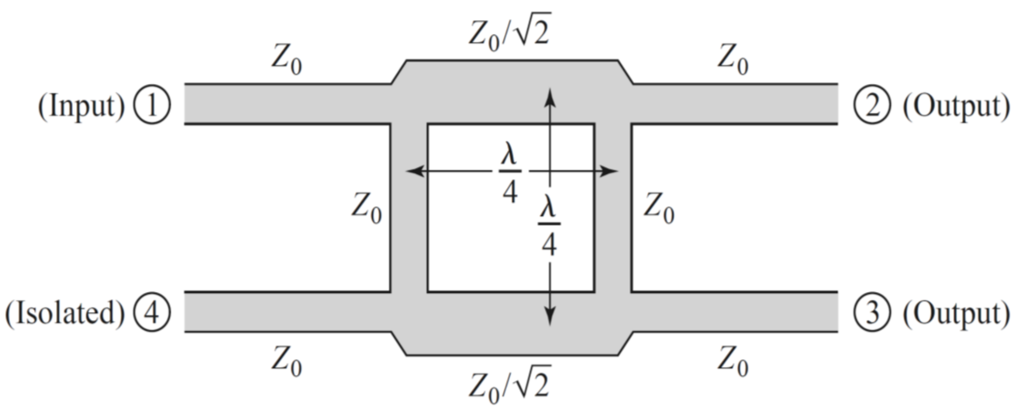


$$[S]=\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

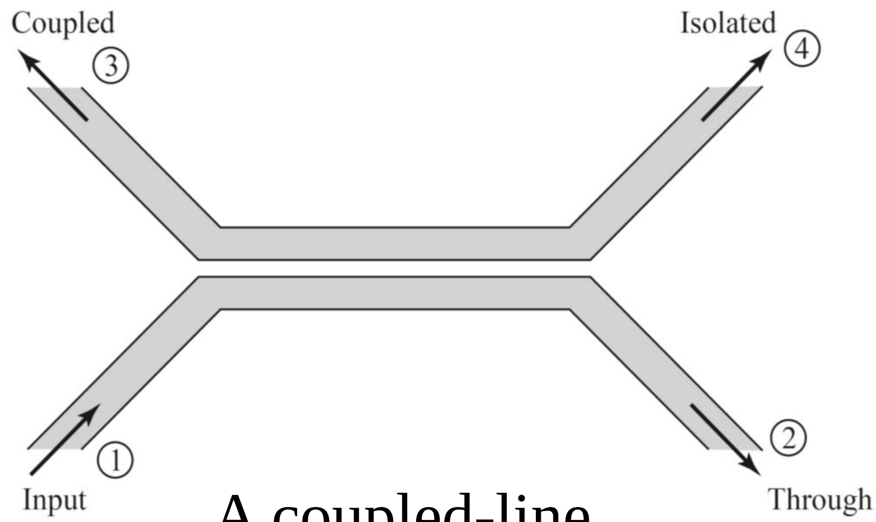
$$[S]=\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$\underline{\underline{\alpha^2+\beta^2=1}}$$

Any 90° coupler can be made into a 180° coupler by adding a 90° transmission to one port, and *vice versa*. The 90° coupler is referred to as a quadrature hybrid, and can be created using directly connected branch lines between two transmission lines or by means of coupled transmission lines.



A branch-line coupler



A coupled-line directional coupler

Coupling, Directionality and Isolation

Coupling = C = 10log(P1/P3) = -20logβ dB

Directivity = D = 10 log (P3/P4) = 20logβ/$|S_{14}|$ dB

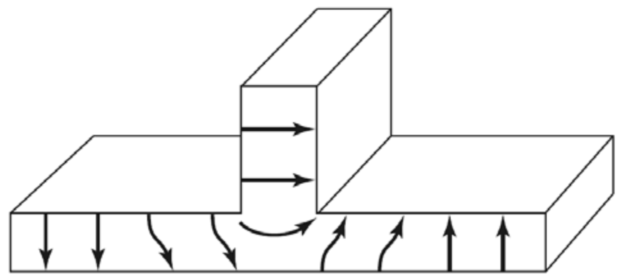
Isolation = I = 10log(P1/P4) = -20log $|S_{14}|$ dB

$$\boxed{I = D + C \text{ dB}}$$

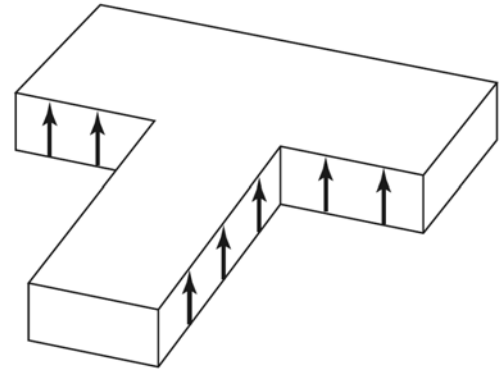
The (3-port) T-Junction Power Divider

Lossless 3-port netowork

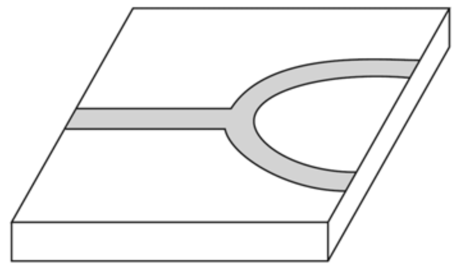
The discontinuity at the junction create fringing fields and higher order modes, and leads to stored energy which can be accounted for by a lumped susceptance.



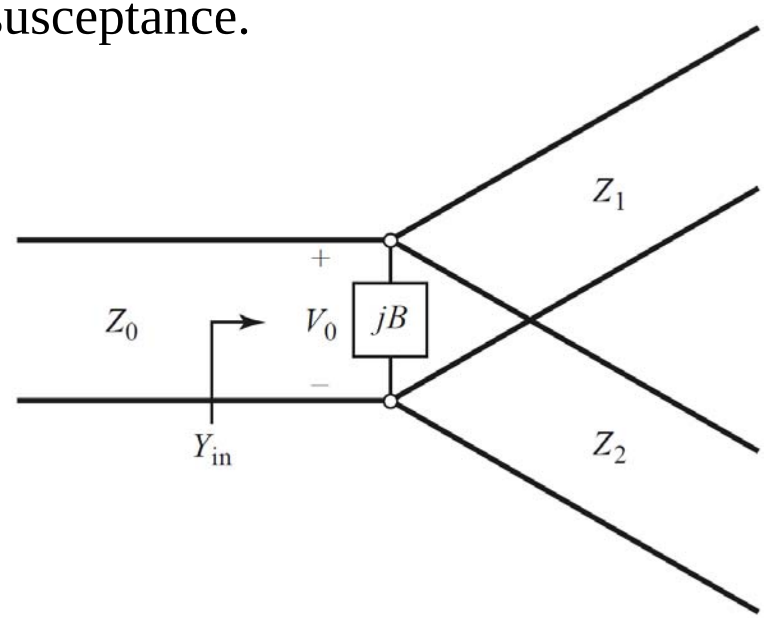
E plane waveguide T



H plane waveguide T



Microstrip T-junction



Transmission line model of a lossless T-junction.

If we relax the requirement for matched ports, we can realize tee networks that divide power and provide a match at one port (typically chosen as the input port).

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

For a lossless network, all the characteristic impedances are real, and we can assume B = 0. Then $\underline{\underline{Y_o = Y_1 + Y_2}}$.

The voltage V_0 at the junction is the same for line 1 and line 2, and the power in the lines is

$\underline{\underline{P_1 = V_0^2 Y_1/2}}$ and $\underline{\underline{P_2 = V_0^2 Y_2/2}}$

so the power division ratio r is

$$r = \frac{P_2}{P_1} = \frac{Y_2}{Y_1} = \frac{Z_1}{Z_2}$$

Substituting $Y_2 = rY_1$ we have $Y_o = Y_1(1+r)$, so $Z_1 = Z_o(1+r)$

and $Z_2 = Z_o \dfrac{1+r}{r}$

The output line impedances Z_1 and Z_2 can be selected to provide various power division ratio. If we let r = 1, we get *even power division* (power split, -3 dB), and $Z_1 = 2\,Z_0$, $Z_2 = 2\,Z_0$. If needed, the quarter-wave transformers can be used to bring the output line impedances back to the desired level.

If *B* is not negligible, some type of tuning element can usually be added to the divider to cancel this susceptance.

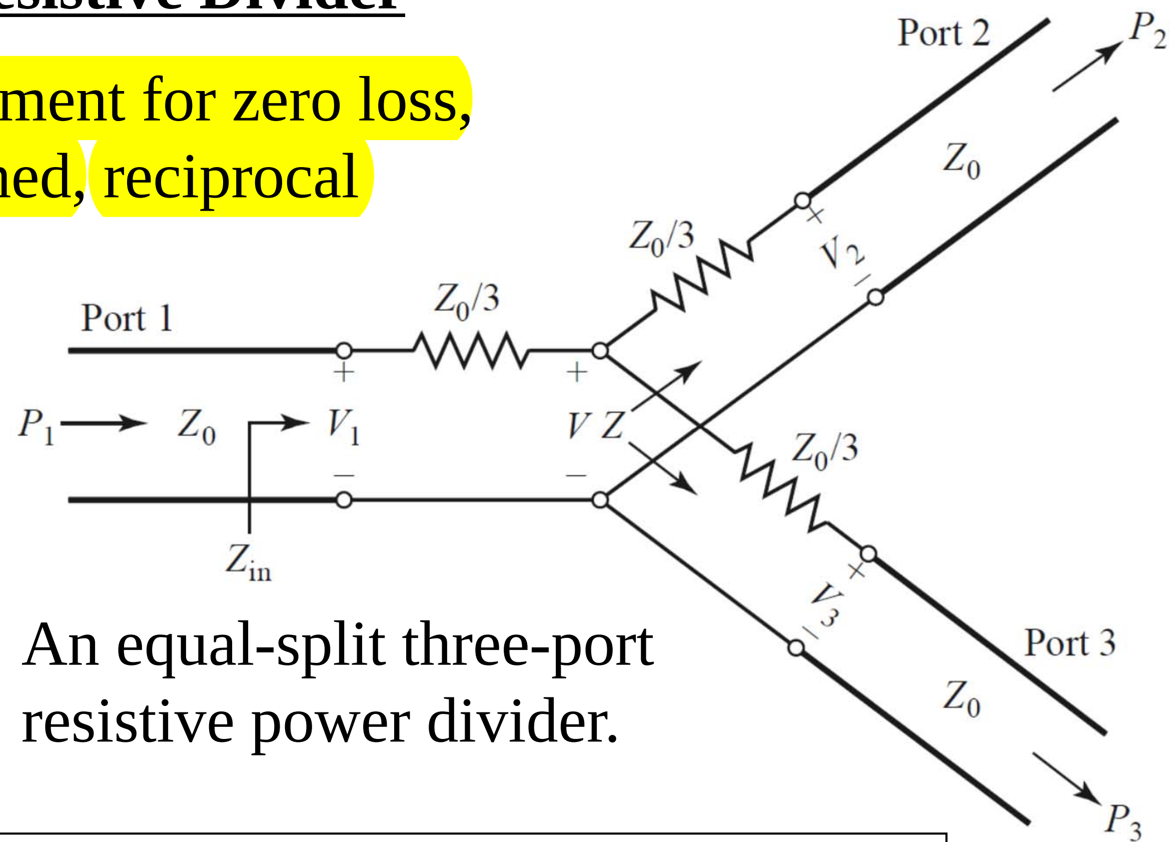
Output ports are not isolated.

Example 7.1 of Pozar!

Resistive Divider

If we relax the requirement for zero loss, we can realize a matched, reciprocal power divider.

Such a divider can be realized by connecting three Zo transmission lines to a star circuit consisting of three resistors $R = Z_o/3$.



An equal-split three-port resistive power divider.

The impedance looking into the resistor followed by the output line, is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

The input impedance of the divider is

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0 \longrightarrow \text{matched}$$

Since the network is symmetric from all three ports, the output ports are also matched.

<u>Output Power</u>

The output voltage from port 2 and 3 are, $V_2 = V_3 = \dfrac{V_1}{2}$

Thus, $S_{21} = S_{31} = S_{23} = 1/2$, which indicates 6 dB below the input power level. We have

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

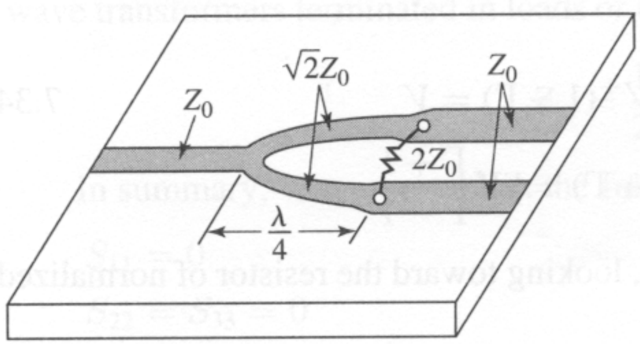
Port 2 and 3 are not isolated.

$$P_2 = P_3 = \frac{1}{4} P_{in}$$

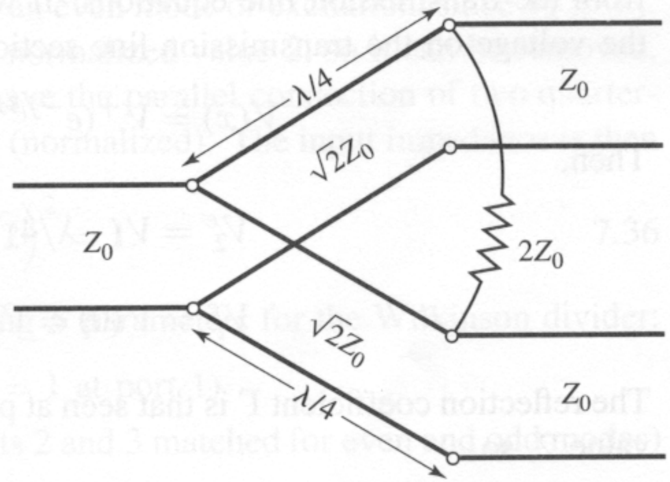
The Wilkinson Power Divider

Motivation: to solve the problem of output isolation

The Wilkinson power divider is a three-port that has all ports matched with isolation between the two output ports.



An equal-split Microstrip Wilkinson power divider



Equivalent circuit

The Wilkinson power divider can be made to give arbitrary power division. But an equal-split one is used here for analysis.

Features of the Wilkinson dividers

1. By choosing the impedance of the $\lambda/4$ lines to be $\sqrt{2}Z_o$, the matched output loads $Z_L = Z_o$ are transformed to $2Z_o$ so they can be placed in parallel to equal $2Z_o/2 = Z_o$ creating a matched condition at the input port.

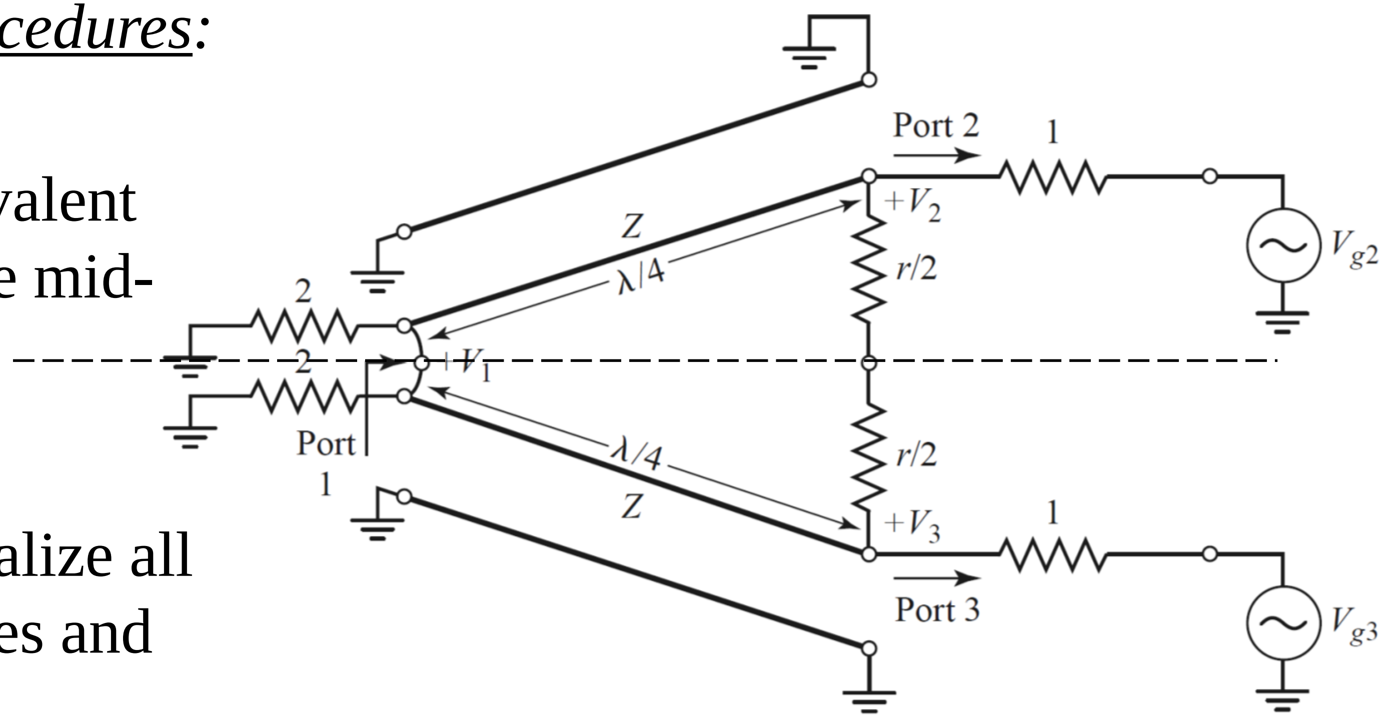
2. Any mismatched power returned from a load at either output port is divided equally between the load on the input port (the generator source impedance, presumed to be $Z_g = Z_o$) and the resistor R.

3. None of the reflected power from a mismatched load is dissipated in the other load, so the output ports are isolated ($S_{23} = S_{32} = 0$).

Even-Odd Mode Analysis

Preparing procedures:

Step 1: Draw a symmetric equivalent circuit across the mid-plane.



Step 2: Normalize all the impedances and resistance.

Step 3: Define even and odd modes

Even mode: $V_{g2} = V_{g3} = 2V$

Odd mode: $V_{g2} = -V_{g3} = 2V$

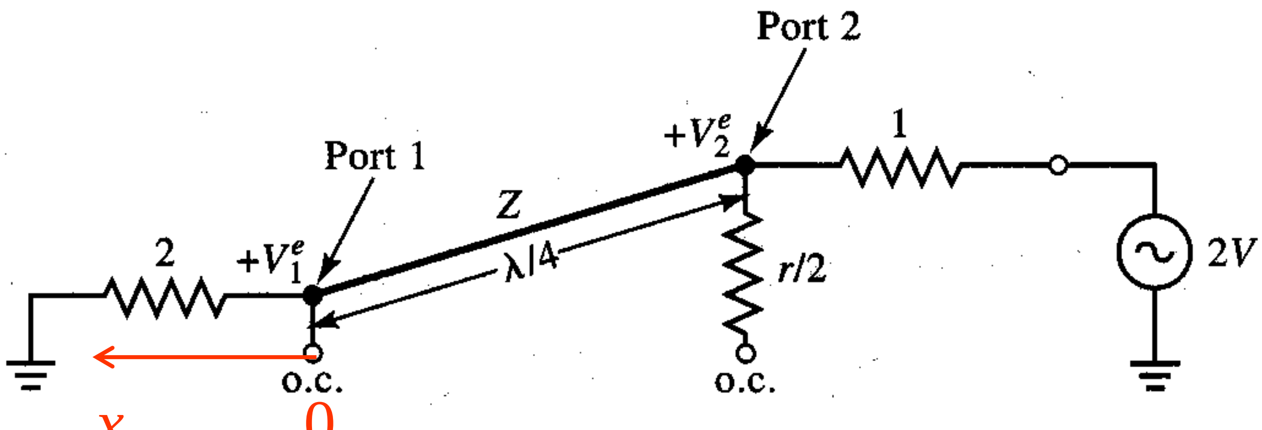
Even mode: $V_{g2} = V_{g3} = 2V$

No current (o.c.) flows through the resistors between port 2 and 3 or the short circuit at port 1.

The impedance looking into port 2 is

$$Z_{in}^{e} = \frac{Z^2}{2}$$

Quarter-wave transformer



Bisection of the equivalent circuit for **<u>even</u>** mode excitation.

For a matched port 2, we have $Z = \sqrt{2}$

so that $V_2^e = V$ and $Z_{in}^e = 1$

Setting up x coordinate as shown in the figure. Looking toward the left from port 1, the transmission line voltage can be written as

$$V(x) = V^{+}(e^{-j\beta x} + \Gamma e^{j\beta x})$$

Then,

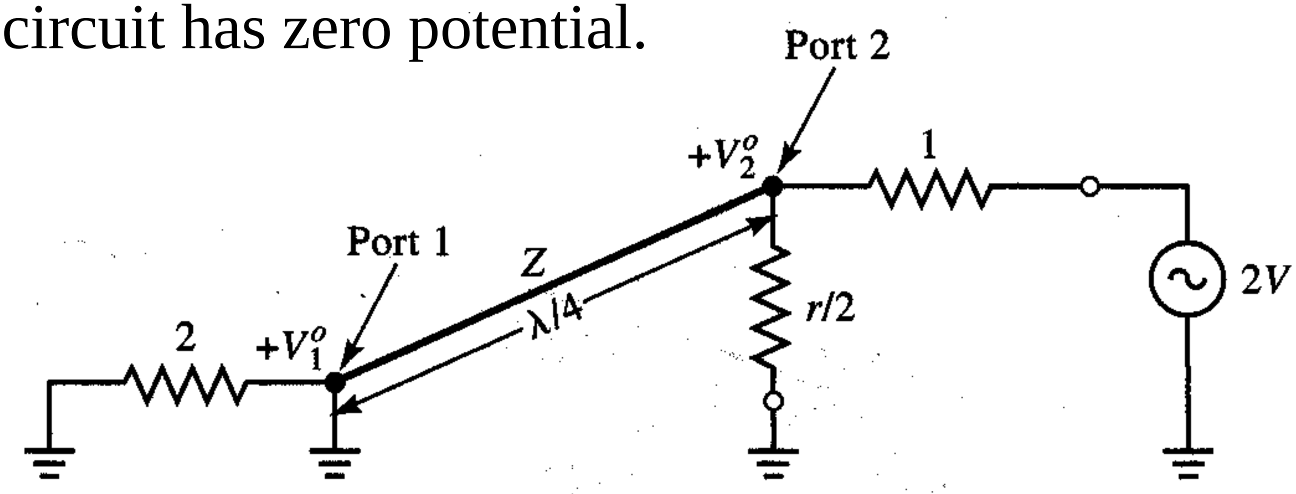
$$V_2^e = V(-\lambda/4) = jV^{+}(1-\Gamma) = V$$

$$V_1^e = V(0) = V^{+}(1+\Gamma) = jV\frac{\Gamma+1}{\Gamma-1}$$

We have $\Gamma = \dfrac{2-\sqrt{2}}{2+\sqrt{2}}$ therefore, $V_1^e = -jV\sqrt{2}$

Odd mode: $V_{g2} = -V_{g3} = 2V$

From symmetry point of view, the middle point of the equivalent circuit has zero potential.



The impedance looking into port 2 is

$$Z_{in}^{o} = r/2$$

A short-circuited quarter-wave line is open.

Bisection of the equivalent circuit for **odd** mode excitation.

For a matched port 2, we have $r = 2$

so that $V_2^o = V$

Then port 2 and 3 are terminated in matched loads, so that we can find S_{11}.

$$Z_{in} = \frac{1}{2}(\sqrt{2})^2 = 1$$

Summary:

$$S_{11} = 0$$

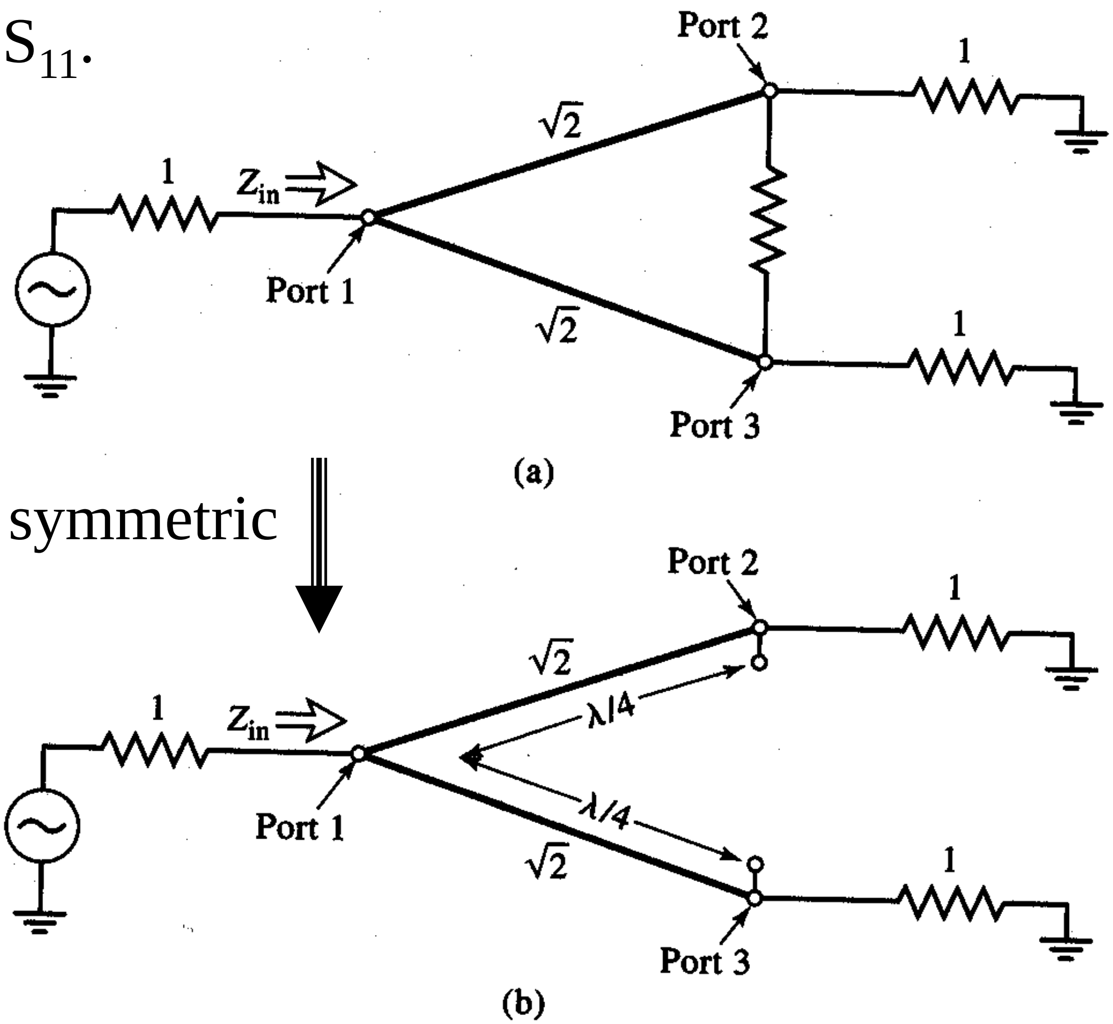
$$S_{22} = S_{33} = 0$$

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o}$$

$$= -j/\sqrt{2}$$

$$S_{13} = S_{31} = -j/\sqrt{2}$$

$$S_{23} = S_{32} = 0$$



When the Wilkinson power divider is driven at port 1 and the outputs are matched, no power is dissipated in the resistor. Thus the divider is lossless when the outputs are matched; only reflected power from ports 2 or 3 is dissipated in the resistor. Since $S_{23} = S_{32} = 0$, ports 2 and 3 are isolated.

Example 7.2 of Pozar.

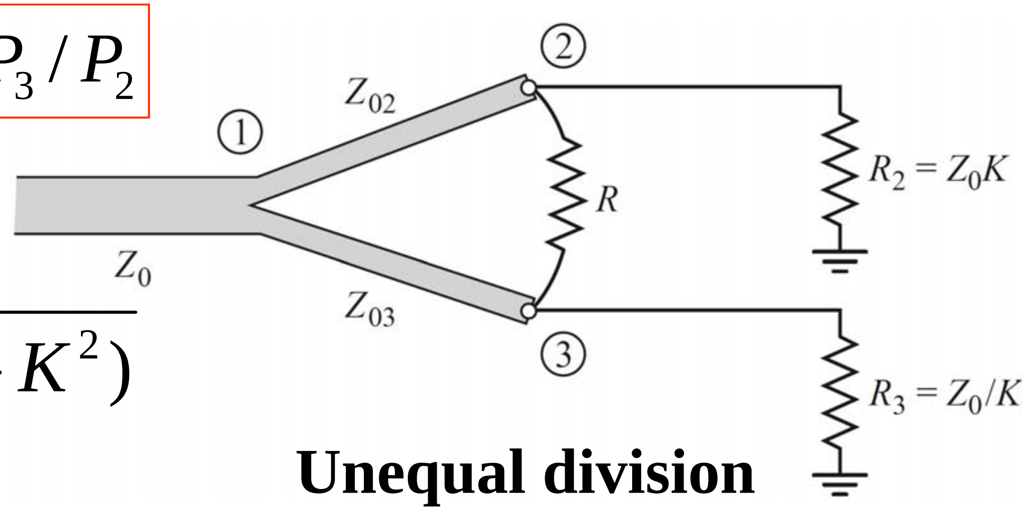
Unequal Power Division and N-Way Wilkinson Dividers

Design equations: $\boxed{K^2 = P_3 / P_2}$

$$Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^3}}$$

$$Z_{02} = K^2 Z_{03} = Z_0 \sqrt{K(1+K^2)}$$

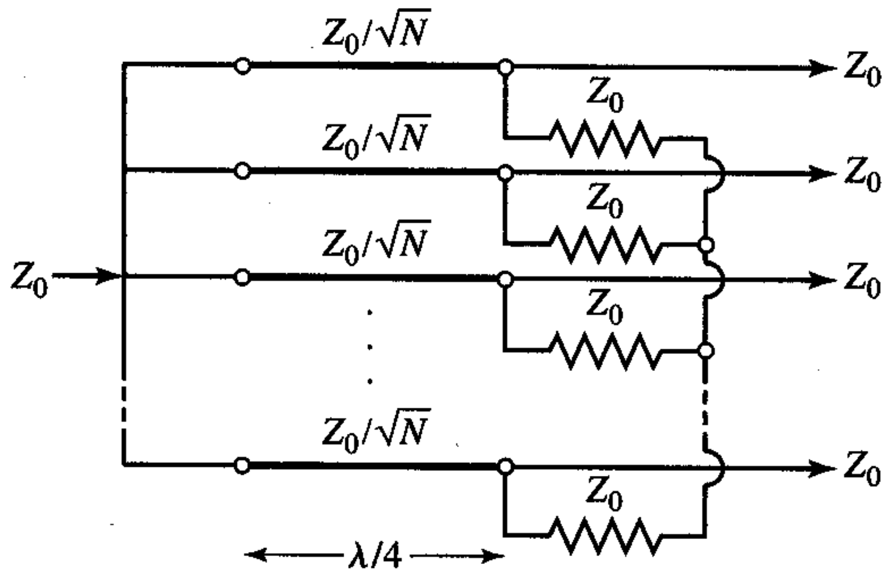
$$R = Z_0\left(K + \frac{1}{K}\right)$$

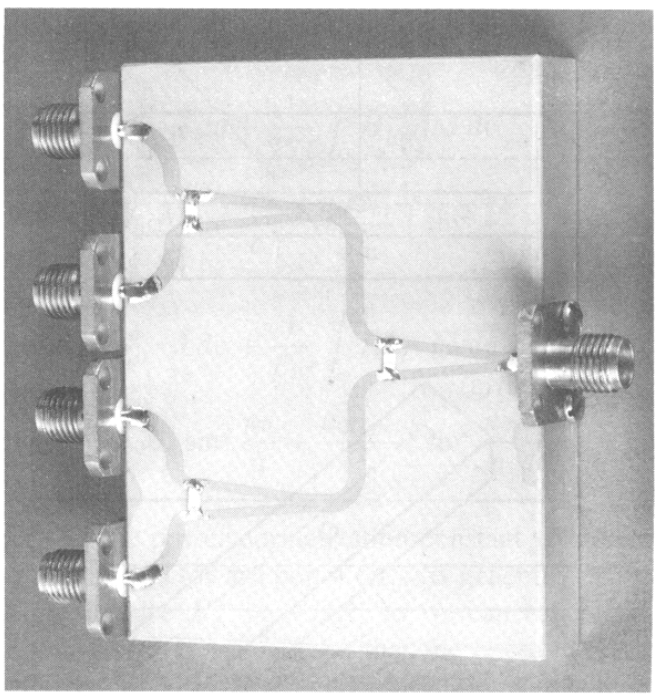


Unequal division

An N-way, equal-split Wilkinson Power Divider

All ports are matched, with isolation between all ports.





Disadvantage: crossovers among the resistors, result in difficult fabrication process.

Solution: using stepped multiple sections.

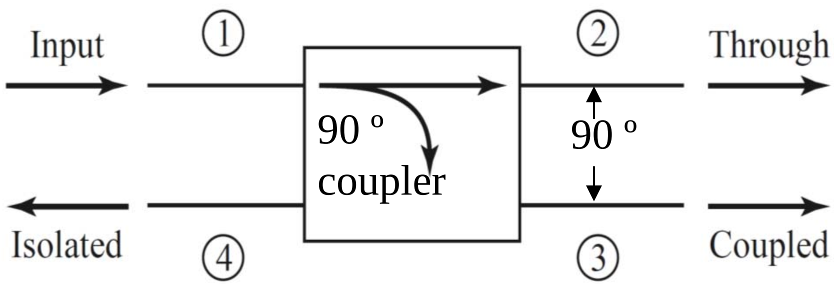
Four Port Networks (Directional Couplers)

Pozar (page 320-322, 4rd Ed.) shows that a matched, reciprocal, lossless four-port network is possible, and that it has directional coupling between pairs of ports.

There are two possible forms of [S] for a directional coupler, one with outputs differing by 90° in phase and the other with outputs differing by 180° in phase.

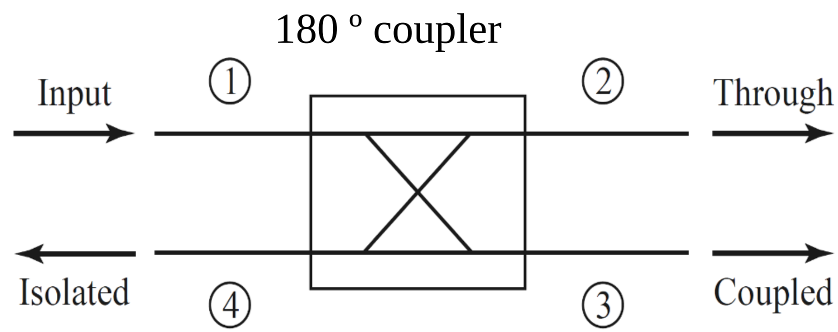
Any 90° coupler can be made into a 180° coupler by adding a 90° transmission to one port, and *vice versa*.. The 90° coupler is referred to as a quadrature hybrid, and can be created using directly connected branch lines between two transmission lines or by means of coupled transmission lines.

Symmetrical coupler



$$[S]=\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

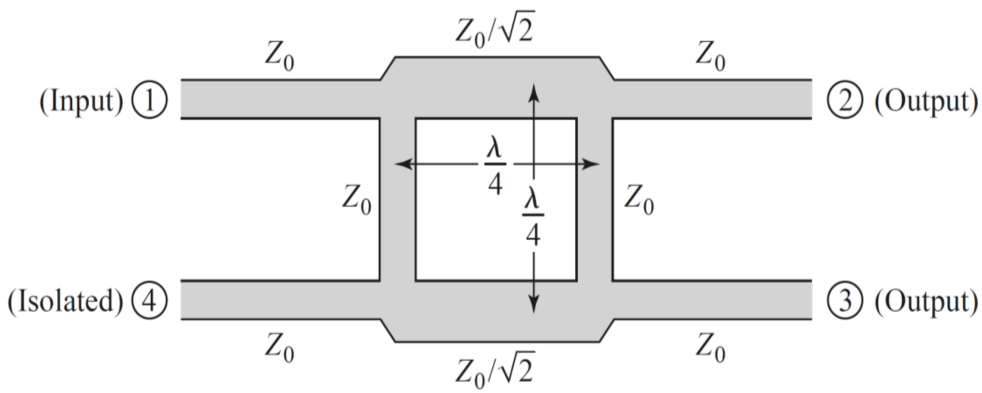
Antisymmetrical coupler



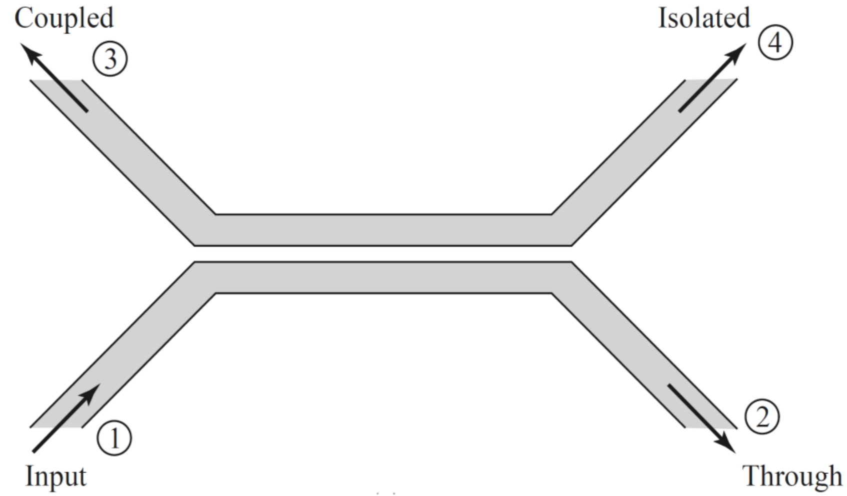
$$[S]=\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$\alpha^2+\beta^2=1$$

Symmetrical Coupler



A branch-line coupler



A coupled-line directional coupler

Coupling, Directionality and Isolation

Coupling = C = 10log(P1/P3) = -20logβ dB

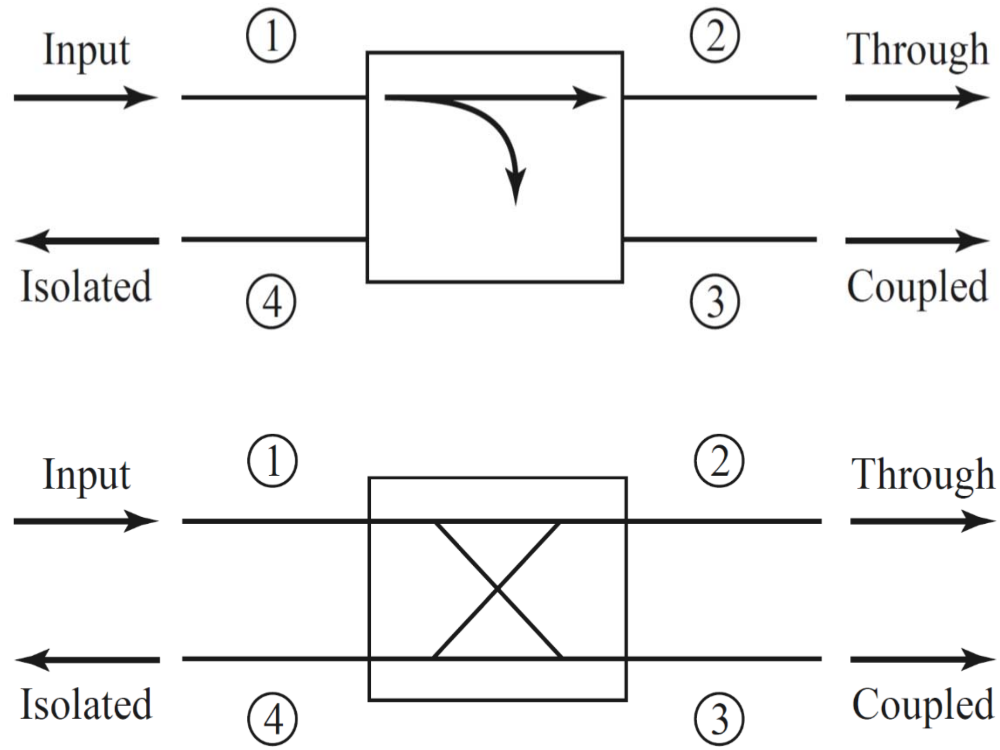
Directivity = D = 10 log (P3/P4) = 20log$\beta/|S_{14}|$ dB

Isolation = I = 10log(P1/P4) = -20log $|S_{14}|$ dB

Insertion loss= IL = 10log(P1/P2) = -20log $|S_{12}|$ dB

$$\boxed{I = D + C \text{ dB}}$$

- A fraction of a wave traveling from port 1 to port 2 via the transmission line is coupled to port 3, but not to port 4.
- A fraction of a wave traveling from port 2 to port 1 is coupled to port 4, but not to port 3.
- The same coupling exists to transmission line 1-2 for waves traveling on line 4-3.



Symbols for directional couplers.

The use of a directional coupler is typically to sample the forward and/or reflected wave on a transmission line.

Recalling the definition of *Coupling*, *Directivity* and *Isolation* factors.

Coupling = C =10log(P_1/P_3)

Directivity = D = 10 log(P_3/P_4)

Isolation = I = 10log(P_1/P_4)

Insertion loss = IL = 10log(P1/P2)

C and *IL* can be measured directly, but the power level at the port 4 **may be small enough to make this** difficult, particularly if the signal coupled from the input to reverse output port 4 is masked by a reflected wave from an imperfectly matched load at port 2.

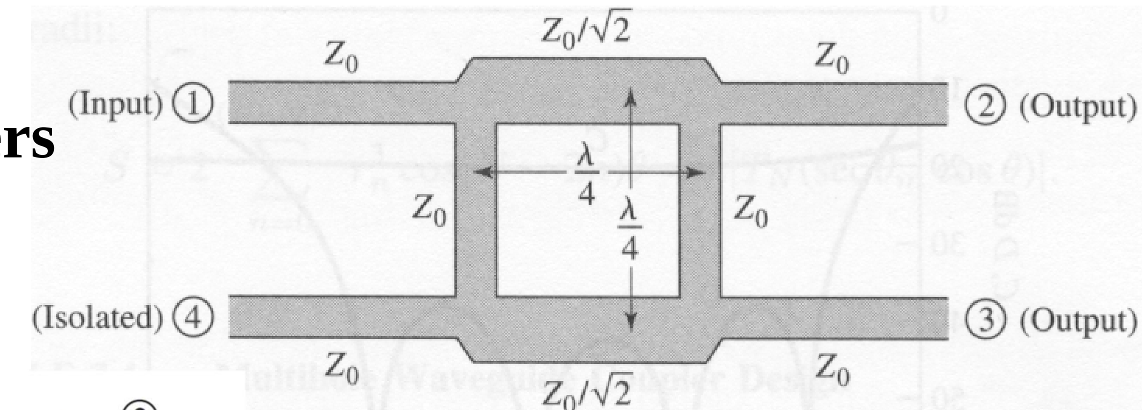
Example: Consider a 20 dB coupler (C = 20 dB) with 30 dB directivity (D = 30 dB) connected to a load with a 15 dB return loss (RL =15 dB).

The power at the reverse coupled port 4 resulting from the incident power at port 1 will be D+C=50 dB below the incident power, while the power at the same port resulting from the load reflection at port 2 will be RL+C=35 dB below the incident power.

Hence the reflected power adequately masks the power that defines the directivity D.

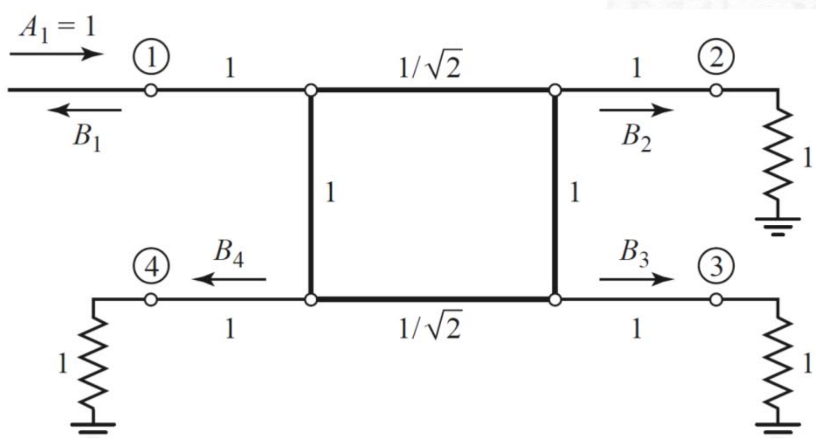
The Quadrature (90º) Hybrid Directional Couplers

- A 3 dB directional coupler with a 90º phase difference in the outputs of the through and coupled arms.
- Exist in the form of branch-line, coupled line and Lange (interdigitated).



Branch-Line Couplers

Highly symmetric

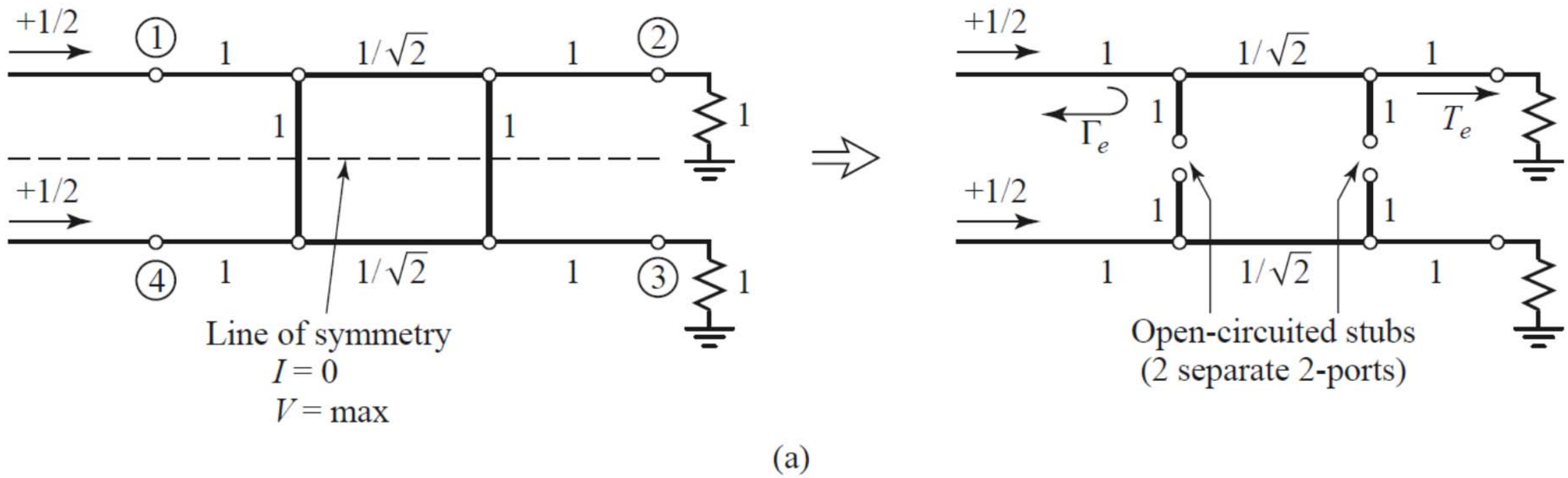


<u>Even-Odd Mode Analysis</u>

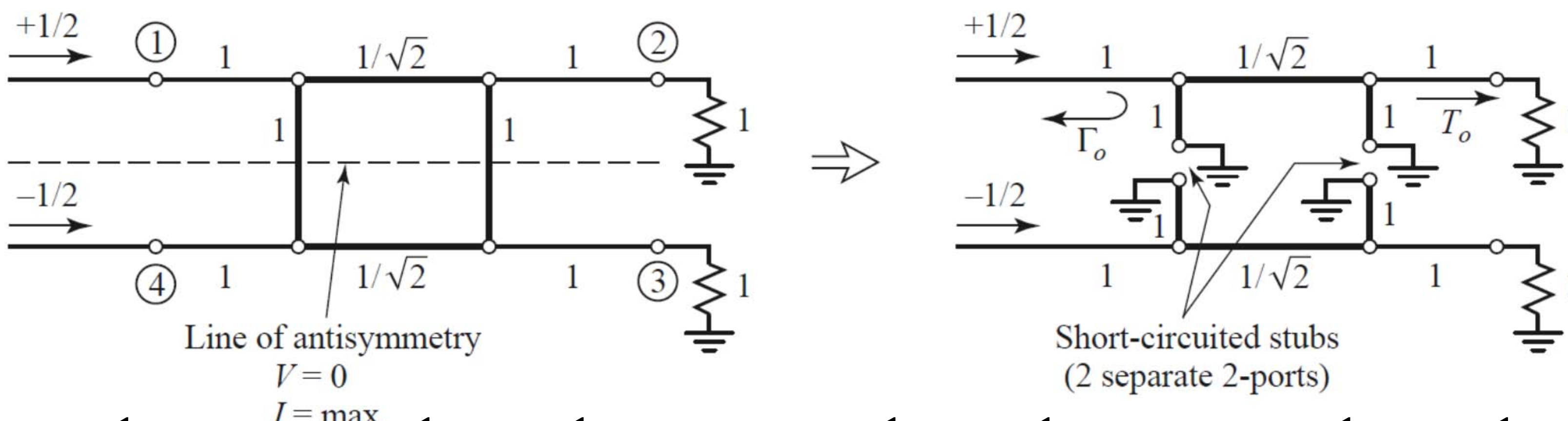
Step 1: Set up the circuit in normalized form.

Decomposition of the coupler into even- and odd-mode excitations.

Even mode



Odd mode



$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

For the even-mode circuit,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \underbrace{\begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}}_{\substack{Shunt \\ Y=j}} \underbrace{\begin{bmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{bmatrix}}_{\substack{\lambda/4 \\ Transmission \\ line}} \underbrace{\begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}}_{\substack{Shunt \\ Y=j}} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

We can then convert the ABCD matrix to S parameters, which are equivalent of the reflection and transmission coefficients. Thus,

$$\Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{(-1+j-j+1)/\sqrt{2}}{(-1+j+j-1)/\sqrt{2}} = 0$$

$$T_e = \frac{2}{A+B+C+D} = \frac{2}{(-1+j+j-1)/\sqrt{2}} = \frac{-1}{\sqrt{2}}(1+j)$$

For the odd-mode circuit, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$

Then, $\Gamma_o = 0$ $\quad T_o = \frac{1}{\sqrt{2}}(1-j)$

So,

$B_1 = 0 \quad$ (port 1 is matched)

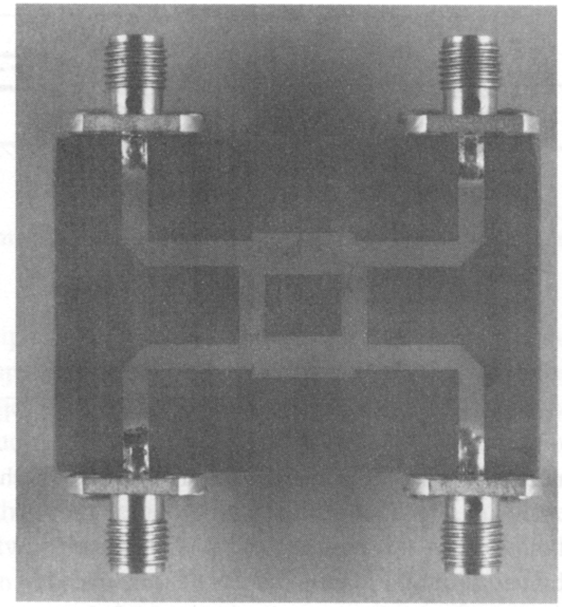
$B_2 = -\frac{j}{\sqrt{2}} \quad$ (half - power, -90° phase shift from port 1 to 2)

$B_3 = -\frac{1}{\sqrt{2}} \quad$ (half - power, -180° phase shift from port 1 to 3)

$B_4 = 0 \quad$ (no power to port 4)

The [S] matrix is given by $\quad [S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$

A practical microstrip quadrature hybrid prototype.



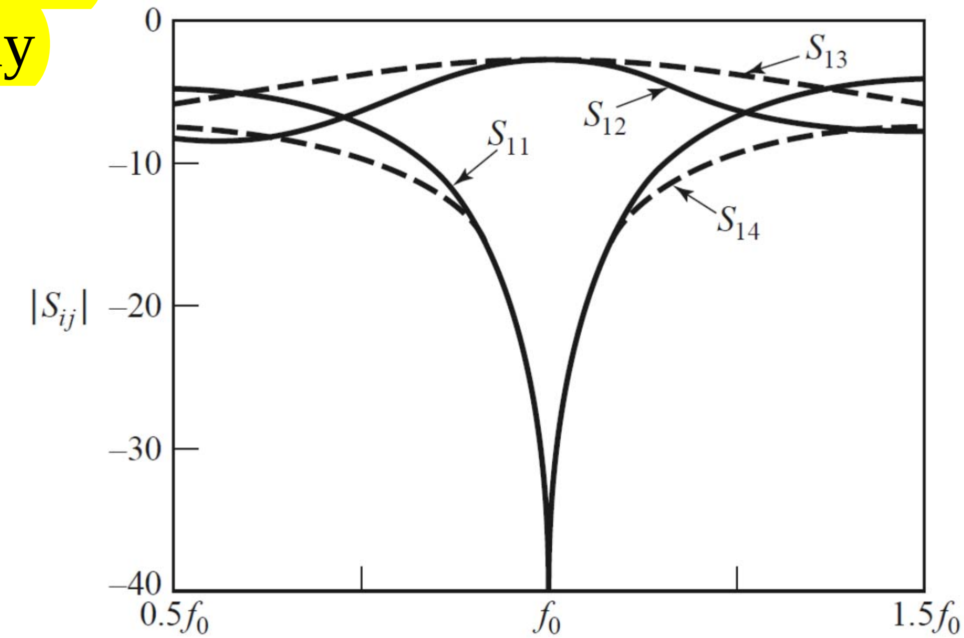
Some practical design issues:

1. Limited bandwidth: due to the nature of quarter-wave line. 10-20% BW Multisection design will help.

2. The effect of discontinuity at the junctions: shunt arms are usually lengthened by 10º - 20º.

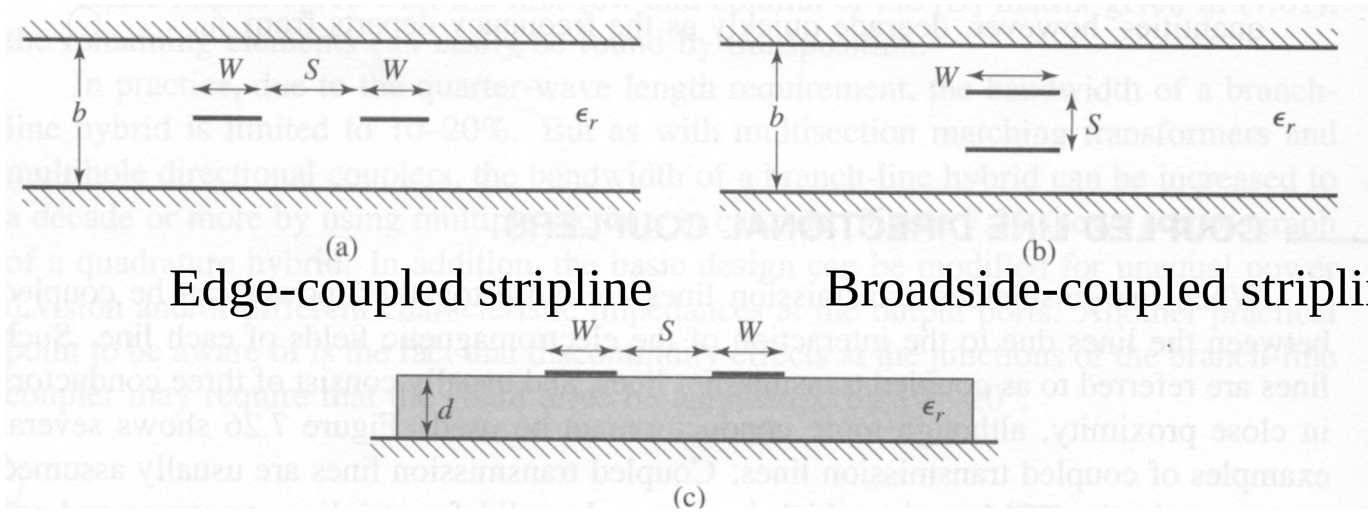
Example 7.5 of Pozar

Frequency response ⟹



Coupled-Line Directional Couplers

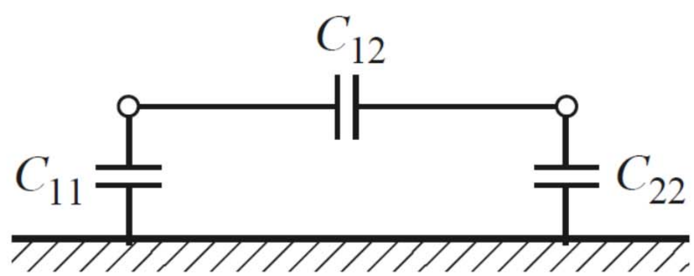
Various geometries



Edge-coupled stripline

Broadside-coupled stripline

Edge-coupled microstrip line.

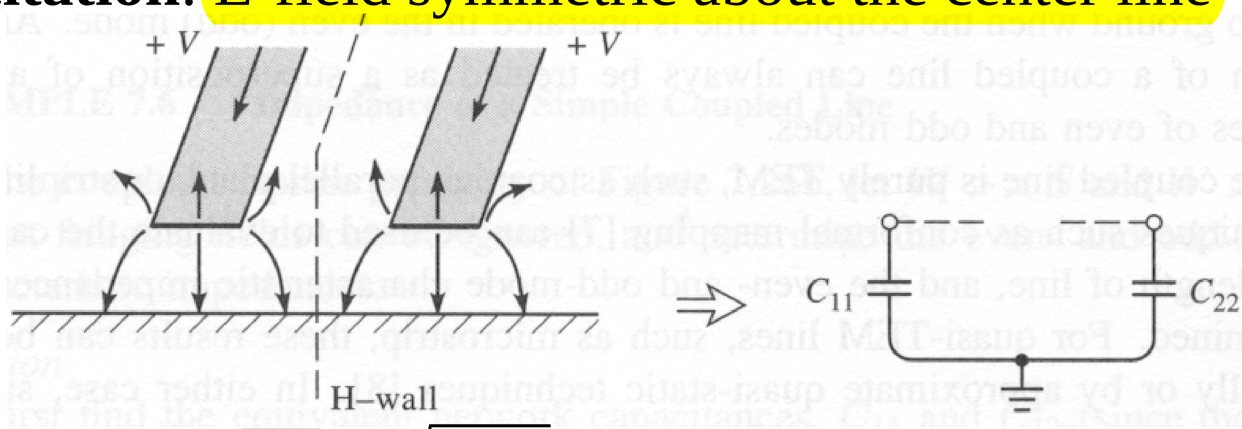


Equivalent circuit

Even mode excitation: E-field symmetric about the center line

C_{12} is open-circuited

$C_e = C_{11} = C_{22}$, assuming the two strips are identical in size and location.

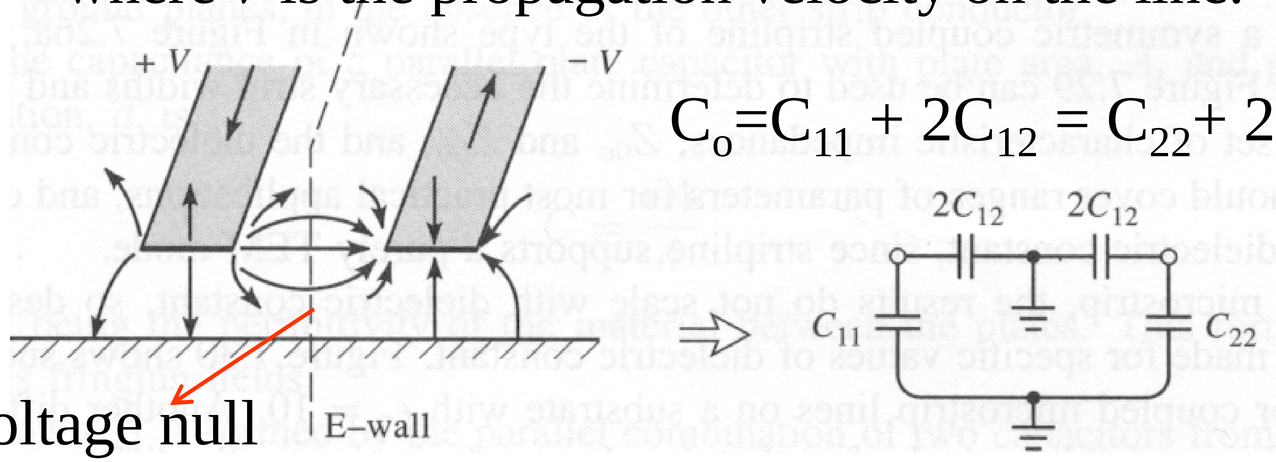


$$Z_{0e} = \sqrt{\frac{L_e}{C_e}} = \frac{\sqrt{L_e C_e}}{C_e} = \frac{1}{v_p C_e},$$

where v is the propagation velocity on the line.

Odd mode excitation

$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$



$$Z_{0o} = \frac{1}{v_p C_o}$$

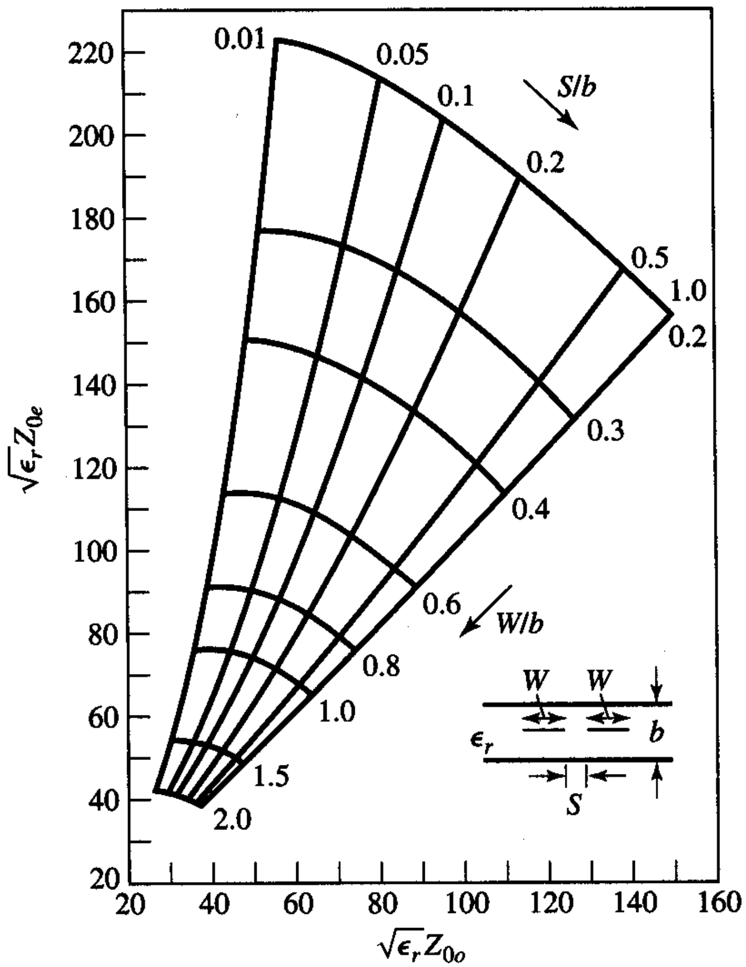
Voltage null

- Z_{0e} (Z_{0o}) is the characteristic impedance of one of the strip conductors relative to ground when the coupled line is operated in the even (odd) mode.

- An *arbitrary* excitation of a coupled line can always be treated as a *superposition* of appropriate amplitudes of even and odd modes.

- For TEM lines, analytical techniques can be used to evaluate the capacitance per unit length of line, and the even- and odd-mode characteristic impedances can then be determined. For quasi-TEM lines, numerical or quasi-static techniques have to be used to find the results.

Design of Coupled-Line Couplers: using design graphs

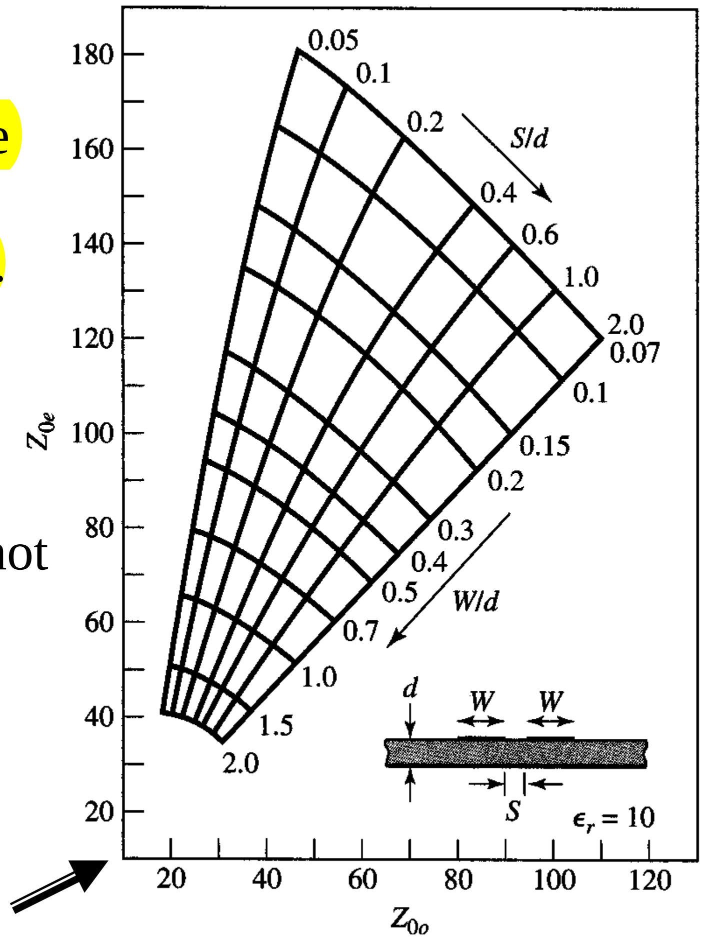


Edge-coupled stripline

TEM lines: scalable with different dielectric constants.

Quasi-TEM lines: not scalable with different dielectric constants

$\varepsilon_r = 10$



coupled microstrip lines

EXAMPLE 7.6 IMPEDANCE OF A SIMPLE COUPLED LINE

For the broadside coupled stripline geometry of Figure 7.26b, assume $W \gg S$ and $W \gg b$, so that fringing fields can be ignored, and determine the even- and odd-mode characteristic impedances.

Solution

We first find the equivalent network capacitances, C_{11} and C_{12} (because the line is symmetric, $C_{22} = C_{11}$). The capacitance per unit length of broadside parallel lines with width W and separation d is

$$\bar{C} = \frac{\epsilon W}{d} \text{ F/m},$$

where ϵ is the substrate permittivity. This formula ignores fringing fields.

C_{11} is formed by the capacitance of one strip to the ground planes. Thus the capacitance per unit length is

$$\bar{C}_{11} = \frac{2\epsilon_r \epsilon_0 W}{b - s} \text{ F/m.}$$

The capacitance per unit length between the strips is

$$\bar{C}_{12} = \frac{\epsilon_r \epsilon_0 W}{S} \text{ F/m.}$$

Then from (7.68) and (7.70), the even- and odd-mode capacitances are

$$\bar{C}_e = \bar{C}_{11} = \frac{2\epsilon_r \epsilon_0 W}{b - S} \text{ F/m,}$$

$$\bar{C}_o = \bar{C}_{11} + 2\bar{C}_{12} = 2\epsilon_r \epsilon_0 W \left(\frac{1}{b - S} + \frac{1}{S} \right) \text{ F/m.}$$

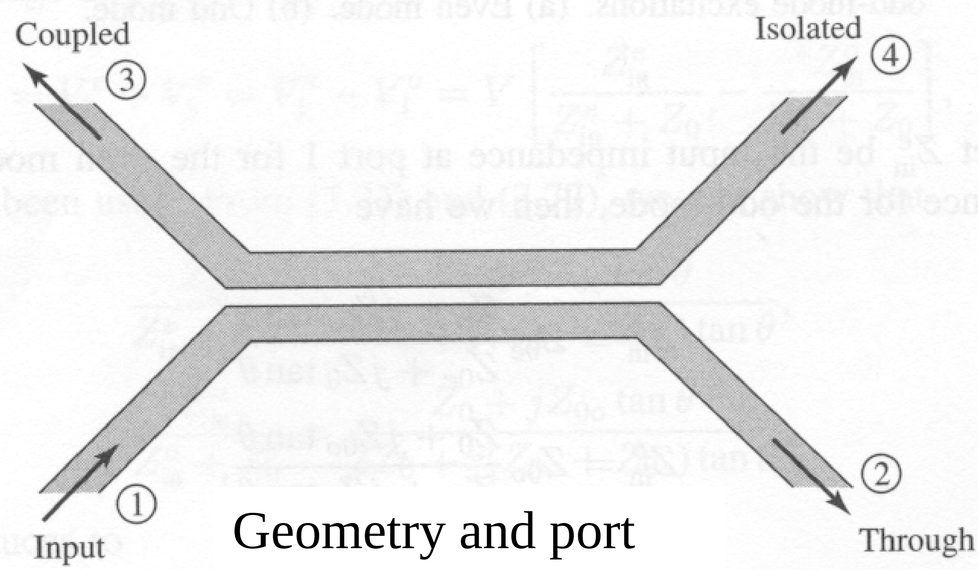
The phase velocity on the line is $v_p = 1/\sqrt{\epsilon_r \epsilon_0 \mu_0} = c/\sqrt{\epsilon_r}$, so the characteristic impedances are

$$Z_{0e} = \frac{1}{v_p \bar{C}_e} = \eta_0 \frac{b - S}{2W\sqrt{\epsilon_r}},$$

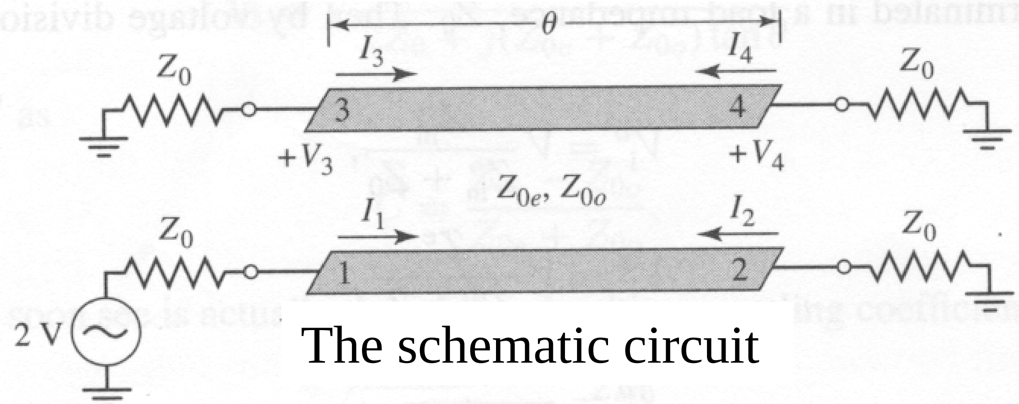
$$Z_{0o} = \frac{1}{v_p \bar{C}_o} = \eta_0 \frac{1}{2W\sqrt{\epsilon_r}[1/(b - S) + 1/S]}. \qquad \blacksquare$$

Design of Coupled Line Couplers

- One port driven with a 2V voltage generator (with internal impedance of Z_0) with other three ports terminated in Z_0.
- A ground conductor is the common ground to both strip conductors.
- Design goals: port 1 is matched and port 4 is isolated.



Geometry and port designations



The schematic circuit

Apply the even-odd mode analysis (avoiding the use of reflection and transmission coefficients).

For symmetry, we have:

<u>Even-mode</u>

$$I_1^e = I_3^e \qquad I_4^e = I_2^e$$

$$V_1^e = V_3^e \qquad V_4^e = V_2^e$$

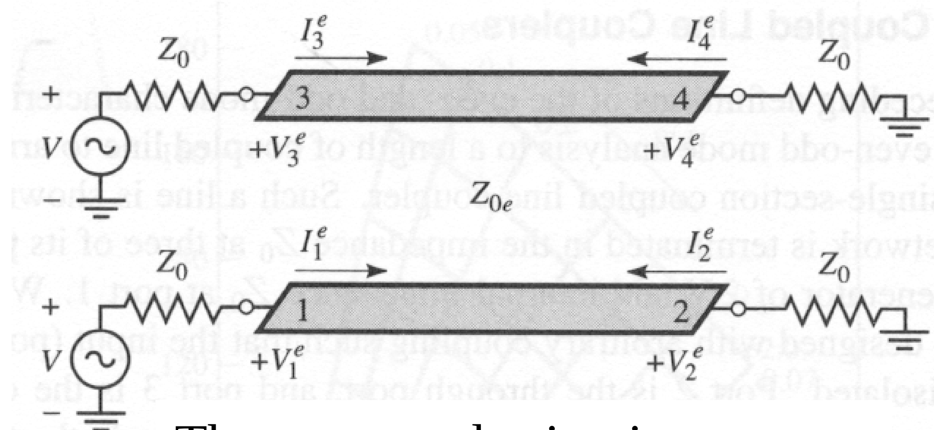
<u>Odd-mode</u>

$$I_1^o = -I_3^o \qquad I_4^o = -I_2^o$$

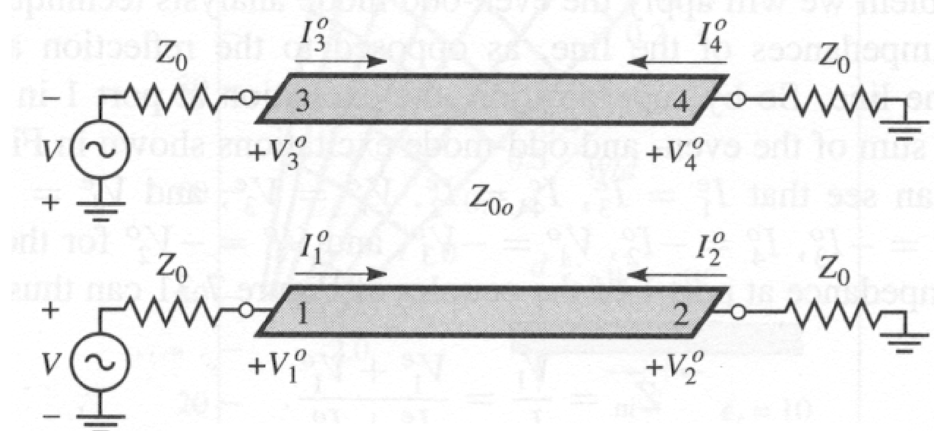
$$V_1^o = -V_3^o \qquad V_4^o = -V_2^o$$

The input impedance at port 1 is:

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o}$$



The even-mode circuits.



The odd-mode circuits.

Task: write the voltages and currents in terms of Z_{0e} and Z_{0o}.

Based on voltage division, we have

$$V_1^o = V\frac{Z_{in}^o}{Z_{in}^o + Z_0} \qquad V_1^e = V\frac{Z_{in}^e}{Z_{in}^e + Z_0}$$

$$I_1^o = \frac{V}{Z_{in}^o + Z_0} \qquad I_1^e = \frac{V}{Z_{in}^e + Z_0}$$

$$Z_{\text{in}} = \frac{Z_{\text{in}}^o(Z_{\text{in}}^e + Z_0) + Z_{\text{in}}^e(Z_{\text{in}}^o + Z_0)}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0} = Z_0 + \frac{2(Z_{\text{in}}^o Z_{\text{in}}^e - Z_0^2)}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0}$$

Meanwhile, the input impedance at port 1 for even and odd modes are:

$$Z_{in}^e = Z_{0e}\frac{Z_0 + jZ_{0e}\tan\theta}{Z_{0e} + jZ_0\tan\theta} \qquad Z_{in}^o = Z_{0o}\frac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta}$$

If we let $Z_0 = \sqrt{Z_{0e}Z_{0o}}$

The expression for Z_{in} is then given as,

$$Z_{in}^e = Z_{0e}\frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta} \qquad Z_{in}^o = Z_{0o}\frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}$$

when $Z_0 = \sqrt{Z_{0e} Z_{0o}}$ We have $Z_{in}^e Z_{in}^o = Z_{0e} Z_{0o} = Z_0^2$

and $Z_{in} = Z_0$ Port 1 is matched.

Now, let us look at the coupling between the two lines, shown at port 3.

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = V\left[\frac{Z_{in}^e}{Z_{in}^e + Z_0} - \frac{Z_{in}^o}{Z_{in}^o + Z_0}\right]$$

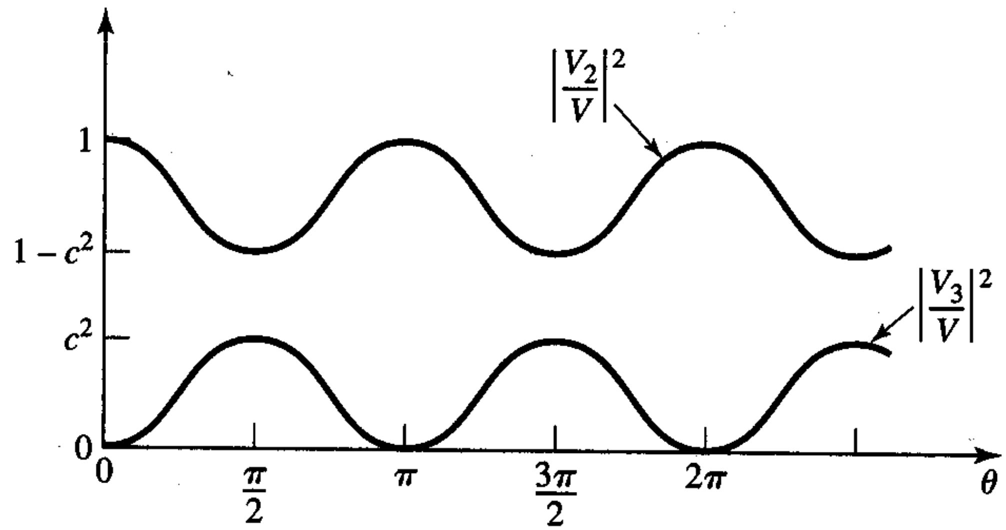
Plug in the expression of Z_{in}^e and Z_{in}^o

$$V_3 = V\frac{j(Z_{0e} - Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}$$

Now, we can define C as $$C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

Then, $\sqrt{1-C^2} = \dfrac{2Z_0}{Z_{0e}+Z_{0o}}$ So that, $V_3 = V \dfrac{jC\tan\theta}{\sqrt{1-C^2}+j\tan\theta}$

So there is coupling between the lines that is periodic in the line electrical length. The coupled line network is generally designed for a line length of λ/4, the first maximum of coupling. For these conditions the analysis yields, for a voltage coupling ratio C, the required impedances



Coupled and through port voltages (squared) versus frequency for the coupled-line coupler.

$Z_{0e} = Z_0\sqrt{\dfrac{1+C}{1-C}}$ $Z_{0o} = Z_0\sqrt{\dfrac{1-C}{1+C}}$

We can show that, $V_4 = V_4^e + V_4^o = V_2^e - V_2^o = 0$

and $$V_2 = V_2^e + V_2^o = V \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

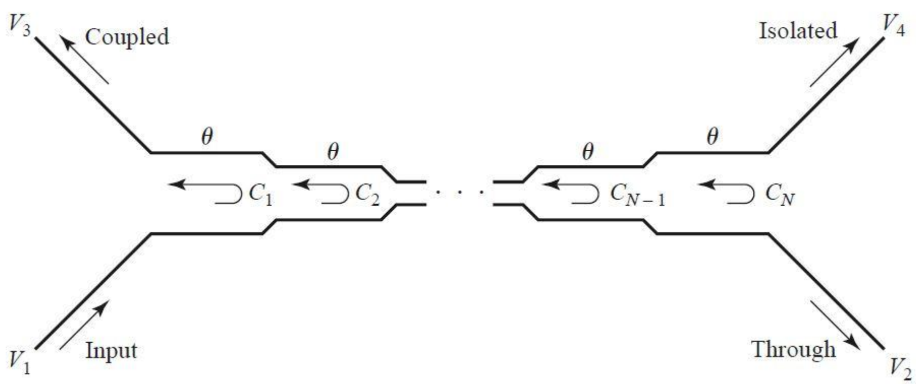
Example 7.7 of Pozar.

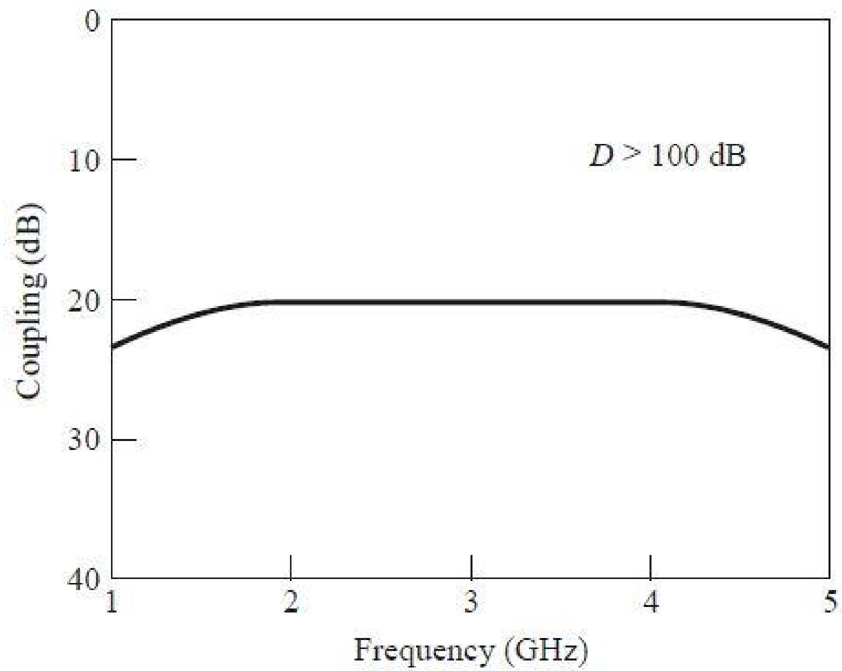
Design notes:

For microstrip, these can be read off a nomogram, simplifying the design of practical couplers. Note that the assumption of equal propagation velocities for the even and odd modes is not accurate for microstrip, so microstrip couplers are generally not as directive as couplers built with TEM transmission lines such as coaxial cable and stripline.

In general, line spacings required for 3 and 6 dB couplers are too close to maintain good tolerances with single edge-coupled transmission lines. Also, the bandwidth limitations of quarter-wave couplers are not suited for practical broadband circuits.

Multisection Coupled Line Couplers





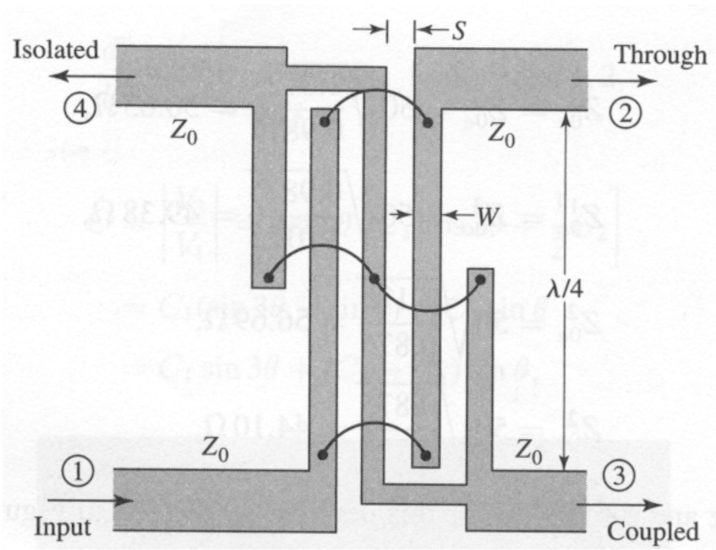
Coupling versus frequency for the three-section binomial coupler

Very Important Self Study:
Pozar, Chapter 7, Section 6

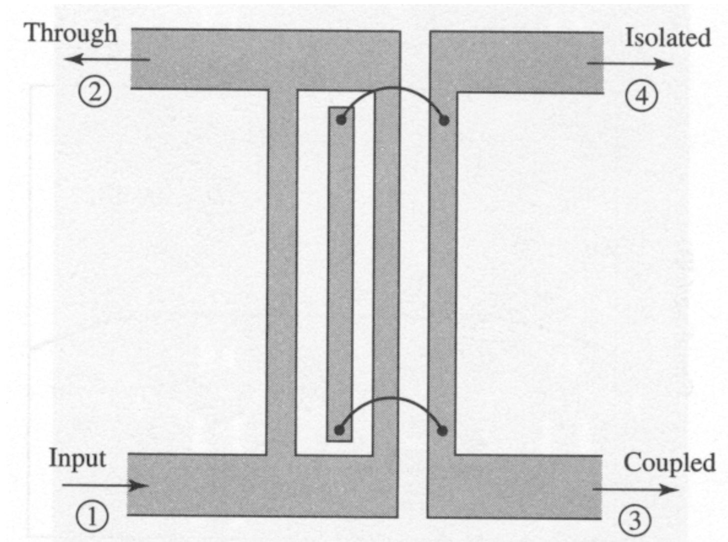
The Lange Couplers

- Coupled-line couplers are not suitable to achieve coupling factors of 3 dB or 6 dB, due to the loose coupling.
- Tight coupling can be achieved by the special layout of the coupled lines, so that the fringing fields at the edge of the lines can contribute to the coupling.

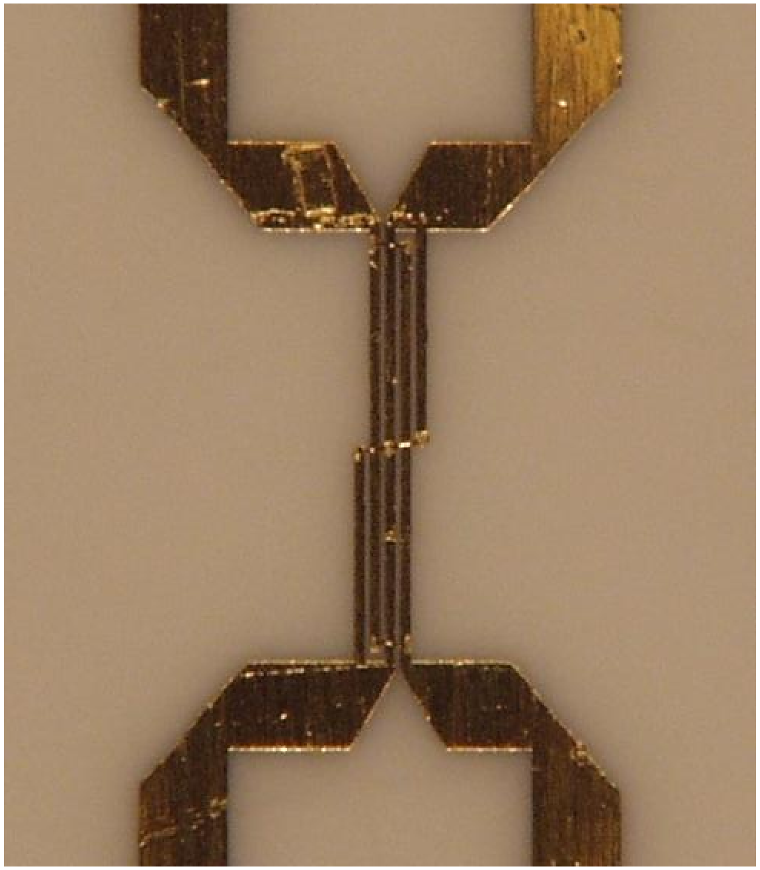
The interdigitated Lange coupler

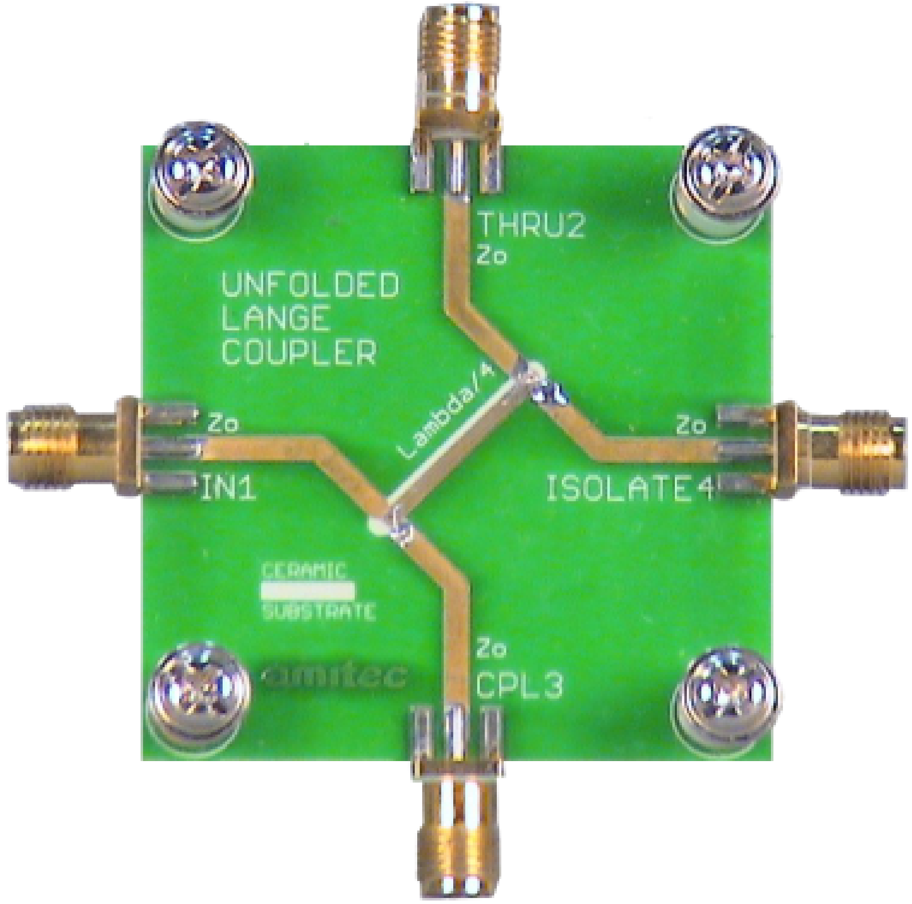


The unfolded Lange coupler



Fabricated Lange Coupler

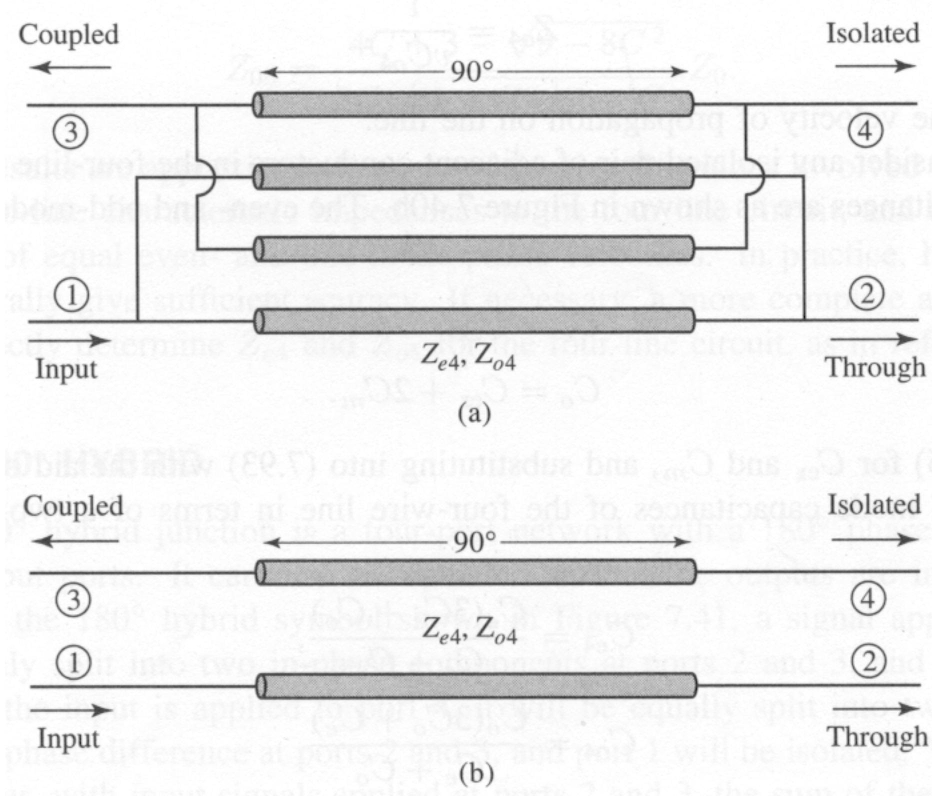




The Lange couplers has the following features:

- There is a 90º phase difference between the output lines (ports 2 and 3)
- Difficult to fabricate the bonding wires due to the narrow spacing between the lines.

For the sake of simplicity, the unfolded Lange coupler is analyzed.



Equivalent circuits for the unfolded Lange Coupler.

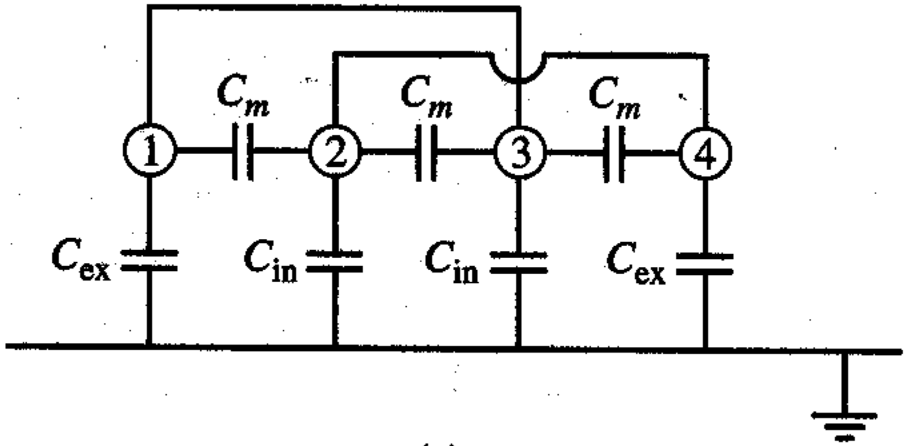
Four-wire coupled line model.

Approximate two-wire coupled line model.

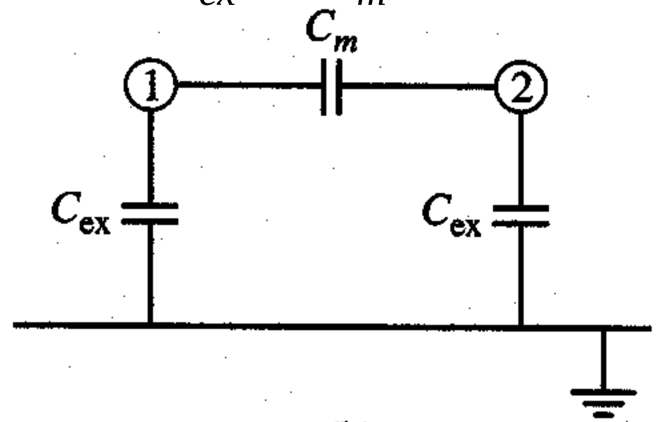
Assumption is made that each line couples only to its nearest neighbor.

Effective capacitance networks

$$C_{in} = C_{ex} - \frac{C_{ex}C_m}{C_{ex} + C_m}$$



Four-wire model (a)



(b)

Two-wire model

Line capacitance:

Even Mode: $C_{e4} = C_{ex} + C_{in}$

$$C_e = C_{ex}$$

Odd Mode: $C_{o4} = C_{ex} + C_{in} + 6C_m$

$$C_o = C_{ex} + 2C_m$$

Then

$$Z_{e4} = \frac{1}{v_p C_{e4}} \qquad Z_{o4} = \frac{1}{v_p C_{o4}}$$

We are to determine Ze4 and Zo4 in terms of Ze and Zo

Solve for C_{e4} and C_{o4} with C_e and C_o, we have

$$C_{e4} = C_e \frac{3C_e + C_o}{C_e + C_o} \qquad C_{o4} = C_o \frac{3C_o + C_e}{C_e + C_o}$$

$$Z_{e4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0o} + Z_{0e}} Z_{0e} \qquad Z_{o4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0e} + Z_{0o}} Z_{0o}$$

Just like the coupled-line couplers, we have

$$Z_0 = \sqrt{Z_{e4} Z_{o4}} = \sqrt{\frac{Z_{0e} Z_{0o} (Z_{0e} + Z_{0o})^2}{(3Z_{0o} + Z_{0e})(3Z_{0e} + Z_{0o})}}$$

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}}$$

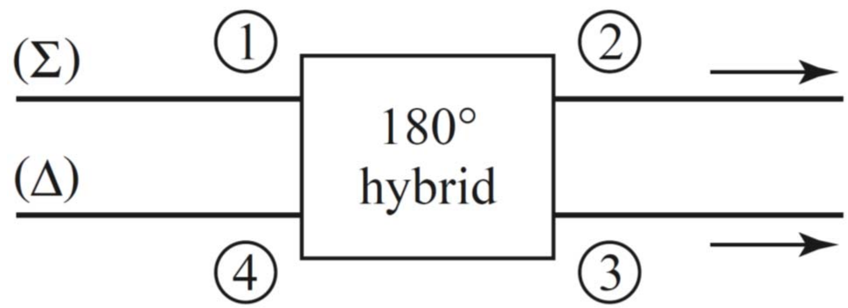
C is the coupling coefficient.

For design purpose, we can obtain

$$Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}} Z_0 \qquad Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}} Z_0$$

The 180º Hybrid

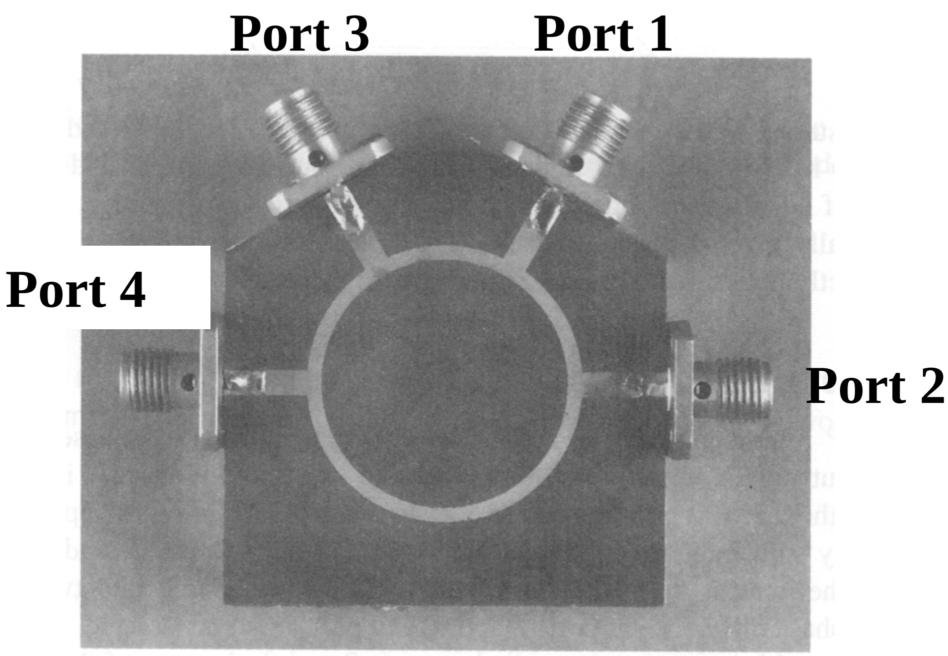
- The two outputs are either in phase or with a **180º** phase difference.
- A signal applied to port 1 is evenly divided into two in-phase components at port 2 and 3, with port 4 isolated.
- A signal applied to port 4 is equally split into two out-of-phase components at port 2 and 3, with port 1 isolated.

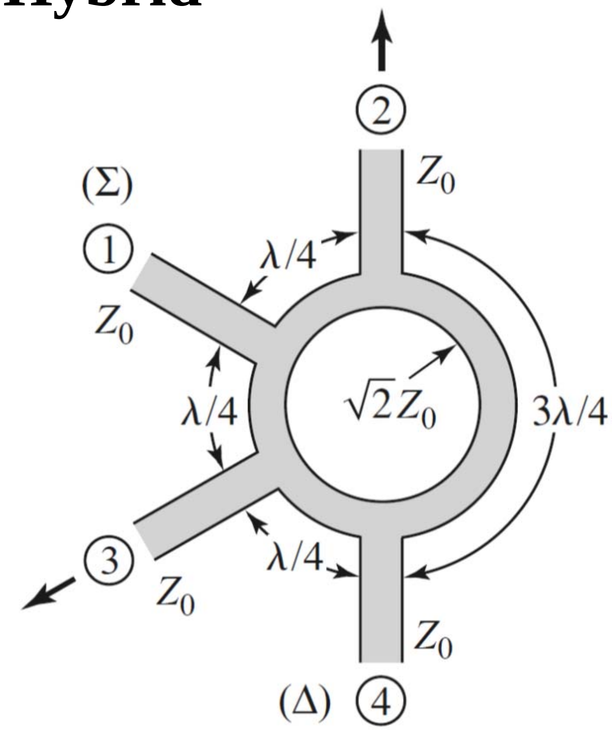


$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

- When operated as a combiner, with inputs at ports 2 and 3, the sum of the inputs will be formed at port 1 (the sum port), while the difference will be formed at port 4 (the difference port).

Ring Hybrid (rat-race): a typical 180º Hybrid

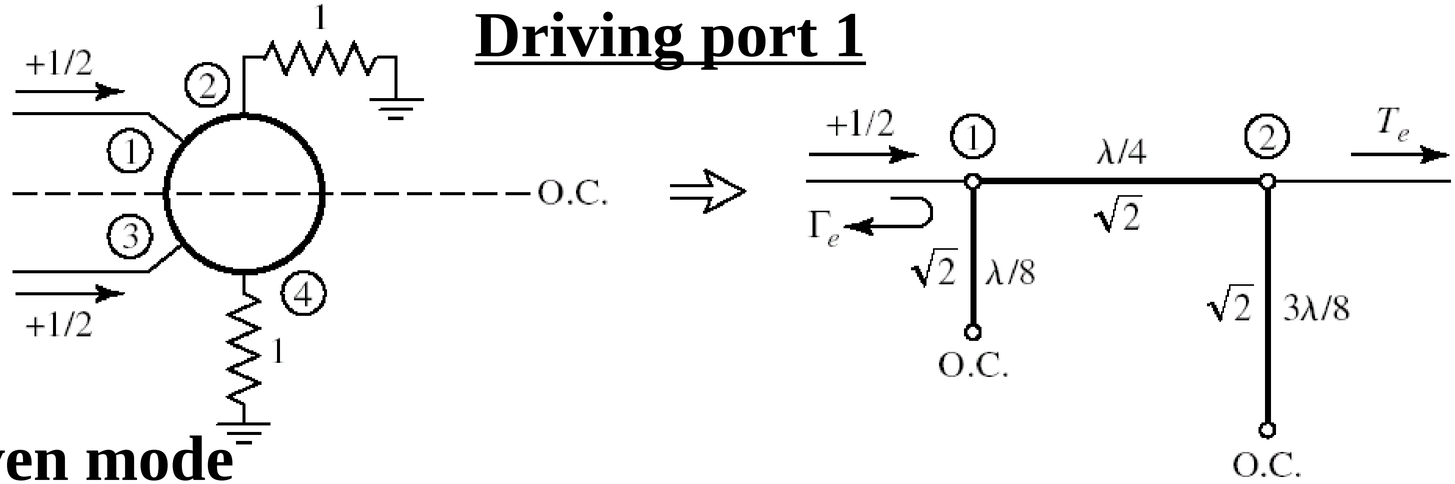




Similar to branch-line coupler. Can be easily constructed in planar (microstrip or stripline) form.

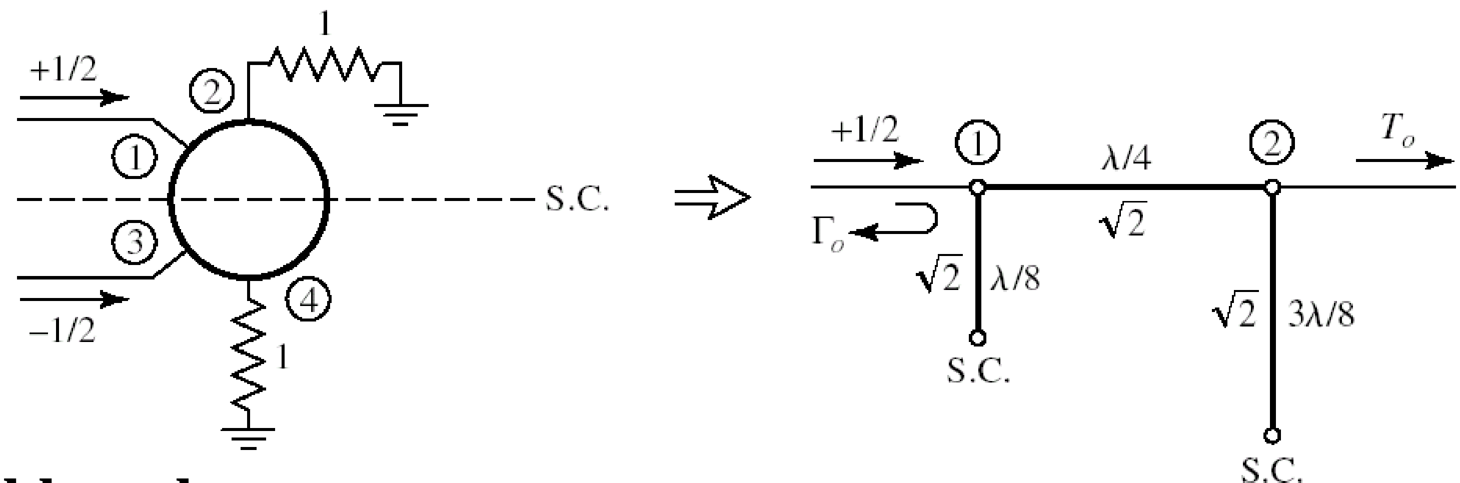
Even-Odd Mode Analysis of the Ring Hybrid

Driving port 1



Even mode

(a)



Odd mode

(b)

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

ABCD matrix for the even- and odd-mode circuits are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

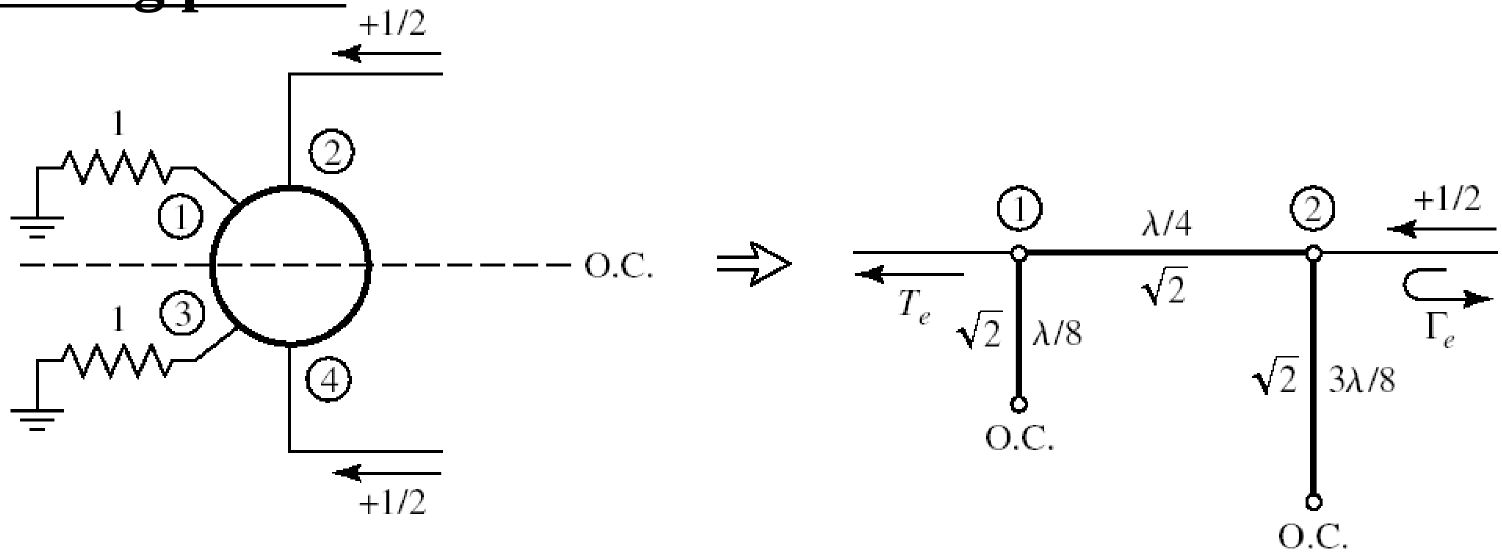
Which lead to,

$$\Gamma_e = \frac{-j}{\sqrt{2}} \qquad T_e = \frac{-j}{\sqrt{2}} \qquad \Gamma_o = \frac{j}{\sqrt{2}} \qquad T_o = \frac{-j}{\sqrt{2}}$$

and,

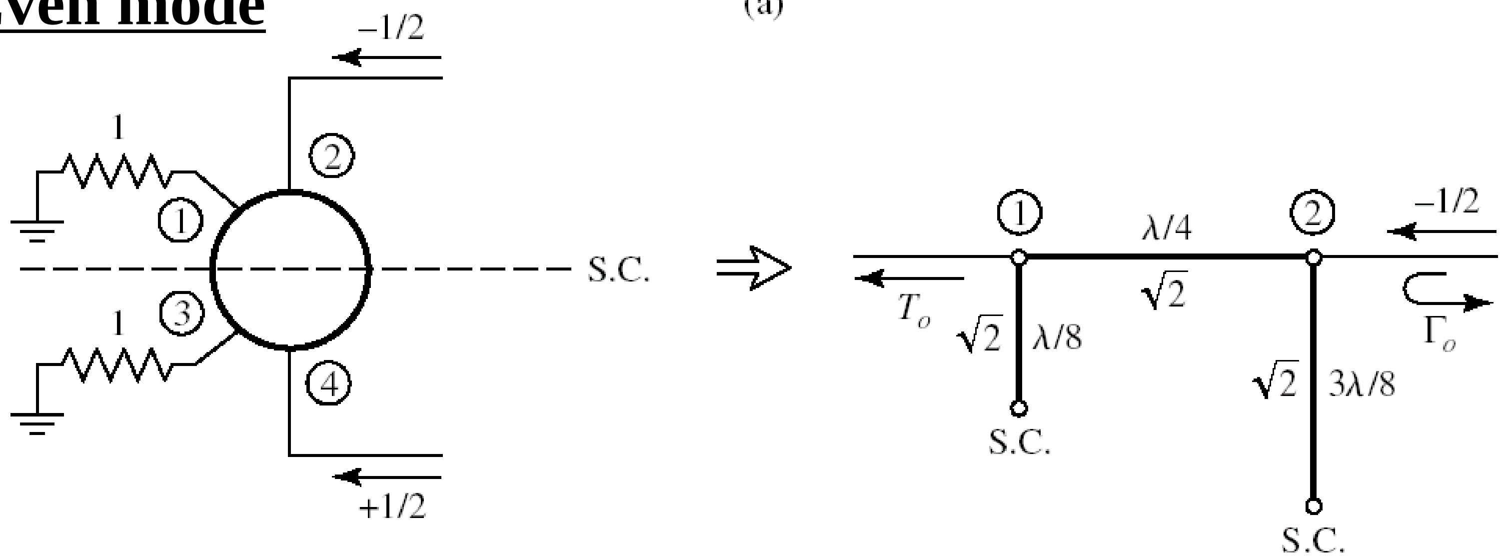
$$B_1 = 0 \qquad B_2 = \frac{-j}{\sqrt{2}} \qquad B_3 = \frac{-j}{\sqrt{2}} \qquad B_4 = 0$$

Driving port 4



Even mode

(a)



Odd mode

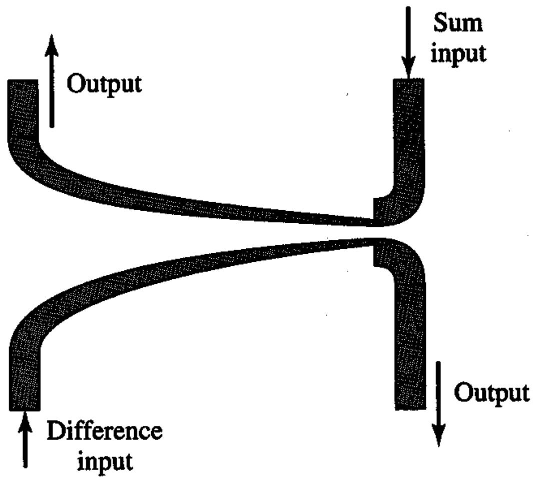
(b)

Following the same ABCD-based analysis

$$B_1 = 0 \qquad B_2 = \frac{j}{\sqrt{2}} \qquad B_3 = \frac{-j}{\sqrt{2}} \qquad B_4 = 0$$

Other Types of Hybrid

A tapered coupled line hybrid



Can be used to achieve any power division ratio

waveguide hybrid junction, or *magic-T*