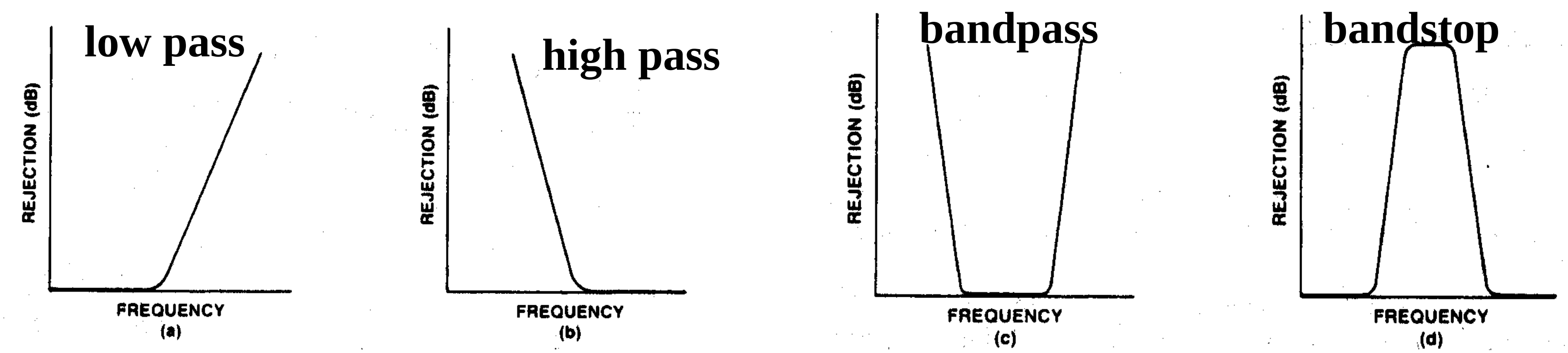
Microwave Filters

- Used to control the frequency response at a certain point in a microwave system by providing transmission at frequencies within the *passband* of the filter and attenuation in the *stopband* of the filter.
- Can be found in any type of microwave communication, radar, or test and measurement system.

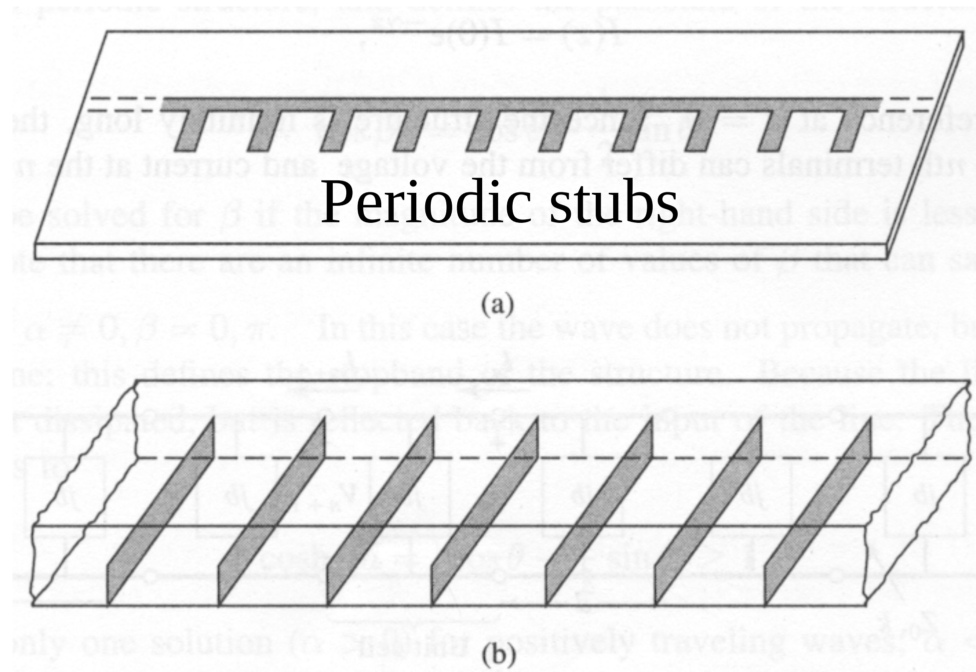


- Periodic structures, which consists of a transmission line or waveguide periodically loaded with reactive elements, exhibit the fundamental passband and stopband behavior --- the analysis follows that of the wave propagation in crystalline lattice structures of semiconductor materials.

Periodic Structures

1. Assume a infinite periodic structure.

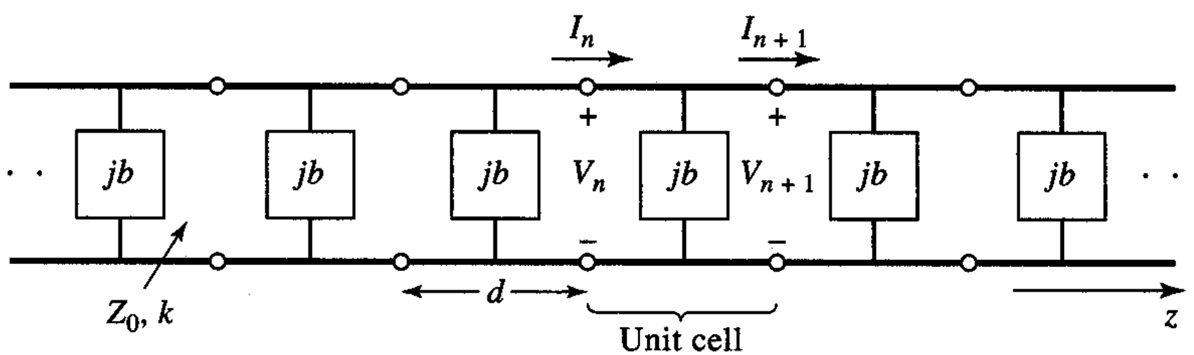
2. Set a unit cell with impedance Z_0, a length of d and a shunt susceptance b.



Periodic stubs

Periodic diaphragms in a waveguide

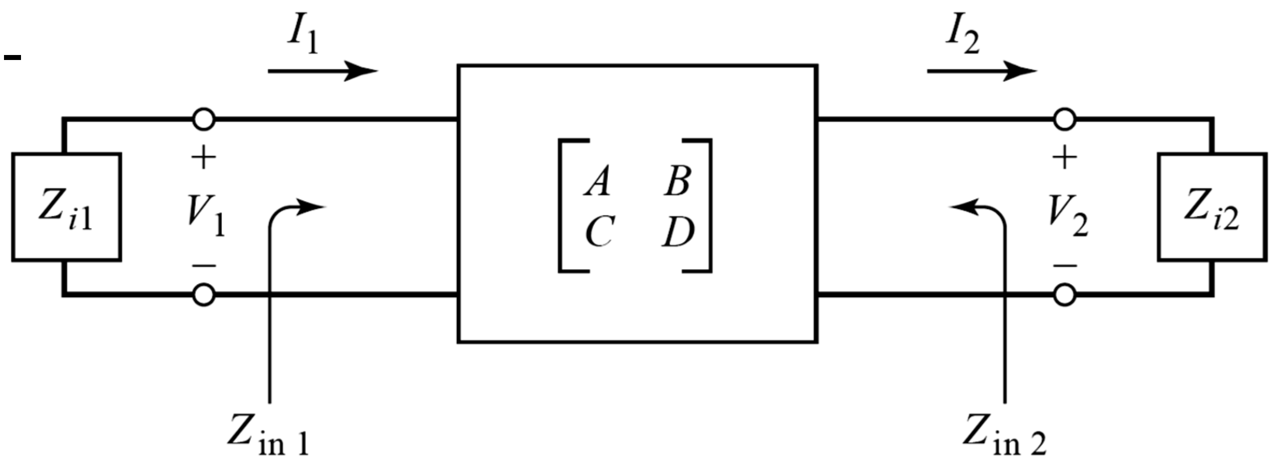
Equivalent circuit of a periodically loaded transmission lines: distributed parameters



Analysis of periodic structures shows that waves can propagate within certain frequency bands (passbands), but will attenuate within other bands (stopbands).

Filter Design By the Image Parameter Method

For a reciprocal two-port network on the right, it can be specified by its *ABCD* parameters.



The image impedances are Z_{i1} and Z_{i2}.

Z_{i1} = input impedance at port 1 when port 2 is terminated with Z_{i2}.

Z_{i2} = input impedance at port 2 when port 1 is terminated with Z_{i1}.

Using ABCD parameters, we have

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

We can derive

$$Z_{in1} = \frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} = \frac{AZ_{i2} + B}{CZ_{i2} + D}$$

Similarly,

$$Z_{in2} = \frac{-V_2}{I_2} = -\frac{DV_1 - BI_1}{-CV_1 + AI_1} = \frac{DZ_{i1} + B}{CZ_{i1} + A}$$

We want to have $Z_{in1} = Z_{i1}$ and $Z_{in2} = Z_{i2}$, which leads to equations for the image impedances:

$$Z_{i1}(CZ_{i2} + D) = AZ_{i2} + B$$

$$Z_{i1}D - B = Z_{i2}(A - CZ_{i1})$$

Solving for Z_{i1} and Z_{i2}

$$Z_{i1} = \sqrt{\frac{AB}{CD}} \qquad Z_{i2} = \sqrt{\frac{BD}{AC}}$$

Then, $Z_{i2} = DZ_{i1} / A$

If the network is symmetric, then A=D and Z_{i1} = Z_{i2} as expected.

The voltage transfer ratio is given by $\dfrac{V_2}{V_1} = \sqrt{\dfrac{D}{A}}(\sqrt{AD} - \sqrt{BC})$

The current transfer ratio is given by $\dfrac{I_2}{I_1} = \sqrt{\dfrac{A}{D}}(\sqrt{AD} - \sqrt{BC})$

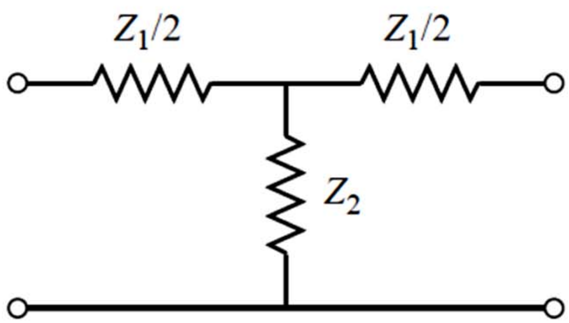
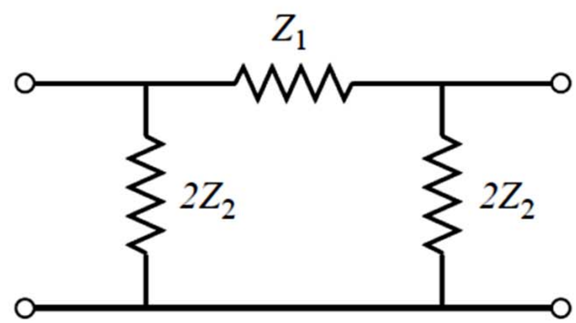
We can define a propagation factor $e^{-\gamma} = \sqrt{AD} - \sqrt{BC}$

Where $\gamma = \alpha + j\beta$

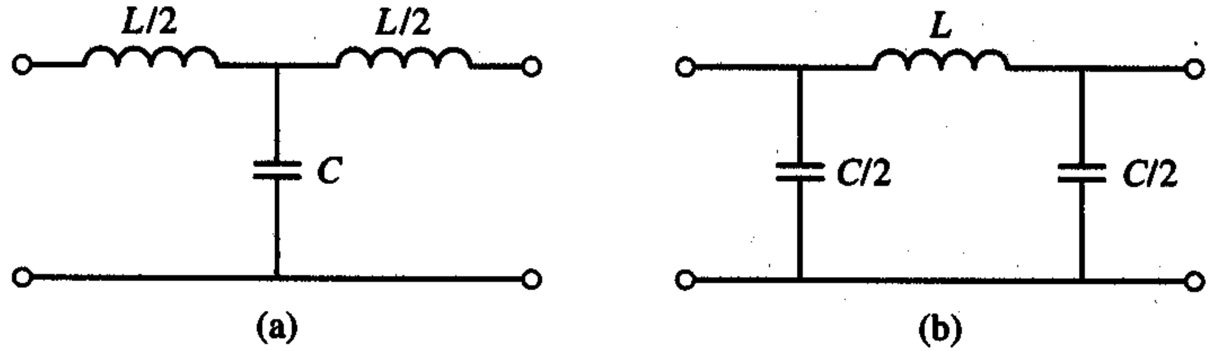
We can also verify that $\cosh\gamma = \dfrac{e^{\gamma} + e^{-\gamma}}{2} = \sqrt{AD}$

Two important types of two-port networks: T and π circuits.

TABLE 8.1 Image Parameters for T- and π-Networks

	
T-Network	π-Network
***ABCD* parameters:**	***ABCD* parameters:**
$A = 1 + Z_1/2Z_2$	$A = 1 + Z_1/2Z_2$
$B = Z_1 + Z_1^2/4Z_2$	$B = Z_1$
$C = 1/Z_2$	$C = 1/Z_2 + Z_1/4Z_2^2$
$D = 1 + Z_1/2Z_2$	$D = 1 + Z_1/2Z_2$
***Z* parameters:**	***Y* parameters:**
$Z_{11} = Z_{22} = Z_2 + Z_1/2$	$Y_{11} = Y_{22} = 1/Z_1 + 1/2Z_2$
$Z_{12} = Z_{21} = Z_2$	$Y_{12} = Y_{21} = 1/Z_1$
Image impedance:	**Image impedance:**
$Z_{iT} = \sqrt{Z_1 Z_2}\sqrt{1 + Z_1/4Z_2}$	$Z_{i\pi} = \sqrt{Z_1 Z_2}/\sqrt{1 + Z_1/4Z_2} = Z_1 Z_2/Z_{iT}$
Propagation constant:	**Propagation constant:**
$e^{\gamma} = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1^2/4Z_2^2}$	$e^{\gamma} = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1^2/4Z_2^2}$

Constant-*k* Filter Sections (low-pass and high-pass filters)



Low-pass constant-k filter sections in *T* and π form.

For the T network, we use the results from the image parameters table, and

$$Z_1 = j\omega L \qquad Z_2 = 1/j\omega C$$

We can derive the image impedance as

$$Z_{iT} = \sqrt{\frac{L}{C}}\sqrt{1-\frac{\omega^2 LC}{4}}$$

The cutoff frequency, ω_c, can be defined as

$$\omega_c = \frac{2}{\sqrt{LC}}$$

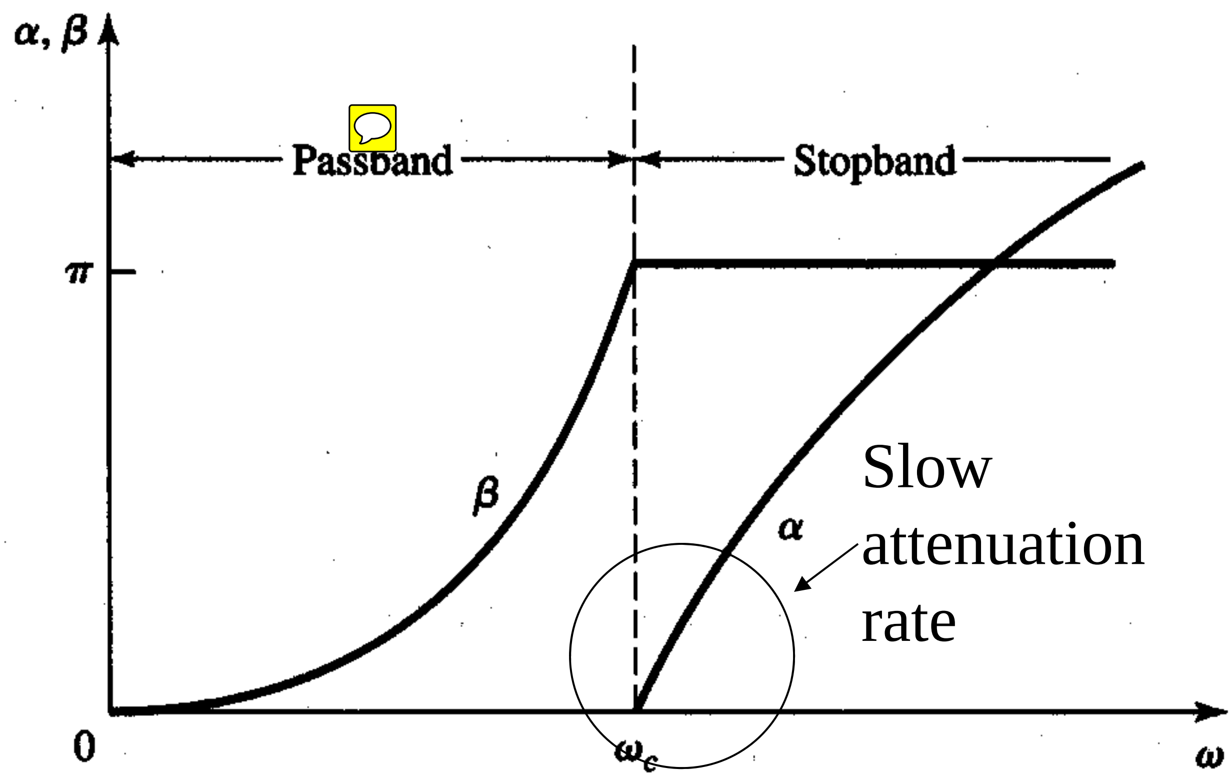
A nominal characteristic impedance, R_0, can be defined as

$$R_0 = \sqrt{\frac{L}{C}} = k \quad \textbf{constant}$$

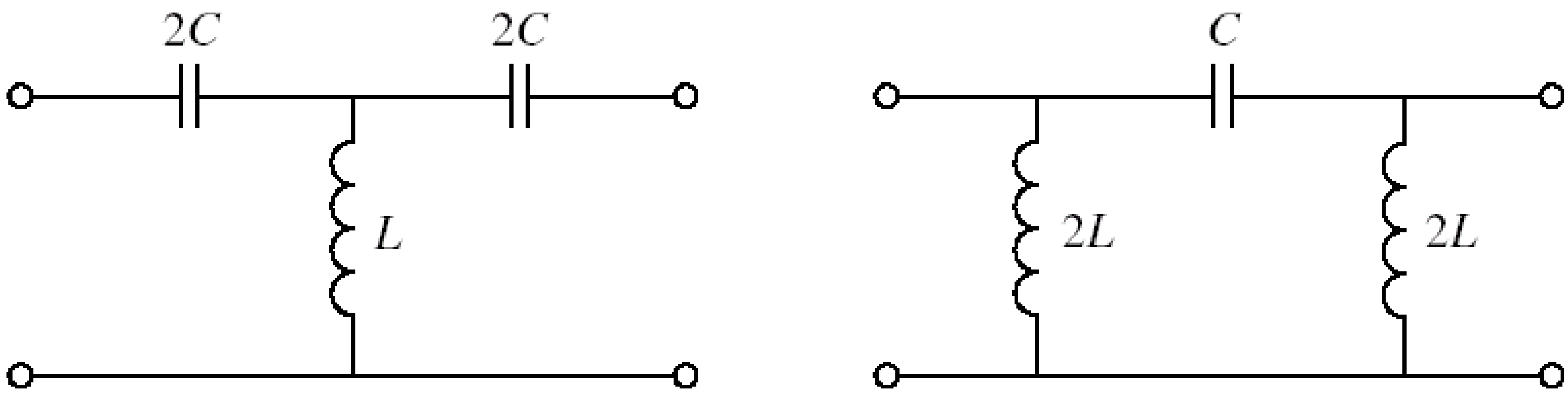
Then, $$Z_{iT} = R_0\sqrt{1-\frac{\omega^2}{\omega_c^2}}$$ For $\omega = 0$, $Z_{iT} = R_0$.

The propagation factor is $$e^{\gamma} = 1-\frac{2\omega^2}{\omega_c^2}+\frac{2\omega}{\omega_c}\sqrt{\frac{\omega^2}{\omega_c^2}-1}$$

Frequency response of the low-pass constant-k sections.



High-pass Constant-k Filter Sections

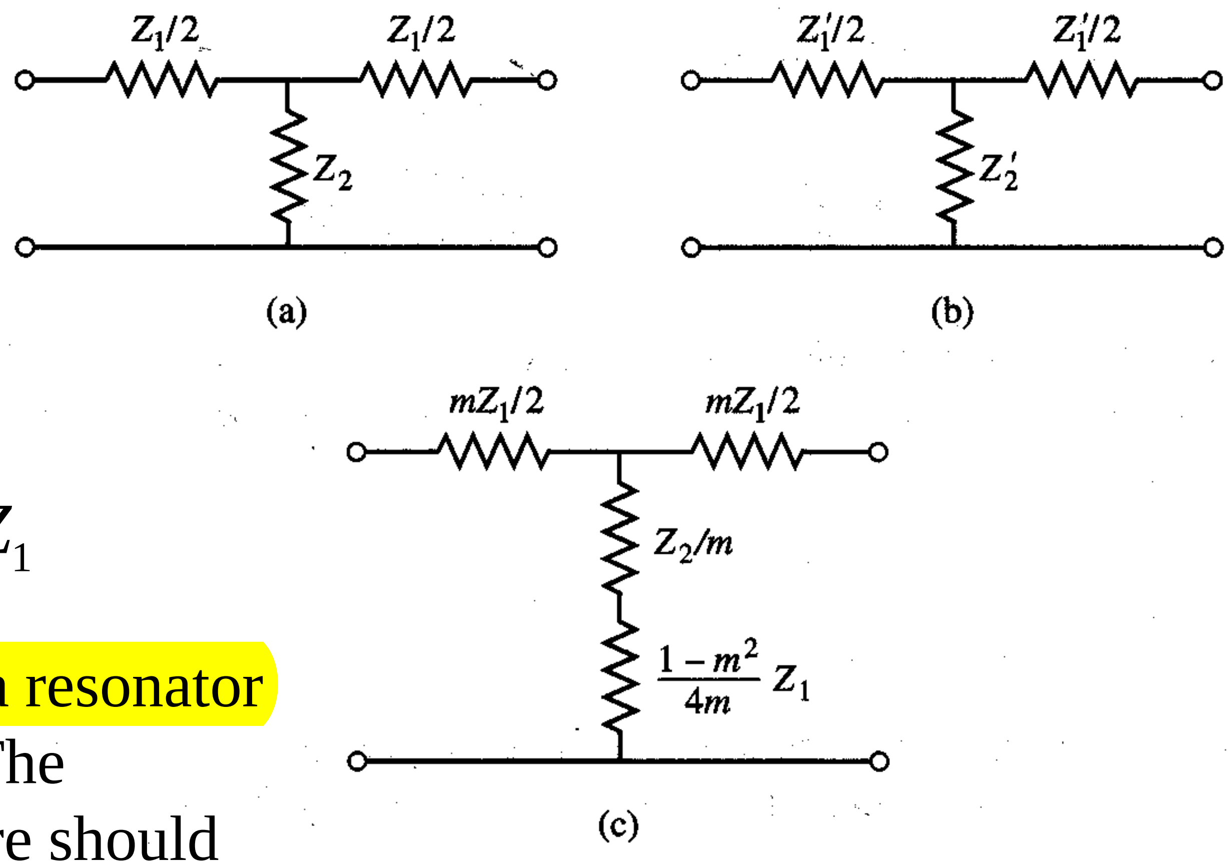


(a) (b)

m-Derived Filter Sections

A modification to overcome the disadvantages of **slow attenuation after cutoff and frequency-dependent image impedances.**

$$Z'_1 = mZ_1$$



In order to have the same Z_{iT}, we have

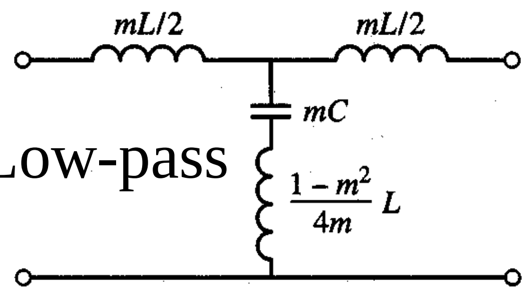
$$Z'_2 = \frac{Z_2}{m} + \frac{(1-m^2)}{4m} Z_1$$

Basic concept: create a resonator along the shunt path. The resonant frequency here should be slightly high than the cutoff frequency.

Cutoff frequency is still

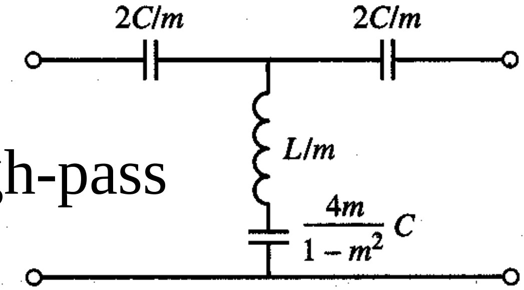
$$\omega_c = \frac{2}{\sqrt{LC}}$$

Low-pass



(a)

High-pass



(b)

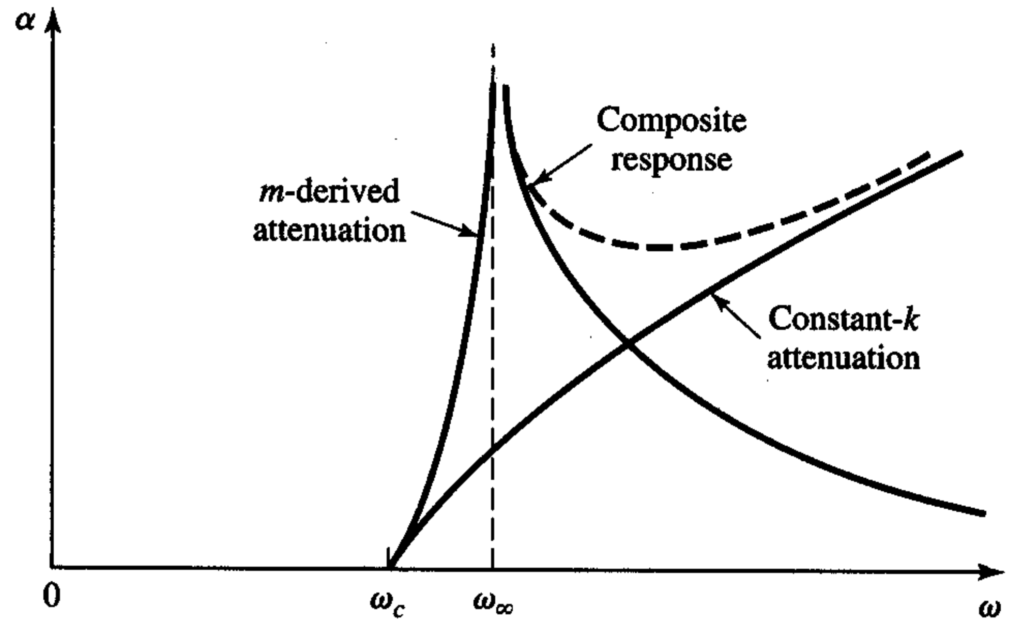
The resonant frequency of the shunt path is

$$\omega_\infty = \frac{1}{\sqrt{mC\left(\frac{1-m^2}{4m}L\right)}}$$

$$= \frac{\omega_c}{\sqrt{1-m^2}}$$

m is restricted into to the range of

0 < m < 1.



Steep decrease of α after $\omega > \omega_\infty$ is not desirable. This problem can be solved by cascading with another constant-k section to give a composite response shown in the figure.

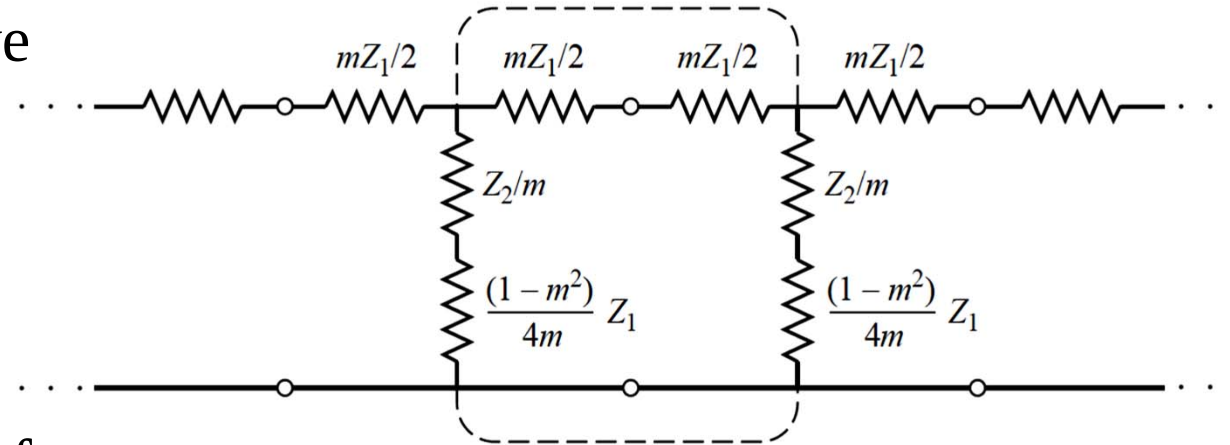
m-Derived π Filter Section

The T-section still have the problem of a nonconstant image impedances.

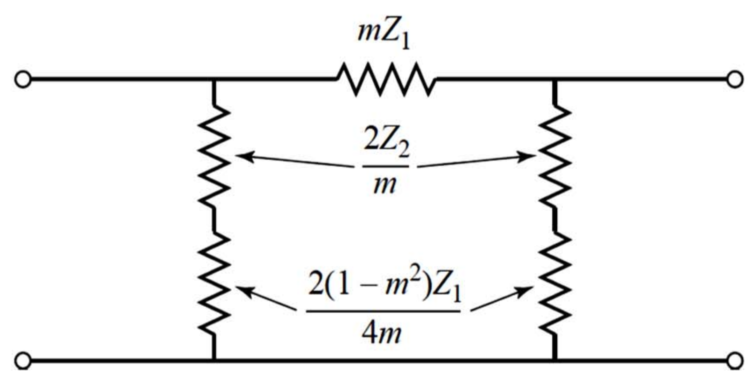
Now consider the π-equivalent as a piece of an infinite cascade of m-derived T-sections. Then,

$$Z_{i\pi} = \frac{Z'_1 Z'_2}{Z_{iT}}$$

$$= \frac{Z_1 Z_2 + Z_1^2(1-m^2)/4}{R_0\sqrt{1-(\omega/\omega_c)^2}}$$



Infinite cascade of *m*-derived *T*-section.



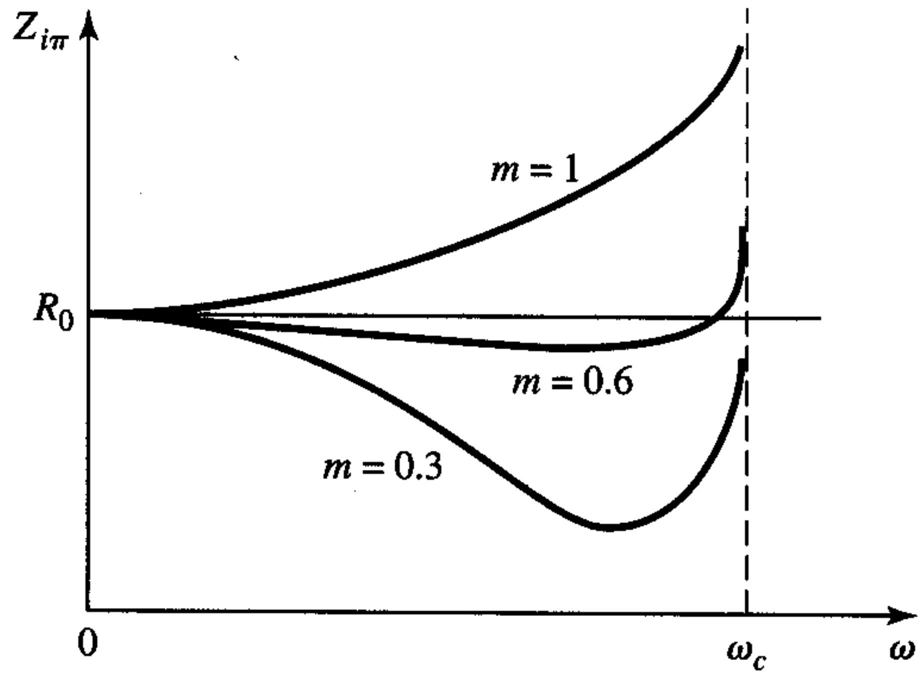
A de-embedded π-equivalent.

Since $Z_1 Z_2 = L/C = R_0^2$ and $Z_1^2 = -\omega^2 L^2 = -4R_0^2 \left(\omega/\omega_c\right)^2$

We have $$Z_{i\pi} = \frac{1-(1-m^2)\left(\omega/\omega_c\right)^2 R_0}{\sqrt{1-\left(\omega/\omega_c\right)^2}}$$

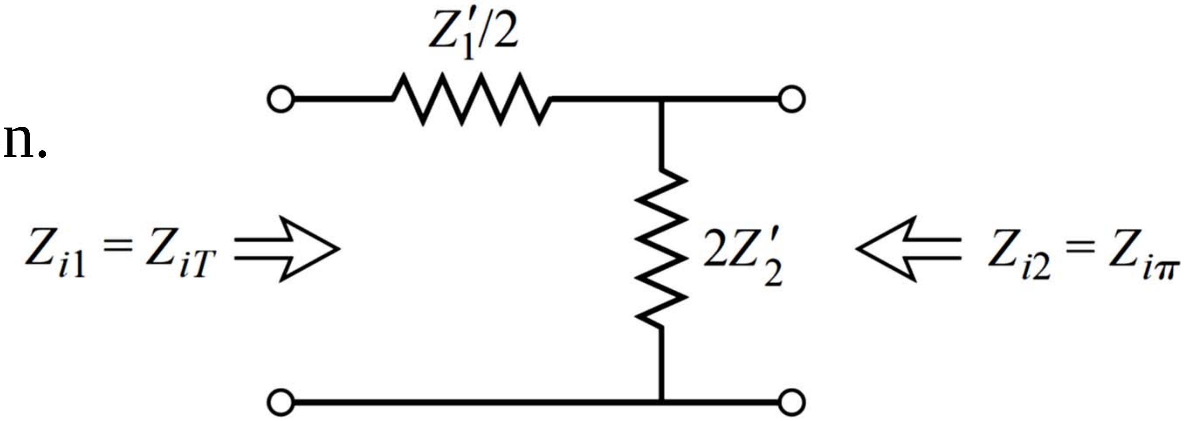
* m provides another freedom to design $Z_{i\pi}$ so that we can minimize the variation of $Z_{i\pi}$ over the passband of the filter.

Variation of $Z_{i\pi}$ in the pass band of a low-pass m-derived section for various values of m. A value of m=0.6 generally gives the best results --- nearly constant impedance match to and from R_0.



How to match the constant-k and m-derived *T*-sections to π-section?

Using bisected π-section.

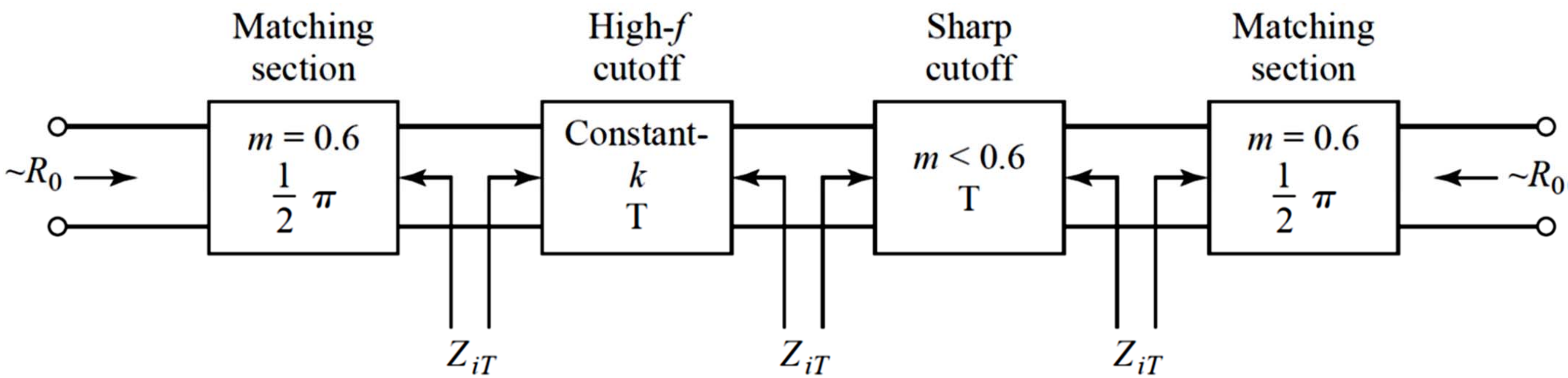


It can be shown that

$$Z_{i1} = \sqrt{Z'_1 Z'_2 + \frac{Z'^2_1}{4}} = Z_{iT}$$

$$Z_{i2} = \sqrt{\frac{Z'_1 Z'_2}{1 + Z'_1 / 4Z'_2}} = \frac{Z'_1 Z'_2}{Z_{iT}} = Z_{i\pi}$$

Composite Filters

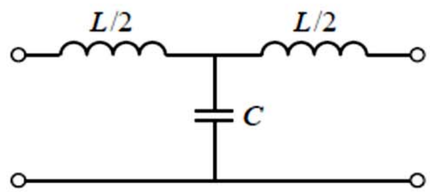


- The sharp-cutoff section, with m< 0.6, places an attenuation pole near the cutoff frequency to provide a sharp attenuation response.
- The constant-k section provides high attenuation further into the stopband.
- The bisected-π sections at the ends match the nominal source and load impedance, R_0, to the internal image impedances.
- The composite filter design is obtained from three parameters: cutoff frequency, impedance, and infinite attenuation frequency ω_∞ (or m).

TABLE 8.2 Summary of Composite Filter Design

Low-Pass

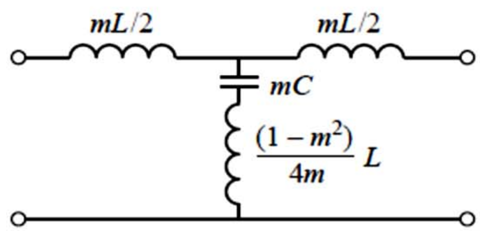
Constant-k T section



$R_0 = \sqrt{L/C}$ $\quad L = 2R_0/\omega_c$

$\omega_c = 2/\sqrt{LC}$ $\quad C = 2/\omega_c R_0$

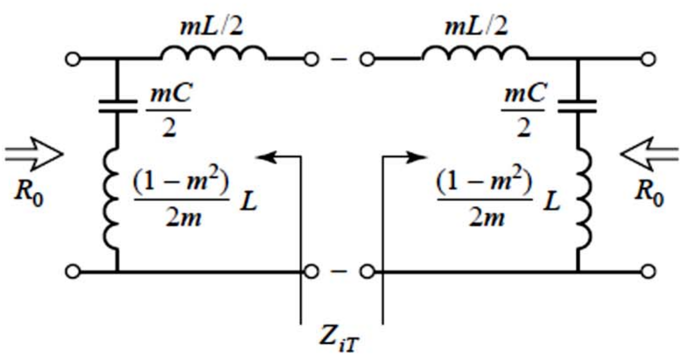
m-derived T section



L, C Same as constant-k section

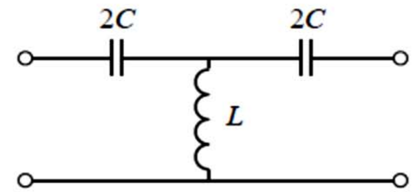
$$m = \begin{cases} \sqrt{1 - (\omega_c/\omega_\infty)^2} & \text{for sharp-cutoff} \\ 0.6 & \text{for matching} \end{cases}$$

Bisected-π matching section



High-Pass

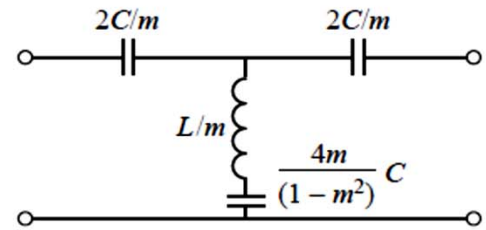
Constant-k T section



$R_0 = \sqrt{L/C}$ $\quad L = R_0/2\omega_c$

$\omega_c = 1/2\sqrt{LC}$ $\quad C = 1/2\omega_c R_0$

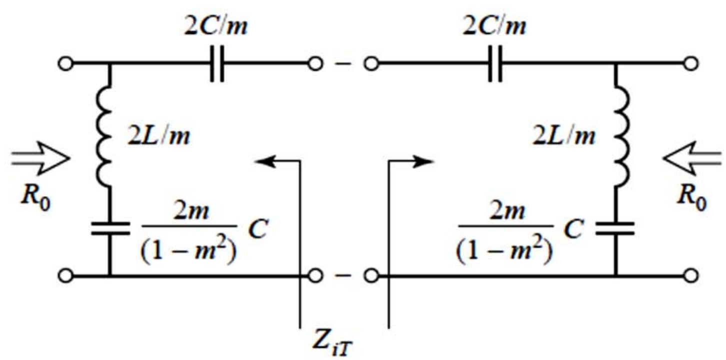
m-derived T section



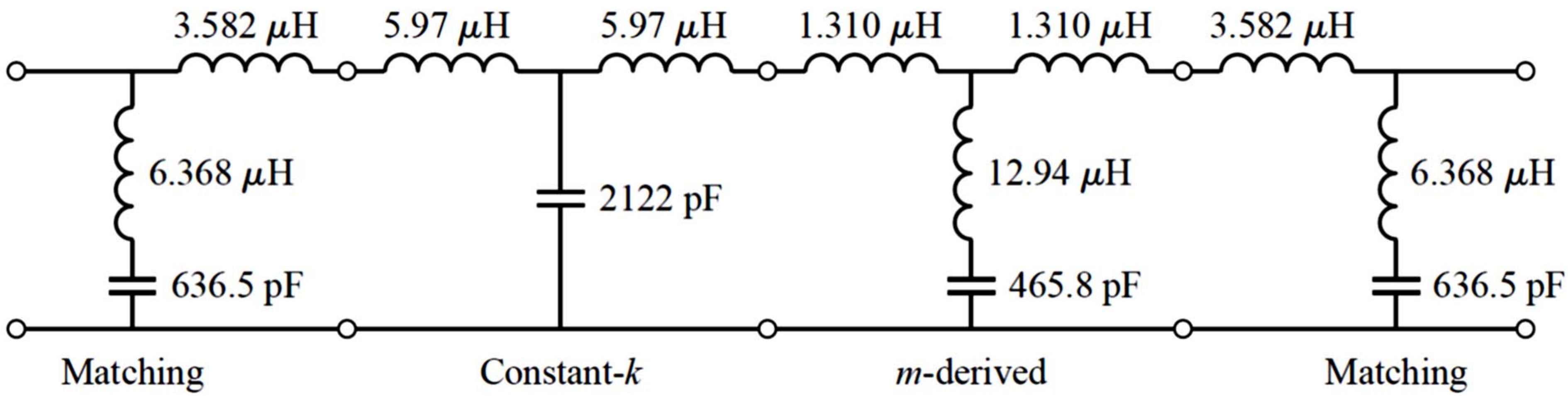
L, C Same as constant-k section

$$m = \begin{cases} \sqrt{1 - (\omega_\infty/\omega_c)^2} & \text{for sharp-cutoff} \\ 0.6 & \text{for matching} \end{cases}$$

Bisected-π matching section

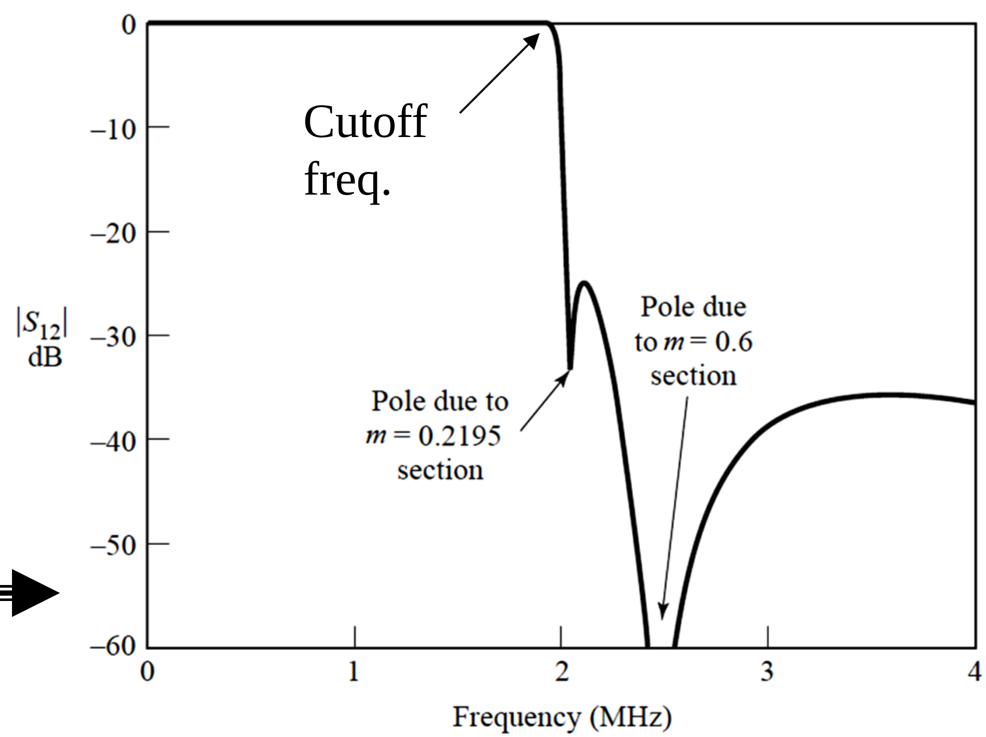


Example 8.2 of Pozar: Low-Pass Composite Filter Design



The series pairs of inductors between the sections can be combined. The self-resonance of the bisected p-section will provide additional attenuation.

Frequency response



Filter Design By the Insertion Loss Method

What is a perfect filter?

• *Zero insertion loss in the passband, infinite attenuation in the stopband, and linear phase response (to avoid signal distortion) in the passband.*

No perfect filters exist, so **compromises** need to be made.

The *image parameter* method have very limited freedom to nimble around.

The *insertion loss* method allows a high degree of control over the passband and stopband amplitude and phase characteristics, with a systematic way to synthesize a desired response.

Characterization by Power Loss Ratio

The power loss ratio and insertion loss of a filter are defined as:

$$P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1-\left|\Gamma(\omega)\right|^2}$$

$$IL = 10 \log P_{LR}$$

When both load and source are matched, $P_{LR} = |S_{21}|$^2

Since $|\Gamma(\omega)|^2$ is an even function of ω, it can be expressed as a polynomial in ω^2.

$$\left|\Gamma(\omega)\right|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)}$$

Where M and N are real polynomials in ω^2. So the power loss ratio can be given as

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

Several Types of Filter Response:

Maximally flat: *binomial or Butterworth response*

Provide the flattest possible passband response. For a low-pass filter, it is specified by

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

Where N is the order of the filter, and ω_c is the cutoff frequency.

At the band edge the power loss ratio is $1+k^2$. Maximally flat means that the the first (2N-1) derivatives of the power loss ratio are zero at $\omega = 0$.

Equal ripple: *A Chebyshev polynomial is used to represent the insertion loss of an N-order low-pass filter*

Provide the sharpest cutoff, with ripples in the passband.

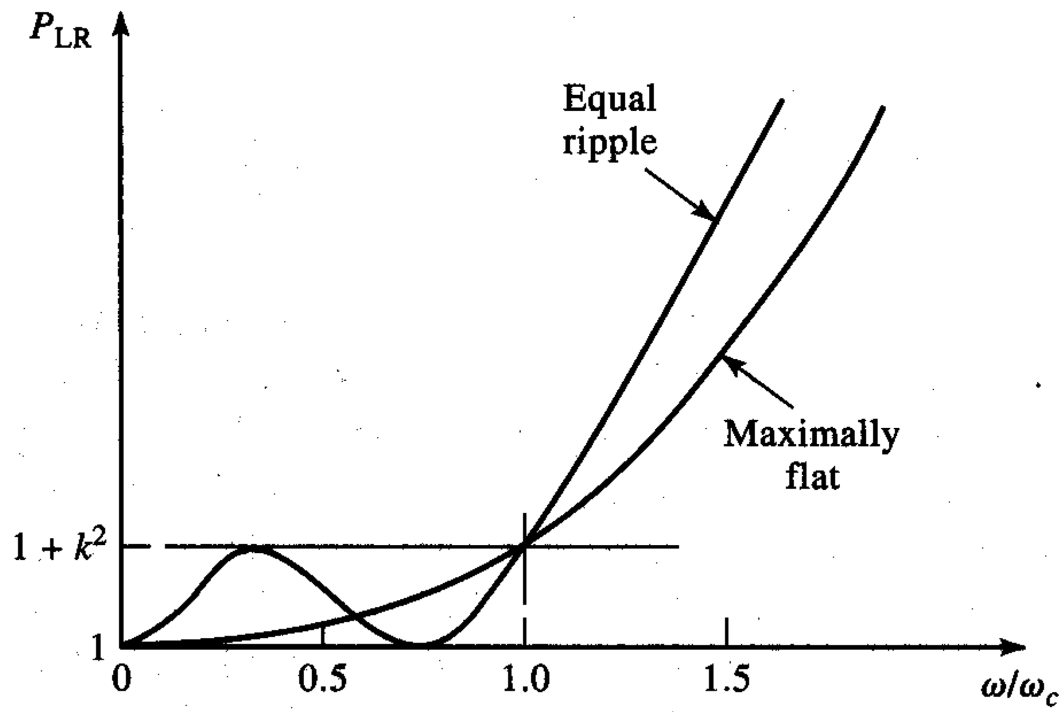
The insertion loss is: $$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

Ripple amplitude: $1 + k^2$

For $\omega >> \omega_c$, the insertion loss is,

$$P_{LR} \approx \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}$$

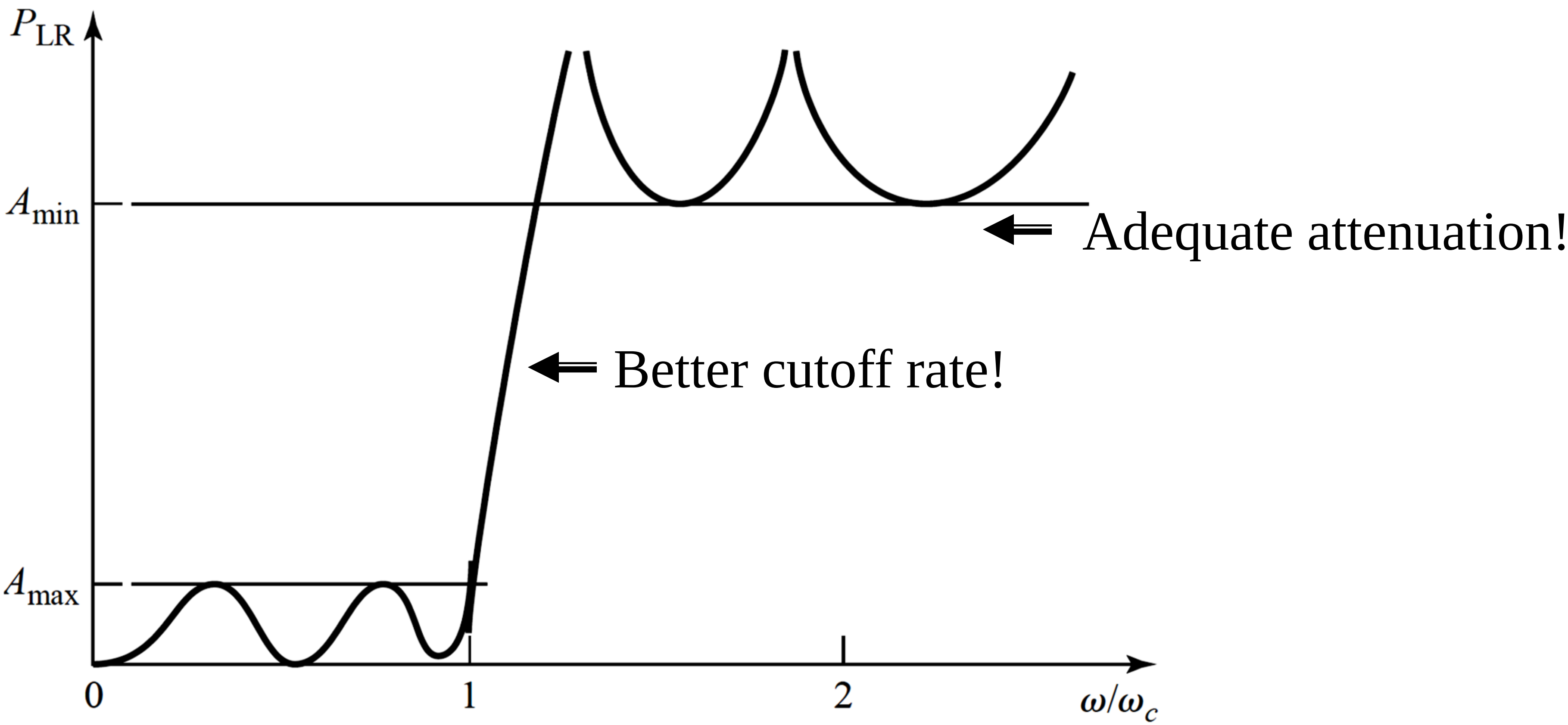
Insertion loss rises faster in the stopband compared to the binomial filters.



Both the maximally flat and equal-ripple responses both have monotonically increasing attenuation in the stopband --- *not necessary in applications*.

***Elliptic function:** Equal ripple responses in the passband and the stopband*

Specified by the maximum attenuation in the passband, A_{max}, as well as the minimum attenuation in the stopband, A_{min}. See Figure 8.22.



Linear phase: *a linear phase response in the passband, to avoid signal distortion, generally incompatible with a sharp cutoff response.*

A linear phase response:

$$\phi(\omega) = A\omega\left[1 + p\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

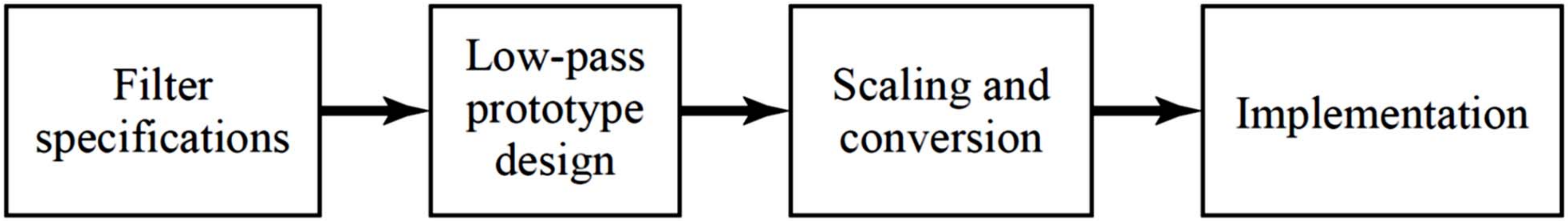
Where p is a constant.

Group delay:

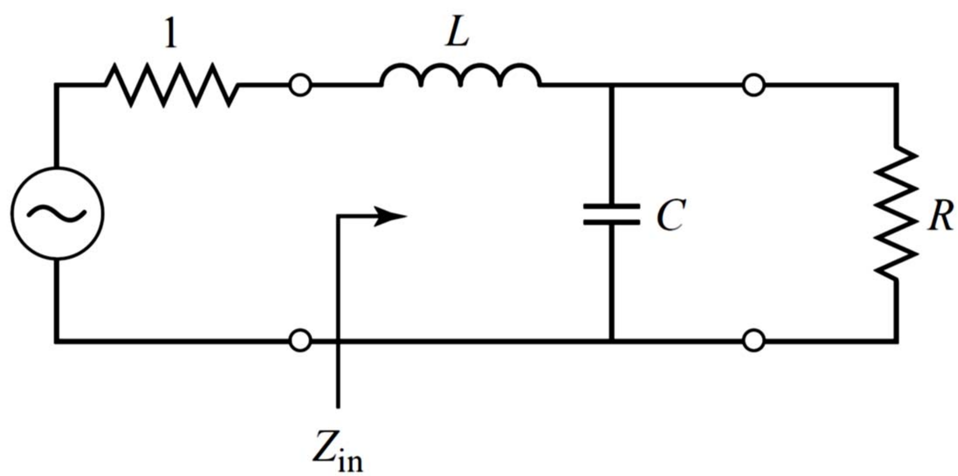
$$\tau_d = \frac{d\phi}{d\omega} = A\left[1 + p(2N+1)\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

Group delay is a maximally flat function.

The process of filter design by the insertion loss method



Maximally Flat Low-Pass Filter Prototype (using normalized element values)



Assume a source impedance of 1Ω, and a cutoff frequency ω_c =1. For a N=2, the desired power loss ratio is 2.

$$P_{LR} = 1 + \omega^4$$

Input impedance

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2} \quad \text{and} \quad \Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$

Power loss ratio is
$$P_{LR} = \frac{1}{1-|\Gamma|^2} = \frac{|Z_{in}+1|^2}{2(Z_{in}+Z^*_{in})}$$

Solve P_{LR} for R, L, C, ω, we have

$$P_{LR} = 1 + \frac{1}{4R}[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2R^2C^2\omega^4]$$

Compare this expression with $P_{LR} = 1+\omega^4$

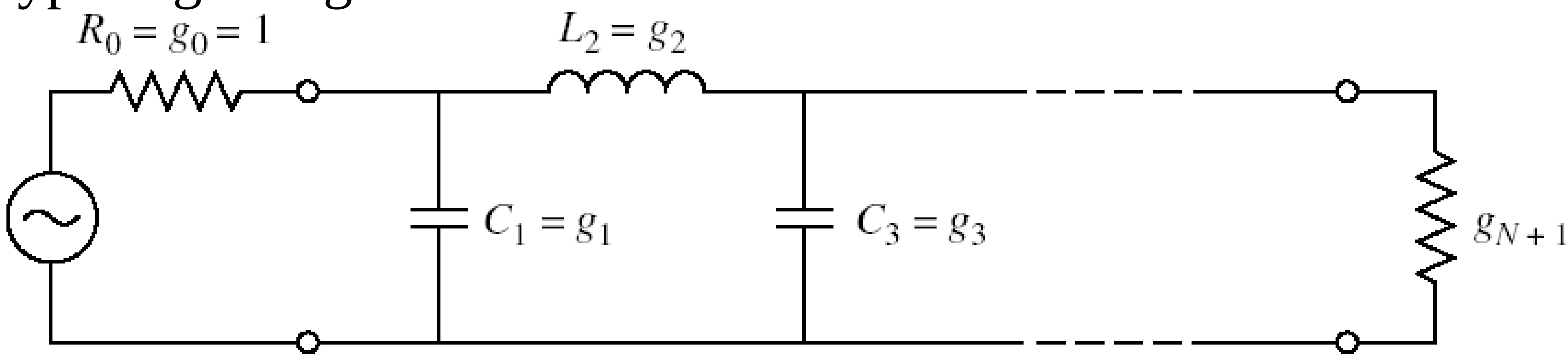
We have, $R=1$ $L=C$ $L=C=\sqrt{2}$

This procedure can be extended to find the element values for filters with an arbitrary number of elements.

Design Table 8.3 of Pozar gives the component value for N=1 to 10.

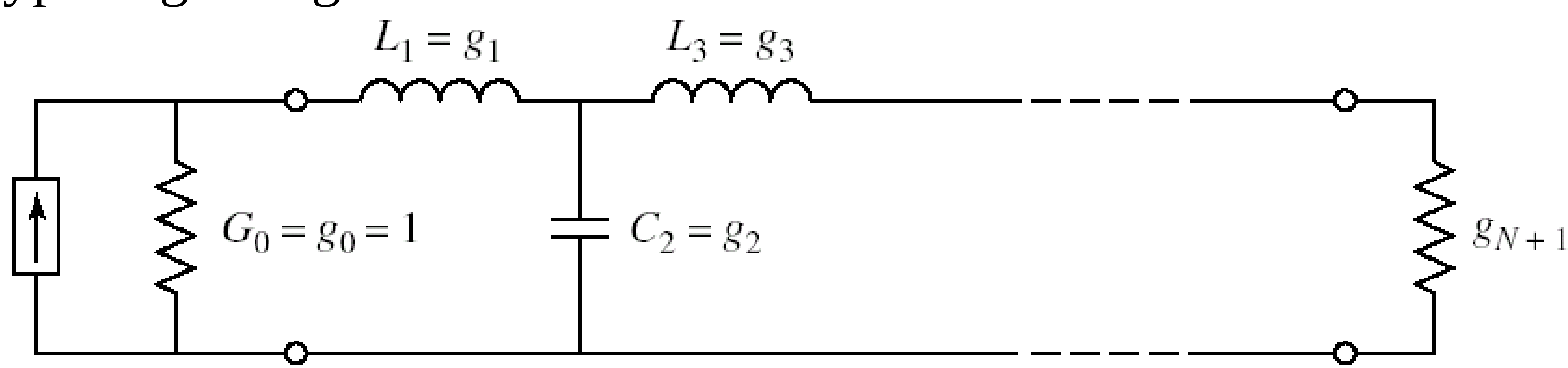
Ladder circuits for low-pass filter prototypes and their element definitions.

Prototype beginning with a shunt element.



(a)

Prototype beginning with a series element.



(b)

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

g_k definition:

$$g_0 = \begin{cases} \text{generator resistance (network of Figure 8.25a)} \\ \text{generator conductance (network of Figure 8.25b)} \end{cases}$$

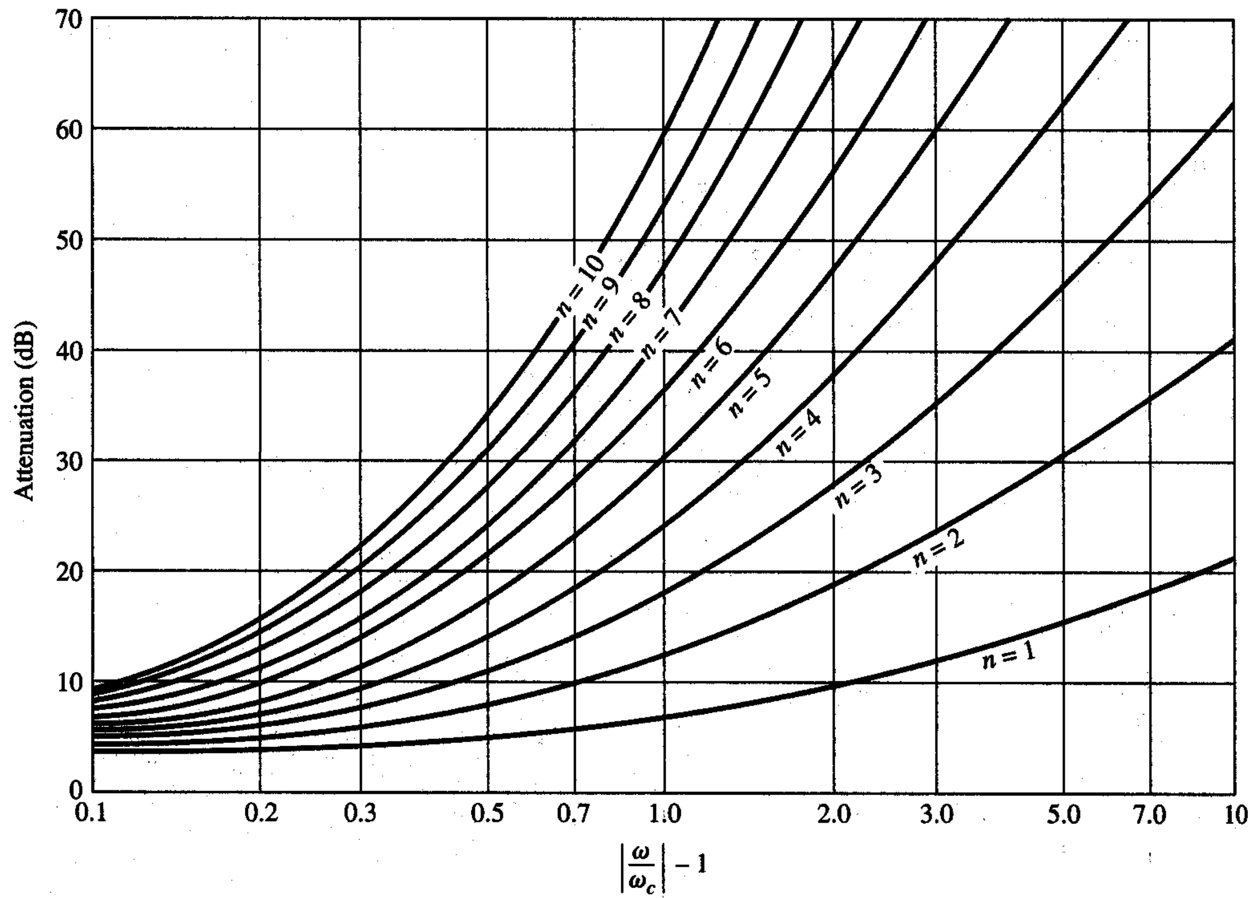
$$\begin{matrix} g_k \\ (k = 1 \text{ to } N) \end{matrix} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases}$$

What to design?

1. The order (size) of the filter N: decided by a specification on the insertion loss at some frequency in the stopband.

2. The value of each component. See table 8.3



Equal-Ripple Low-Pass Filter Prototype

Ripple level has to be specified.

Table 8.4 and Figure 8.27.

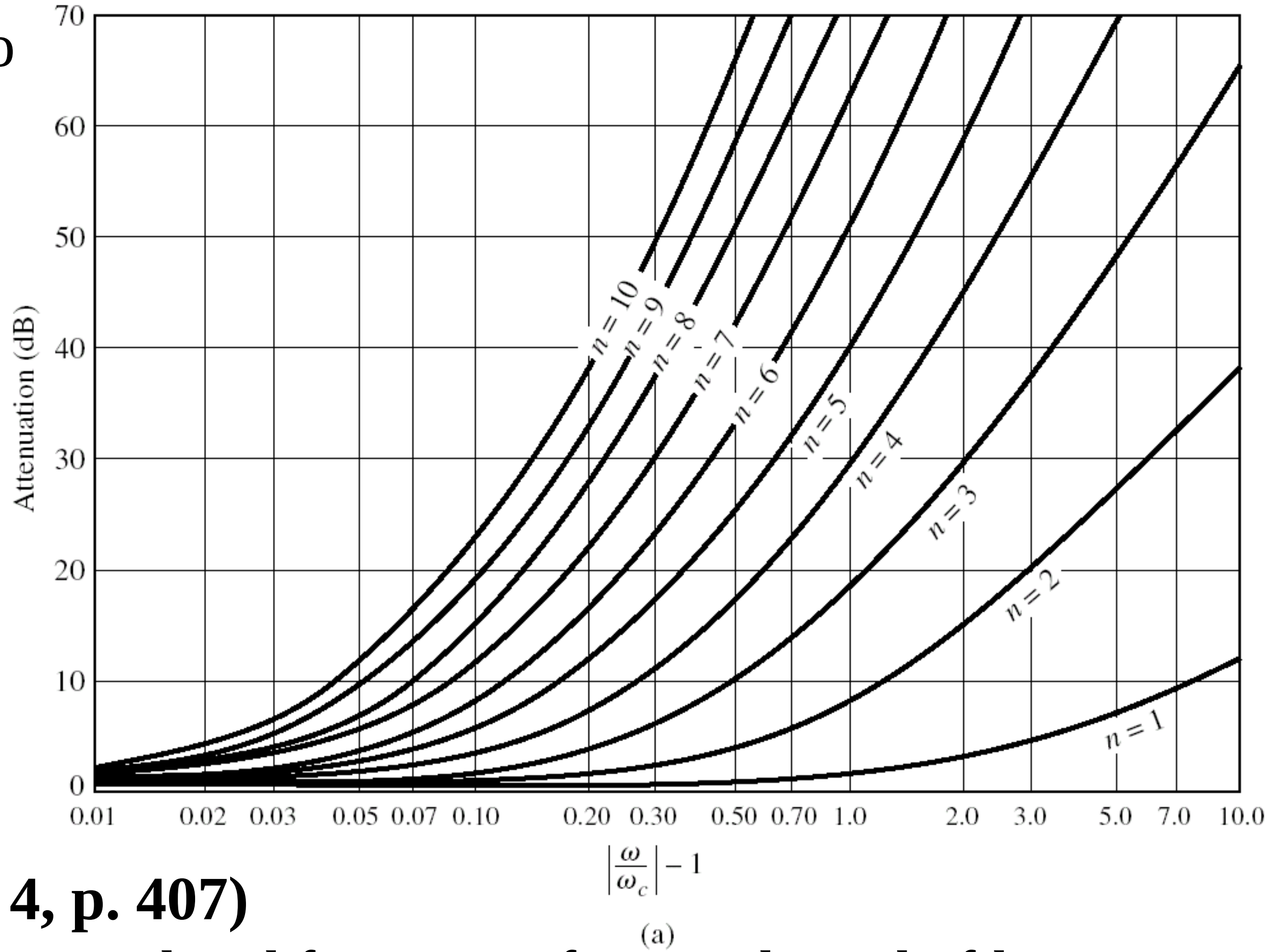


Figure 8.27a (Ed. 4, p. 407)

Attenuation versus normalized frequency for equal-ripple filter prototypes. (*a*) 0.5 dB ripple level.

Adapted from G.L. Mattaei et al., *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Artech House, 1980)

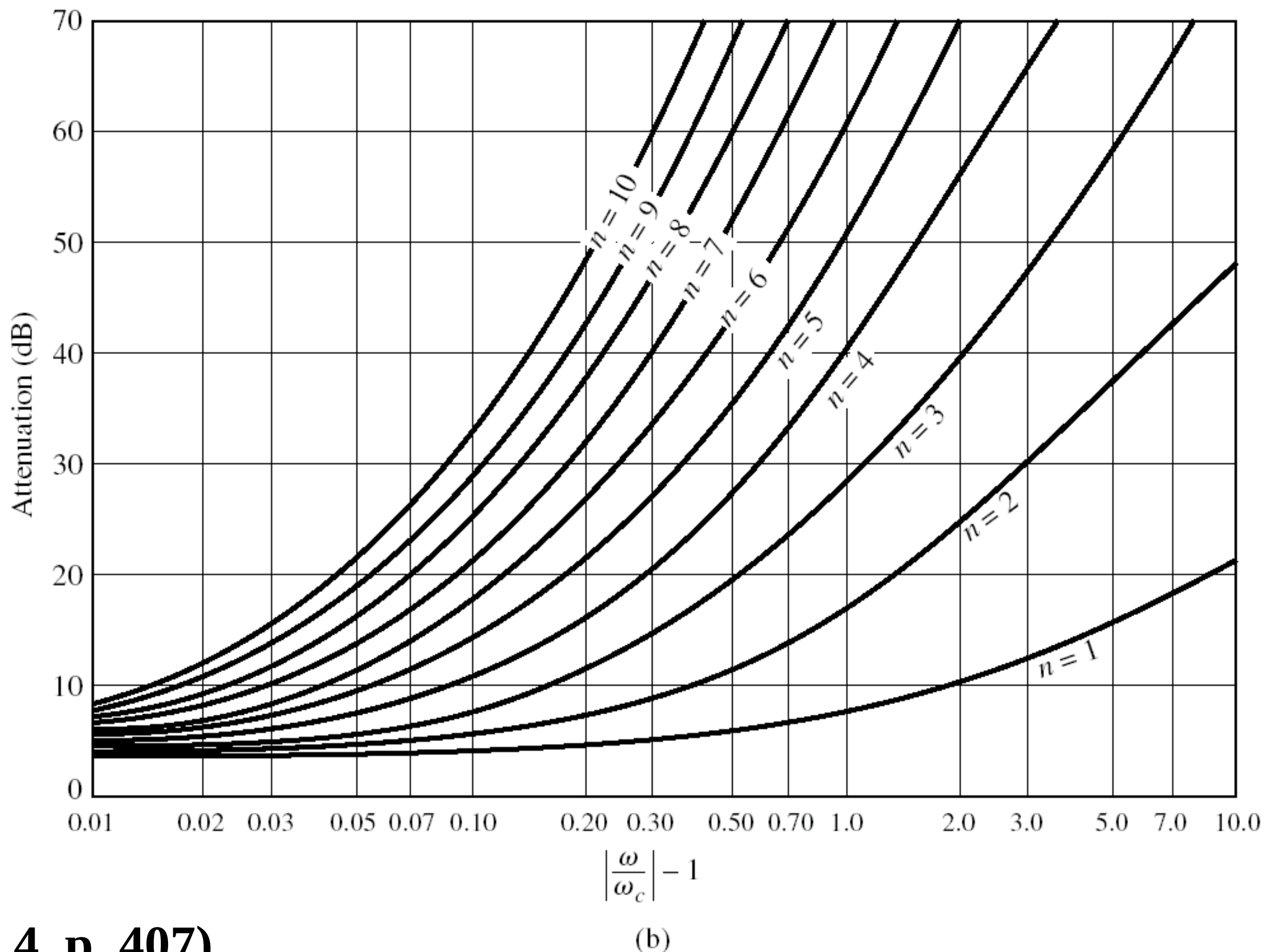


Figure 8.27b (Ed. 4, p. 407)

Attenuation versus normalized frequency for equal-ripple filter prototypes. (*b*) 3.0 dB ripple level.

Adapted from G.L. Mattaei et al., *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Artech House, 1980)

Equal-Ripple Low-Pass Filter Prototype

Ripple level has to be specified.

Table 8.4 and Figure 8.27.

Linear Phase Low-Pass Filter Prototypes

Table 8.5.

Filter Transformations:

Scaling in terms of impedance and frequency

Conversion to high-pass, bandpass, or bandstop filters.

Impedance and Frequency Scaling

With a source resistance, R_0, we have the scaling rule given by

$$L' = R_0 L \qquad C' = C / R_0 \qquad R_S' = R_0 \qquad R_L' = R_0 R_L$$

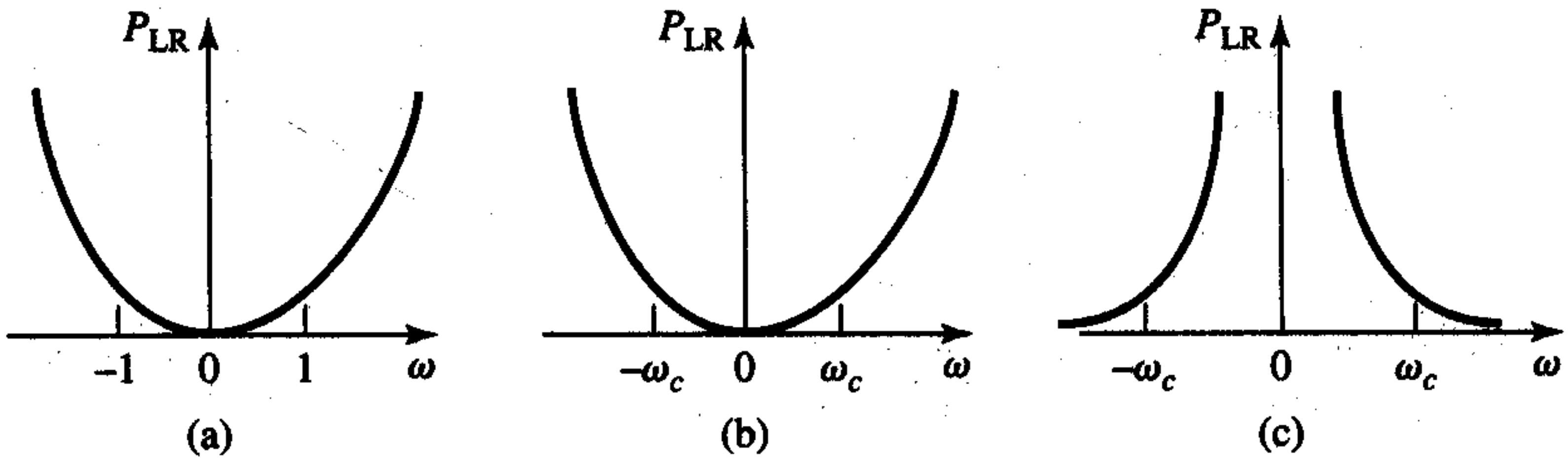
Frequency Scaling for Low-Pass Filters

Scale the frequency response dependence by the factor $1/\omega_c$.

$$P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right) \qquad L'_k = L_k / \omega_c \qquad C'_k = C_k / \omega_c$$

Combining impedance and frequency scaling, we have

$$L'_k = R_0 L_k / \omega_c \qquad C'_k = C_k / (R_0 \omega_c)$$



Low pass filter for ω_c=1

Frequency scaling for low-pass

Transformation to high-pass response.

Low-Pass to High-Pass Transformation: $\omega \leftarrow -\dfrac{\omega_c}{\omega}$

The negative sign is needed to convert inductors (and capacitors) to realizable capacitors (and inductors).

The reactance and susceptance become:

$$jX_k = -j\frac{\omega_c}{\omega}L_k = \frac{1}{j\omega C'_k} \qquad jB_k = -j\frac{\omega_c}{\omega}C_k = \frac{1}{j\omega L'_k}$$

We obtain the conversion rules given by:

$$C'_k = 1/(\omega_c L_k) \qquad L'_k = 1/(\omega_c C_k)$$

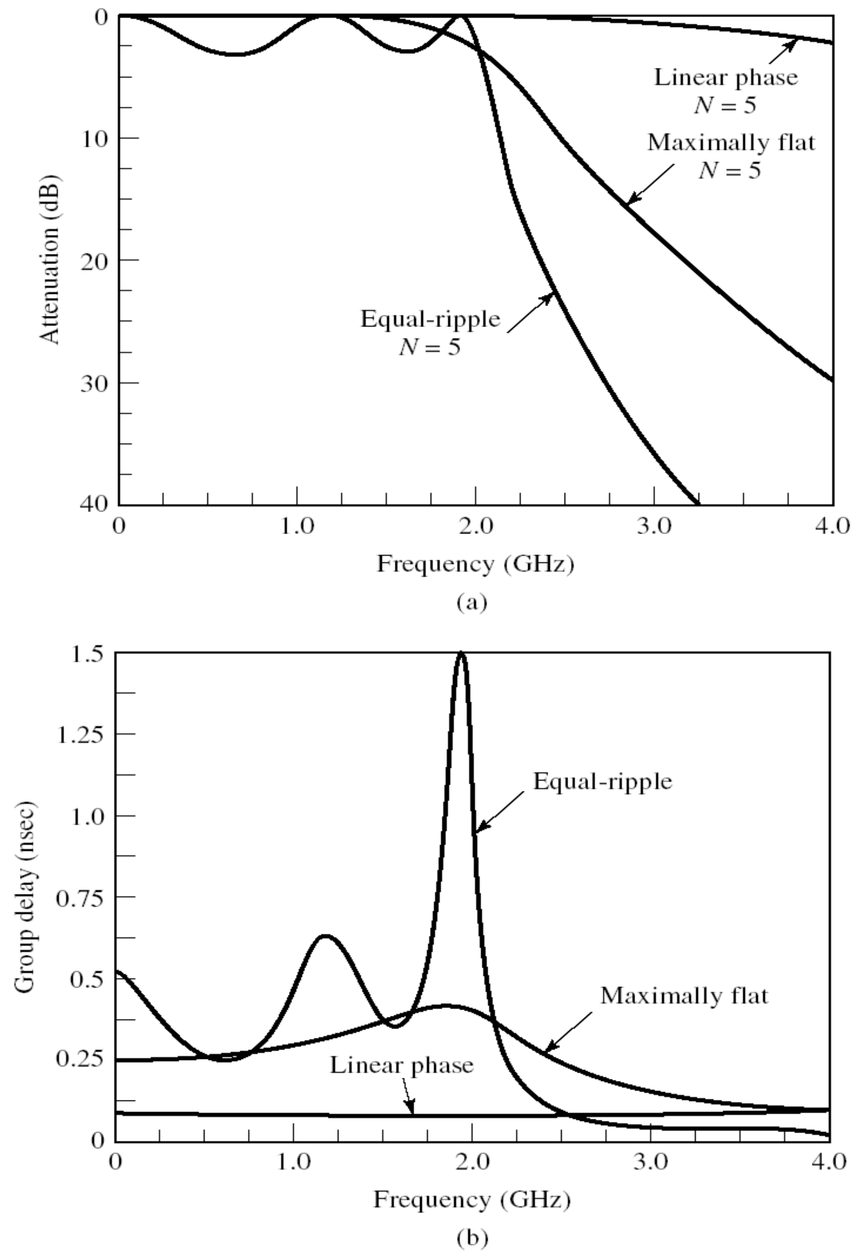
So the series inductors are replaced with capacitors and shunt capacitors are replaced with inductors.

After including the impedance scaling, we have

$$C'_k = 1/(R_0 \omega_c L_k)$$

$$L'_k = R_0/(\omega_c C_k)$$

Example 8.3 of Pozar.



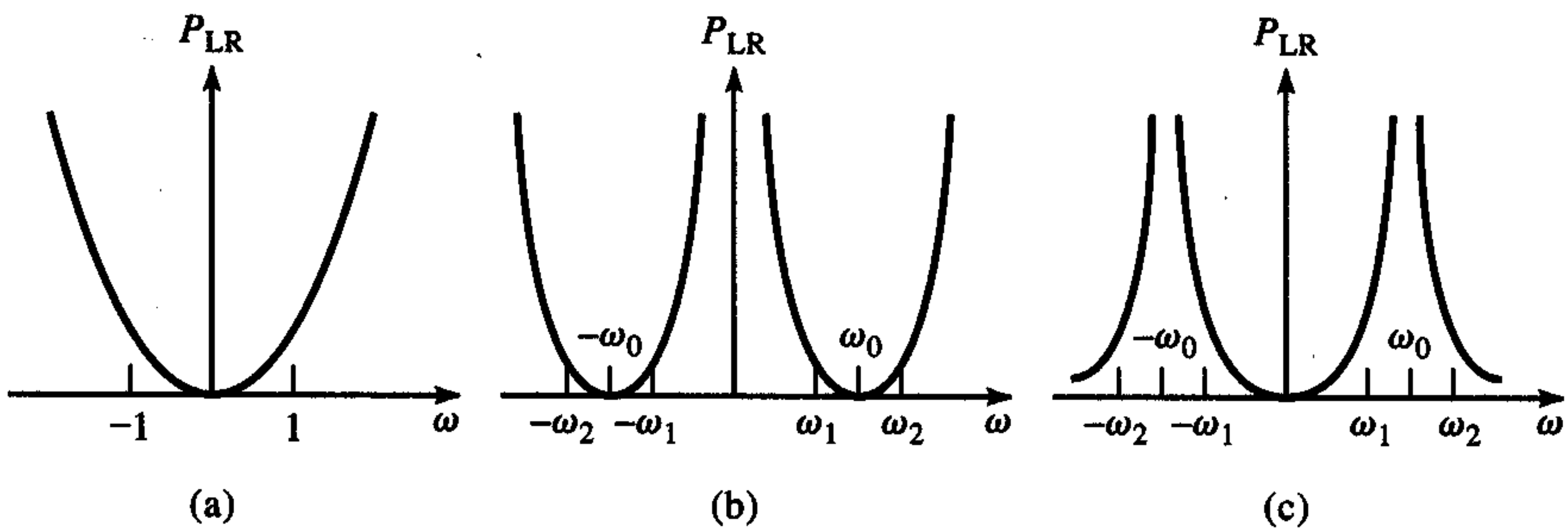
Bandpass and Bandstop Transformation

Conversion Rules for ***bandpass*** filters:

$$\omega \leftarrow \frac{\omega_0}{\omega_2-\omega_1}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

where $$\Delta = \frac{\omega_2-\omega_1}{\omega_0}$$

Δ is the fractional bandwidth of the passband.



(a) Low-pass filter prototype.

(b) Transformation to bandpass response.

(c) Transformation to bandstop response.

If we take the geometric mean for the center frequency ω_0,

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

We have,

when $\omega = \omega_0$,

$$\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = 0$$

when $\omega = \omega_1$,

$$\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0 \omega_1}\right) = -1$$

when $\omega = \omega_2$,

$$\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0 \omega_2}\right) = 1$$

The mapping between the low-pass prototype and the bandpass filter is complete.

Then the new filter elements are given by performing the conversion

$$jX_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)L_k = j\frac{\omega L_k}{\Delta\omega_0} - j\frac{\omega_0 L_k}{\Delta\omega} = j\omega L'_k - j\frac{1}{\omega C'_k}$$

This indicates that, a series inductor, L_k, is transformed to a series LC circuit with element values,

$$L'_k = \frac{L_k}{\Delta\omega_0} \qquad C'_k = \frac{\Delta}{\omega_0 L_k}$$

Resonance at ω_0

Similarly, for a shunt susceptance, we have

$$jB_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)C_k = j\frac{\omega C_k}{\Delta\omega_0} - j\frac{\omega_0 C_k}{\Delta\omega} = j\omega C'_k - j\frac{1}{\omega L'_k}$$

The shunt capacitor, is transformed to a shunt (parallel) LC circuit with element values,

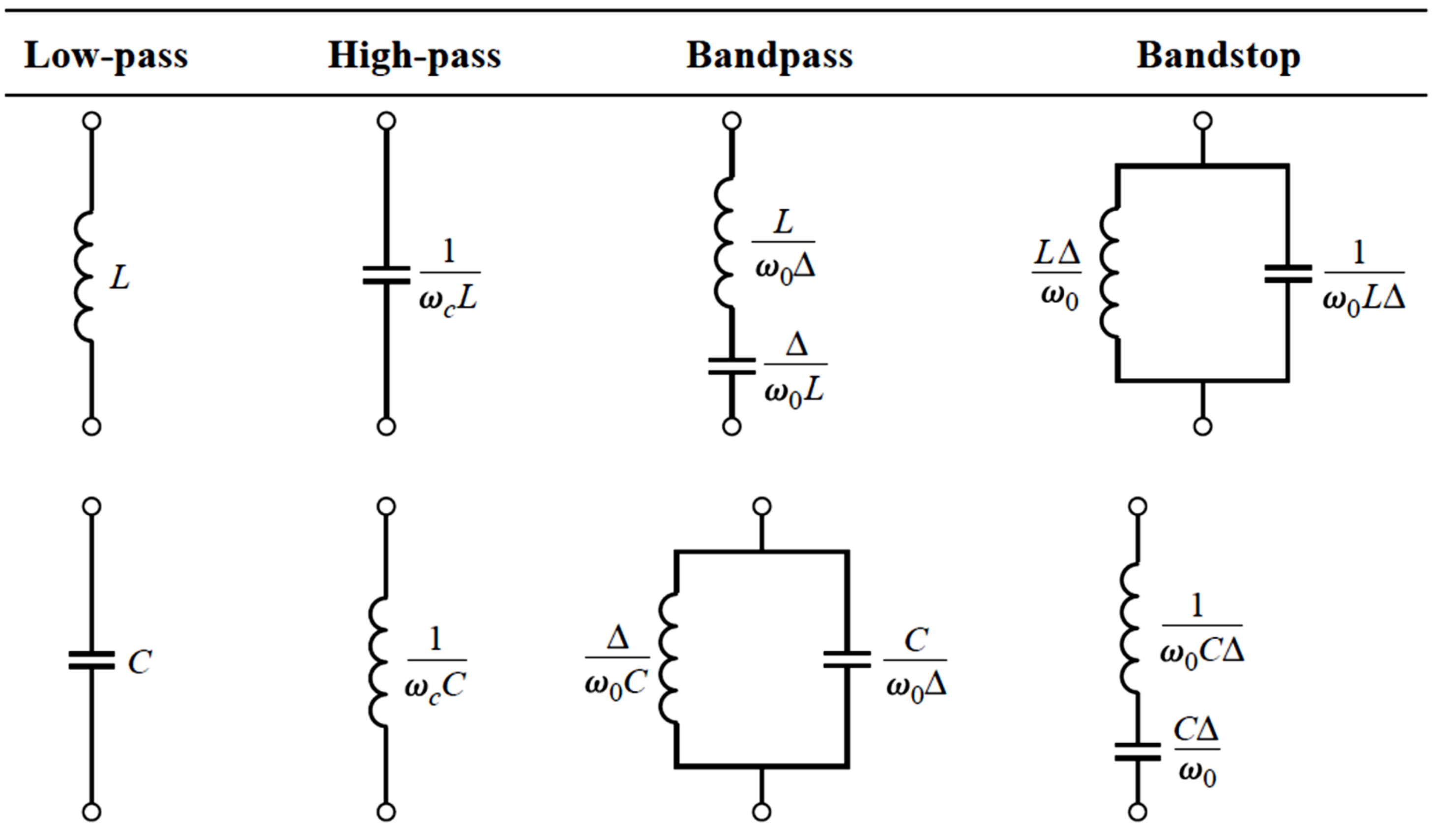
$$L'_k = \frac{\Delta}{\omega_0 C_k} \qquad C'_k = \frac{C_k}{\Delta\omega_0}$$

Resonance at ω_0

Conversion Rules for ***bandstop*** filters:

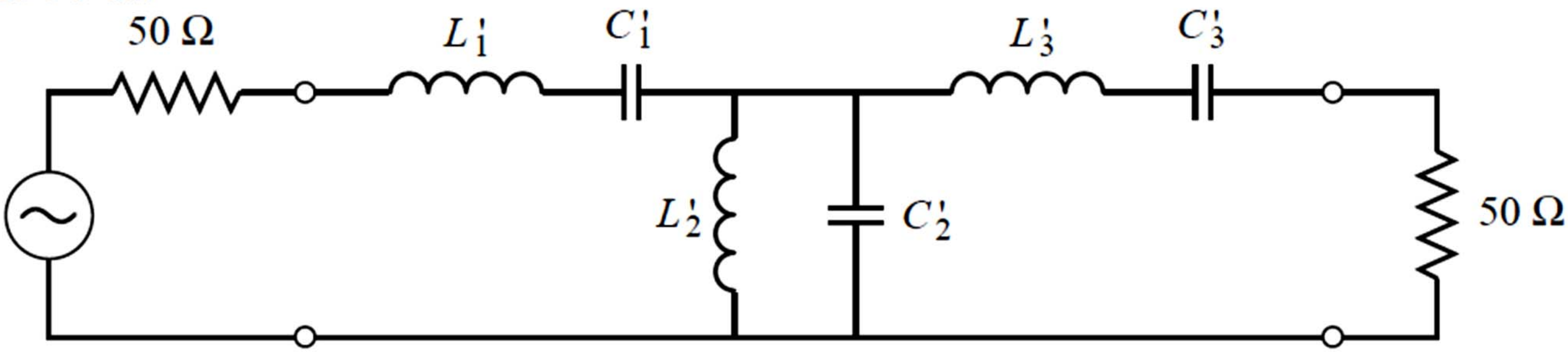
$$\omega \leftarrow -\Delta\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^{-1}$$

TABLE 8.6 Summary of Prototype Filter Transformations $\left(\Delta = \dfrac{\omega_2 - \omega_1}{\omega_0}\right)$



Example 8.4 of Pozar: Bandpass Filter Design

Design a bandpass filter having a 0.5 dB equal-ripple response, with $N = 3$. The center frequency is 1 GHz, the bandwidth is 10%, and the impedance is 50 Ω.



Solution

From Table 8.4 the element values for the low-pass prototype circuit of Figure 8.25b are given as

$$
\begin{aligned}
g_1 &= 1.5963 = L_1,\\
g_2 &= 1.0967 = C_2,\\
g_3 &= 1.5963 = L_3,\\
g_4 &= 1.000 = R_L.
\end{aligned}
$$

Equations (8.64) and (8.74) give the impedance-scaled and frequency-transformed element values for the circuit of Figure 8.32 as

$$L'_1 = \frac{L_1 R_0}{\omega_0 \Delta} = 127.0 \text{ nH},$$

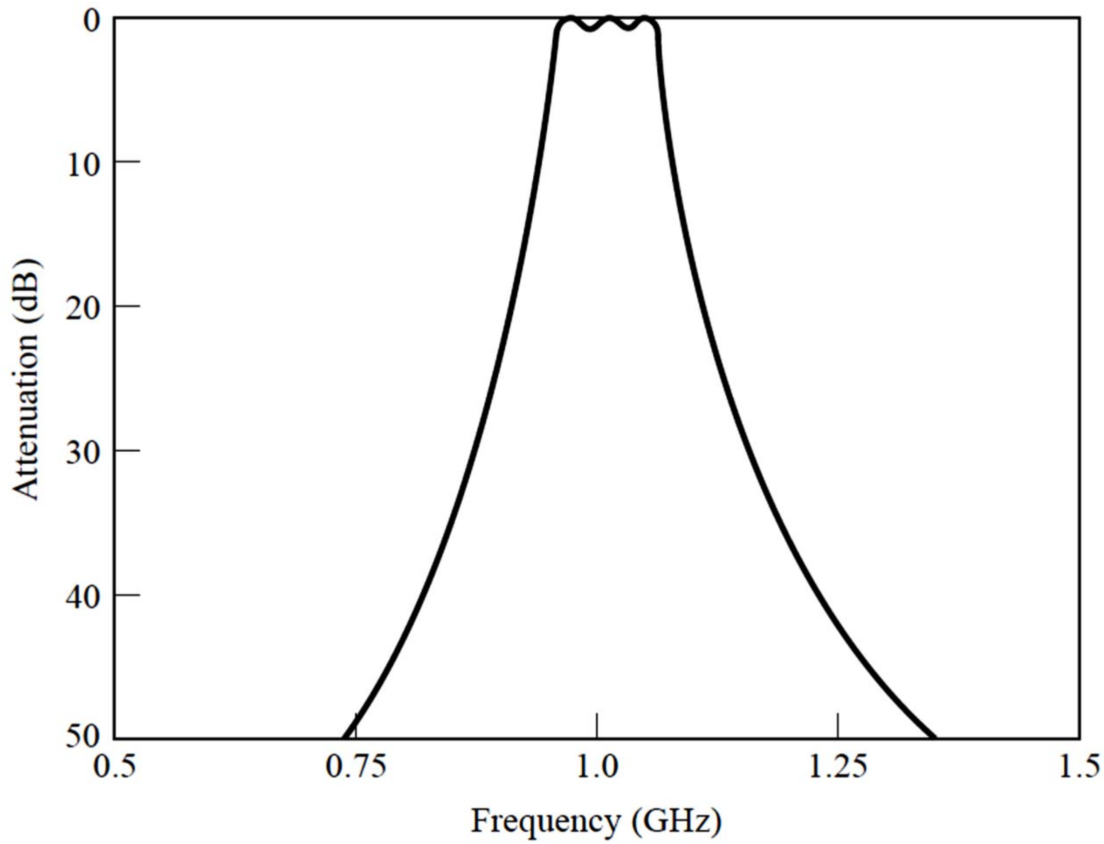
$$C'_1 = \frac{\Delta}{\omega_0 L_1 R_0} = 0.199 \text{ pF},$$

$$L'_2 = \frac{\Delta R_0}{\omega_0 C_2} = 0.726 \text{ nH},$$

$$C'_2 = \frac{C_2}{\omega_0 \Delta R_0} = 34.91 \text{ pF},$$

$$L'_3 = \frac{L_3 R_0}{\omega_0 \Delta} = 127.0 \text{ nH},$$

$$C'_3 = \frac{\Delta}{\omega_0 L_3 R_0} = 0.199 \text{ pF}.$$



Amplitude response for the bandpass filter of Example 8.4.

Filter Implementation

- The lumped-element filter design works well at low frequencies, but imposes problems at microwave frequencies.
- Lumped inductors and capacitors are available only for a limited range of values and are difficult to implement at microwave frequencies which requires smaller inductance and capacitance values.
- At microwave frequencies, the distances between filter components are not negligible.
- The conversion from lumped elements to transmission line sections is needed --- ***Richard's Transformation***.

Richard's Transformation and Kuroda's Identities

Richard's transformation: $$\Omega = \tan\beta l = \tan\frac{\omega l}{v_p}$$

This transformation maps the ω plane to the Ω plane, which repeats with a period of $\omega l/v_p = 2\pi$. The transformation is used to synthesize an LC network using open- and short-circuited transmission lines.

If we replace the frequency variable ω with Ω, the reactance of an ***inductor*** can be written as $$jX_L = j\Omega L = jL\tan\beta l$$

This is equivalent to a short-circuited stub of length βl and characteristic impedance L.

And the susceptance of a ***capacitor*** can be written as

$$jB_C = j\Omega C = jC\tan\beta l$$

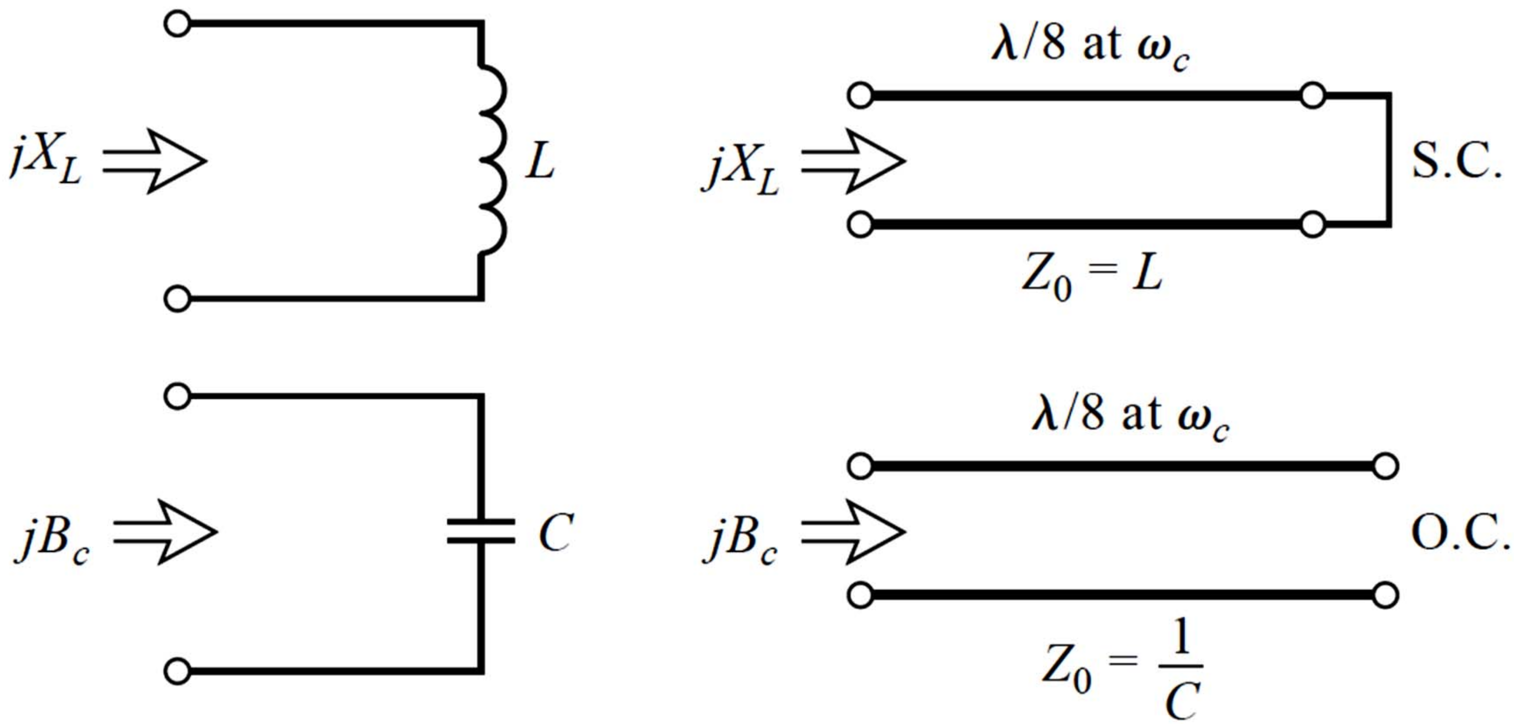
This is equivalent to a open-circuited stub of length βl and characteristic impedance *1/C*.

For a low-pass filter prototype, cutoff frequency is unity. According to Richard's transformation, we have

$$\Omega = 1 = \tan\beta l$$

Which gives a stub size of $l = \lambda/8$, where λ is the wavelength of the line at cutoff frequency, ω_C.

Inductors and capacitors can be replaced with λ/8 lines:



The λ/8 transmission line sections are called *commensurate* lines, since they are all the same length in a given filter.

At the frequency $\omega_0 = 2\omega_C$, the lines will be $\lambda/4$ long, and an attenuation pole will occur. At frequencies away from ω_C, the impedances of the stubs will no longer match the original lumped-element impedances, and the filter response will differ from the desired prototype response. Also, the response will be periodic in frequency, repeating every 4 ω_C.

Richard's Transformation and **Kuroda's** Identities focus on uses of $\lambda/8$ lines, for which the reactance $jX = jZ_o$. Richard's idea is to use variable Z_o (width of microstrip, for example) to create lumped elements from transmission lines. A lumped low-pass prototype filter can be implemented using $\lambda/8$ lines of appropriate Z_o to replace lumped L and C elements.

So if we need an inductance of L for a prototype filter normalized to cutoff frequency $\omega_c = 1$ and admittance $g_o = 1$, we can substitute a $\lambda/8$ transmission line stub that has $Z_o = L$. The last step of the filter design will be to scale the design to the desired ω_c and Z_o (typically 50Ω).

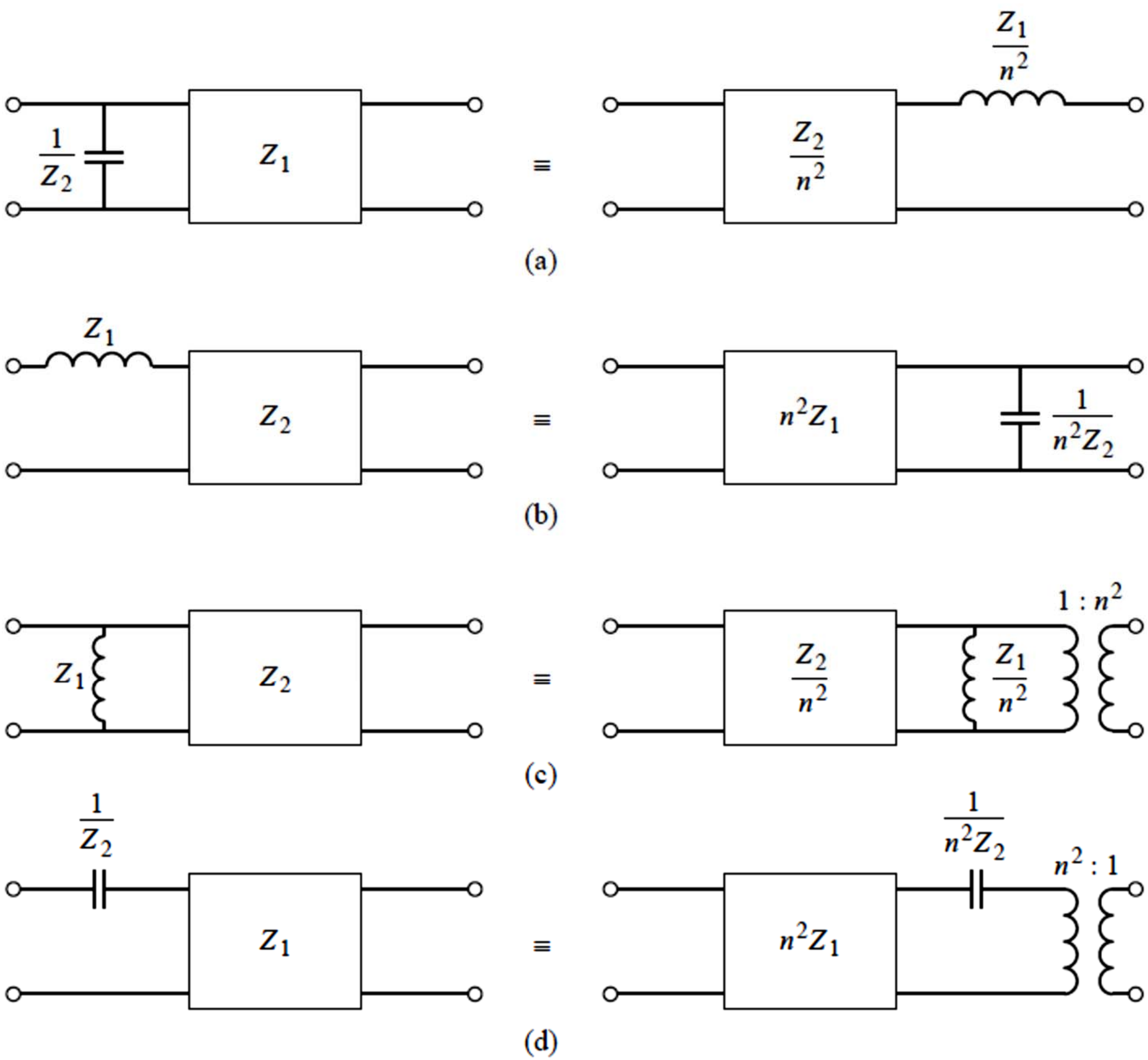
Kuroda's idea is use the redundant λ/8 line of appropriate Z_o to transform awkward or unrealizable elements to those with more tractable values and geometry. As an example, the series inductive stub in the diagram here can be replaced by a shunt capacitive stub on the other end of the λ/8 line, with different values of characteristic impedance determined by

$$\boldsymbol{k} = \boldsymbol{n}^2 = 1 + \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_1}$$

Kuroda's identities can do the following operations:

- Physically separate transmission line stubs
- Transform series stubs into shunt stubs, or vice versa
- Change impractical characteristic impedances into more realizable ones

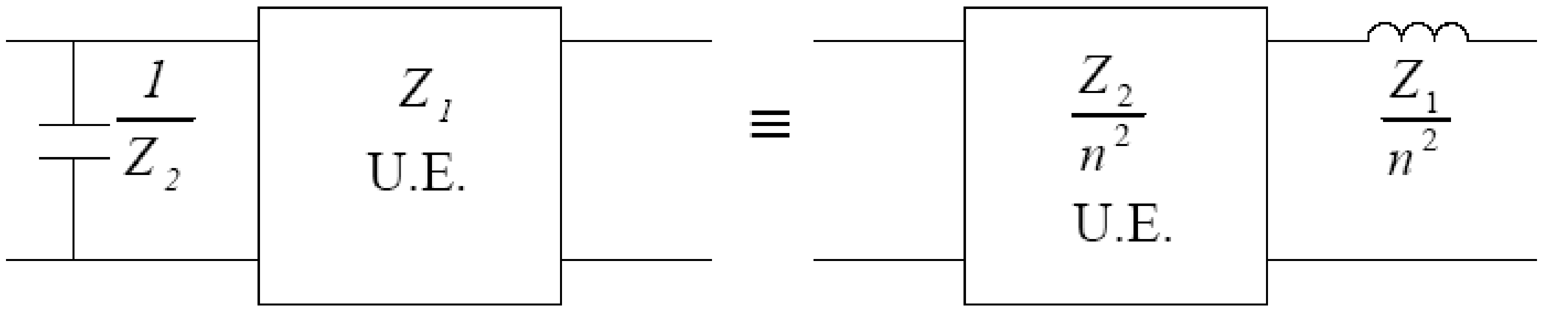
TABLE 8.7 The Four Kuroda Identities ($n^2 = 1 + Z_2/Z_1$)



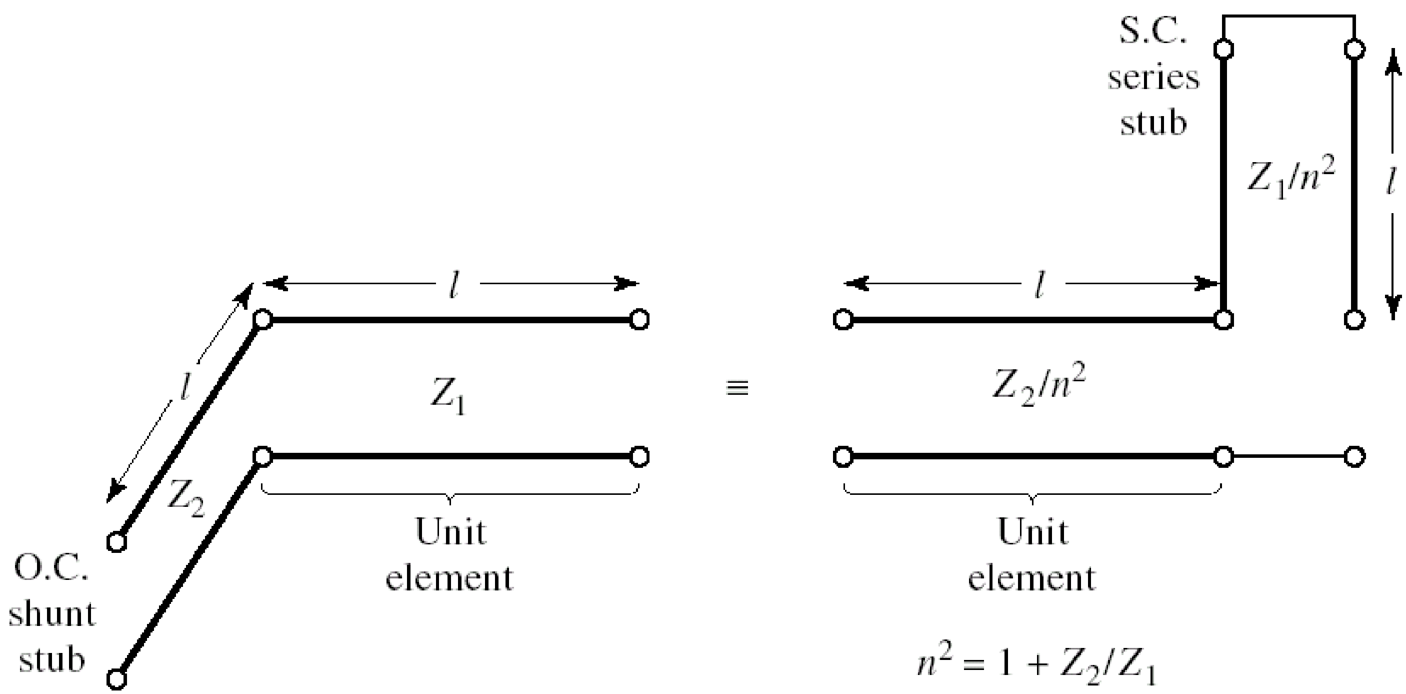
L represents short-circuit stub; *C* represents open-circuit stub

8

Illustration of Kuroda identity for stub conversion



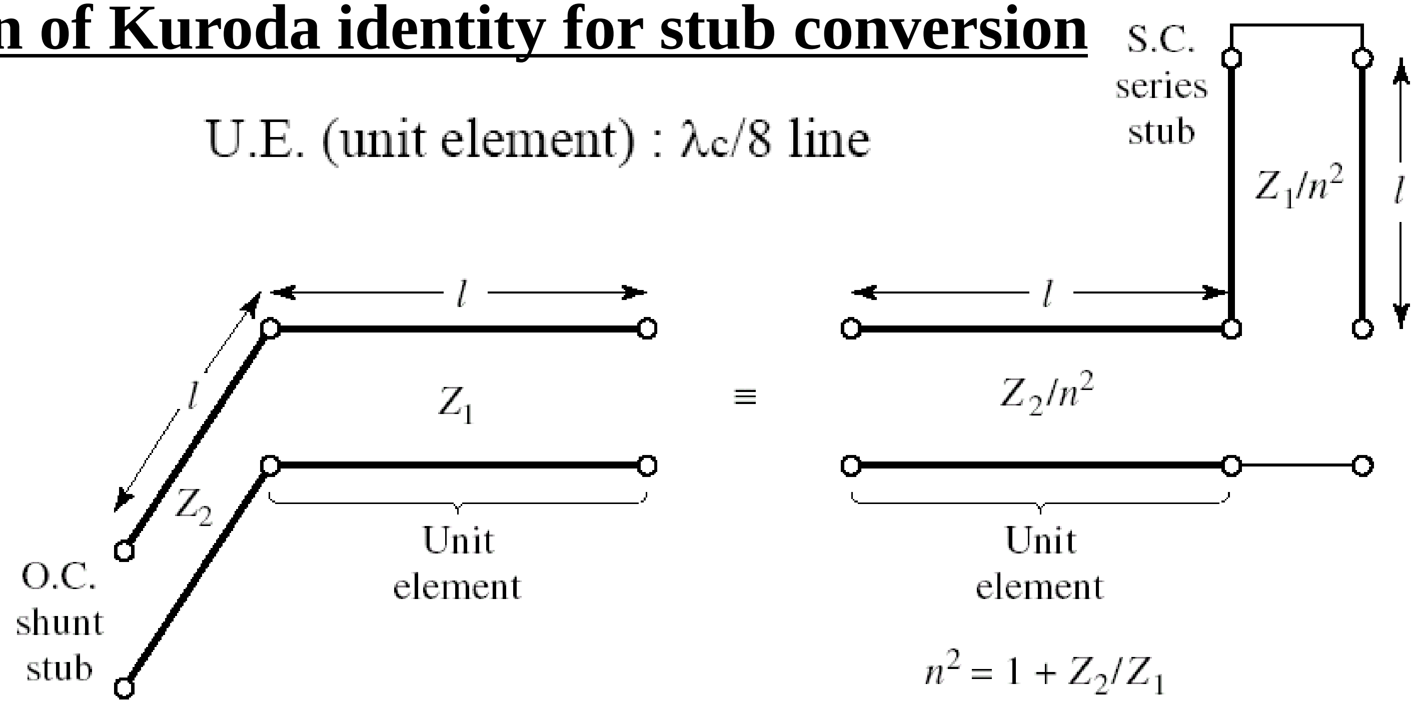
U.E. (unit element) : $\lambda c/8$ line



$n^2 = 1 + Z_2/Z_1$

Illustration of Kuroda identity for stub conversion

U.E. (unit element) : λc/8 line



From Table 4.1, the *ABCD* matrix of a length ℓ of transmission line with characteristic impedance Z_1 is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\ell & jZ_1 \sin\beta\ell \\ \dfrac{j}{Z_1}\sin\beta\ell & \cos\beta\ell \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_1 \\ \dfrac{j\Omega}{Z_1} & 1 \end{bmatrix}, \qquad (8.79)$$

where $\Omega = \tan\beta\ell$. The open-circuited shunt stub in the first circuit in Figure 8.35 has an impedance of $-jZ_2 \cot\beta\ell = -jZ_2/\Omega$, so the $ABCD$ matrix of the entire circuit is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_L = \begin{bmatrix} 1 & 0 \\ \dfrac{j\Omega}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & j\Omega Z_1 \\ \dfrac{j\Omega}{Z_1} & 1 \end{bmatrix} \frac{1}{\sqrt{1+\Omega^2}}$$

$$= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_1 \\ j\Omega\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2}\right) & 1-\Omega^2\dfrac{Z_1}{Z_2} \end{bmatrix}. \quad (8.80a)$$

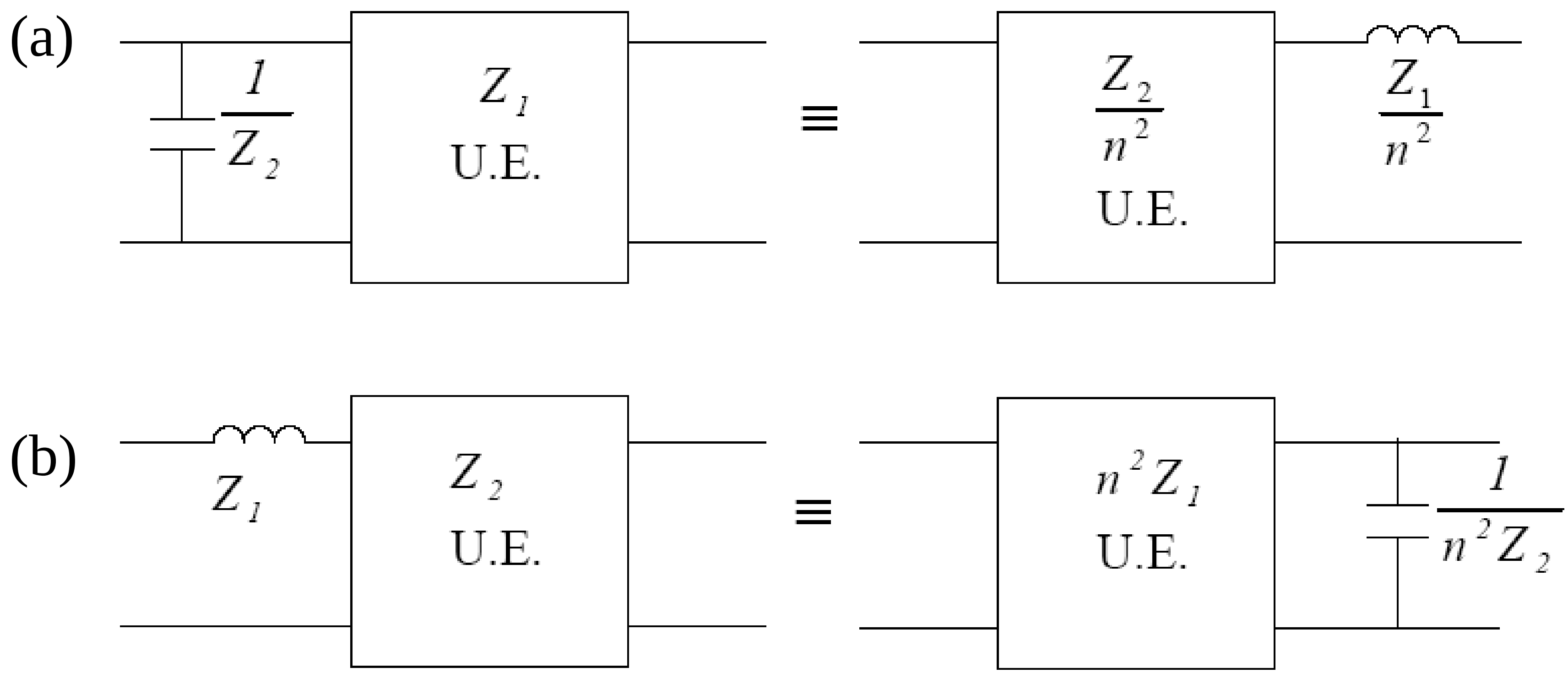
The short-circuited series stub in the second circuit in Figure 8.35 has an impedance of $j(Z_1/n^2)\tan\beta\ell = j\Omega Z_1/n^2$, so the $ABCD$ matrix of the entire circuit is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_R = \begin{bmatrix} 1 & j\dfrac{\Omega Z_2}{n^2} \\ \dfrac{j\Omega n^2}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \dfrac{j\Omega Z_1}{n^2} \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{1+\Omega^2}}$$

$$= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & \dfrac{j\Omega}{n^2}(Z_1+Z_2) \\ \dfrac{j\Omega n^2}{Z_2} & 1-\Omega^2\dfrac{Z_1}{Z_2} \end{bmatrix}. \quad (8.80b)$$

Results in (8.80a) and (8.80b) are identical if **$n^2 = 1 + Z_2/Z_1$** 11

- Kuroda's identites



U.E. (unit element) : $\lambda_c/8$ line

(b) is the most commonly used identity, which removes a *series stub* (difficult to implement in microstrip line form)by transforming it to a shunt stub along with adjustment of characteristic impedances of the $\lambda/8$ lines.

Low Pass Filter Using Stubs

The prototype lowpass LC structure employs series inductors, so a direct conversion to transmission line stubs by Richard's transformation would result in series stubs. However, we can use the Kuroda identity for series inductors to create a structure that has only series transmission line sections and shunt open stubs.

In order to do this we must be aware that we should begin by **adding unit elements (λ/8 transmission lines of Z_o = 1) at each end of the filter**, so that there will be structures that are of the form of the Kuroda identities.

The filter is designed by the following steps:

- Lumped element low pass prototype (from tables, typically)
- Convert series inductors to series stubs, shunt capacitors to shunt stubs
- Add λ /8 lines of $Z_o = 1$ at input and output
- Apply Kuroda identity for series inductors to obtain equivalent with shunt open stubs with λ /8 lines between them
- Transform design to 50Ω and f_c to obtain physical dimensions (all elements are λ/8).

EXAMPLE 8.5 LOW-PASS FILTER DESIGN USING STUBS

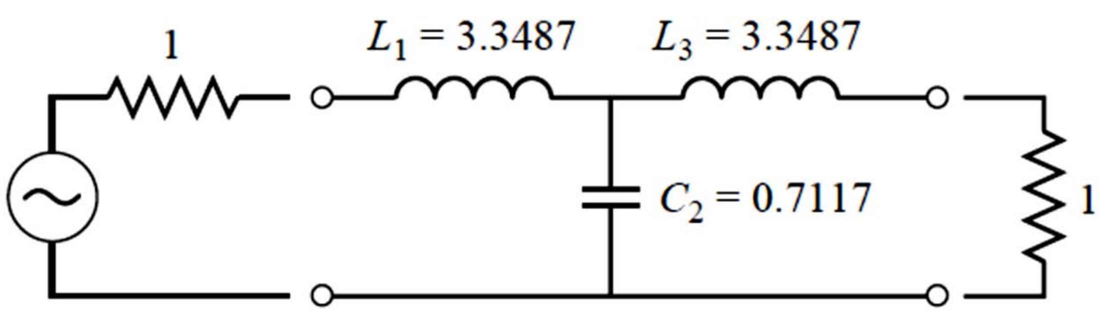
Design a low-pass filter for fabrication using microstrip lines. The specifications include a cutoff frequency of 4 GHz, an impedance of 50 Ω, and a third-order 3 dB equal-ripple passband response.

Solution

From Table 8.4 the normalized low-pass prototype element values are

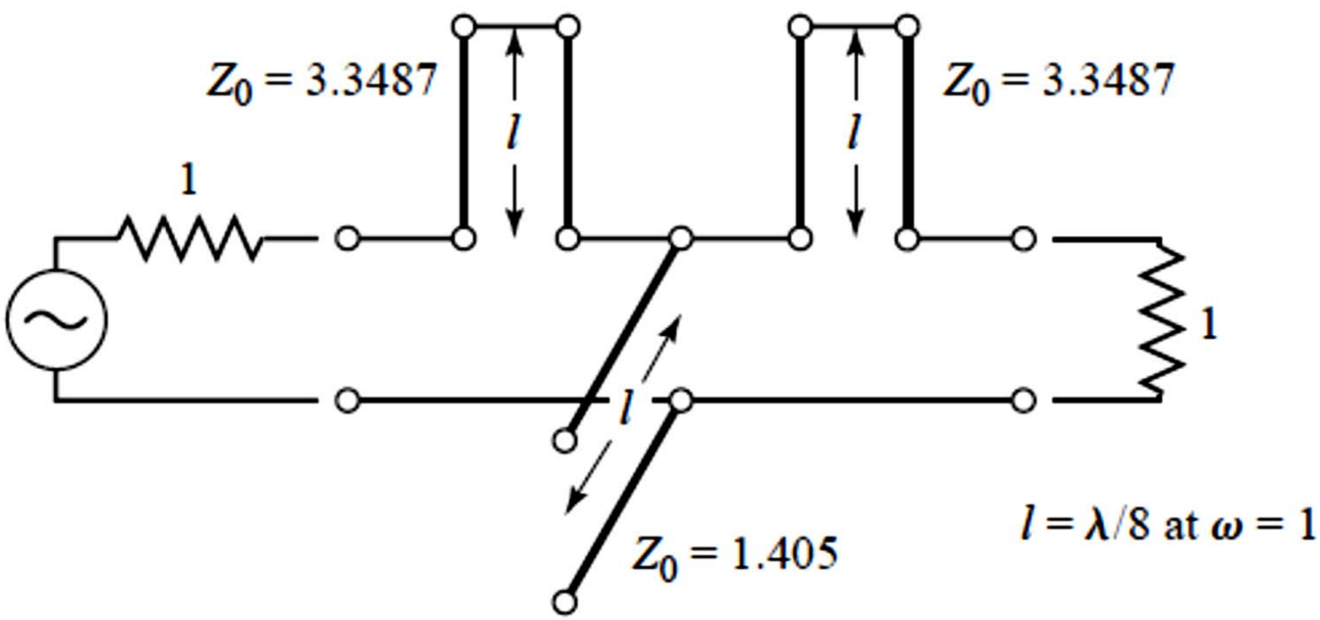
$$g_1 = 3.3487 = L_1,$$
$$g_2 = 0.7117 = C_2,$$
$$g_3 = 3.3487 = L_3,$$
$$g_4 = 1.0000 = R_L,$$

with the lumped-element circuit shown in Figure 8.36a.



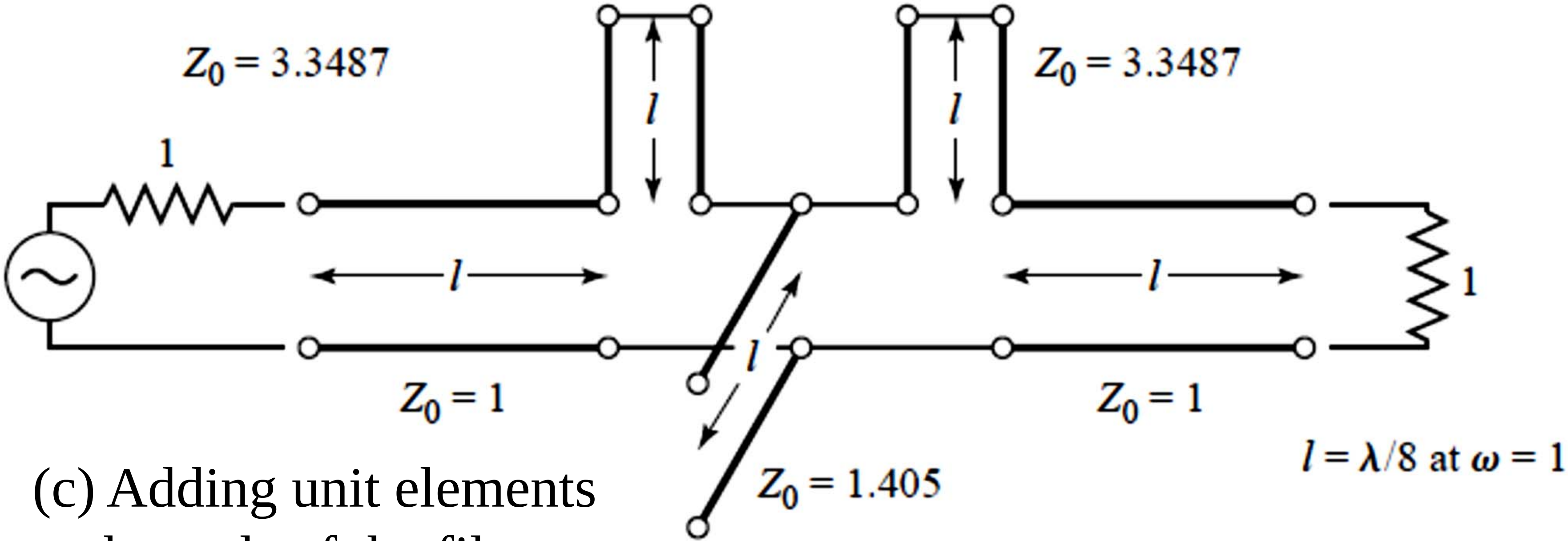
(a)

We now use Richards' transformations to convert series inductors to series stubs, and shunt capacitors to shunt stubs, as shown in Figure 8.36b. According to (8.78), the characteristic impedance of a series stub (inductor) is L, and the characteristic impedance of a shunt stub (capacitor) is $1/C$. For commensurate line synthesis, all stubs are $\lambda/8$ long at $\omega = \omega_c$. (It is usually most convenient to work with normalized quantities until the last step in the design.)



(b)

(b) Using Richards' transformations to convert inductors and capacitors to series and shunt stubs.



(c) Adding unit elements at the ends of the filter.

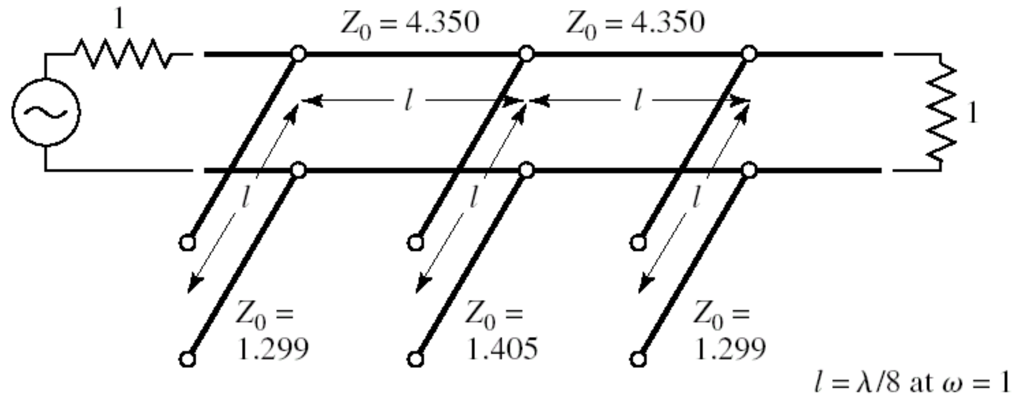
(c)

The series stubs of Figure 8.36b would be very difficult to implement in microstrip line form, so we will use one of the Kuroda identities to convert these to shunt stubs. First we add unit elements at either end of the filter, as shown in Figure 8.36c. These redundant elements do not affect filter performance since they are matched to the source and load ($Z_0 = 1$). Then we can apply Kuroda identity (b) from Table 8.7 to both ends of the filter. In both cases we have that

$$n^2 = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{3.3487} = 1.299.$$

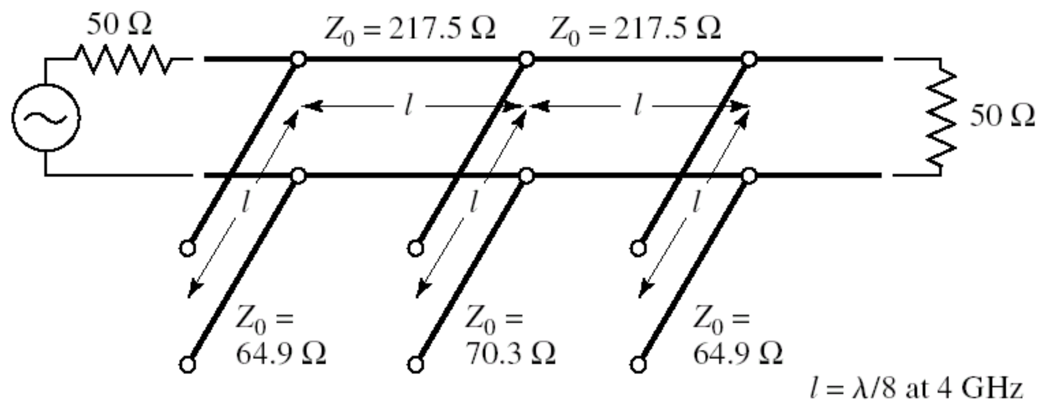
The result is shown in Figure 8.36d.

Figure 8.36 (p. 410)
(*d*) Applying the second Kuroda identity.



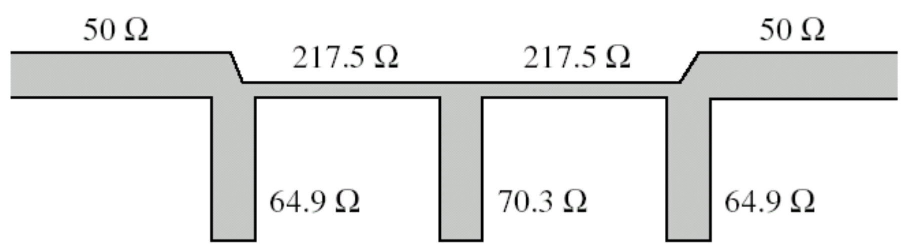
(d)

(*e*) After impedance and frequency scaling.

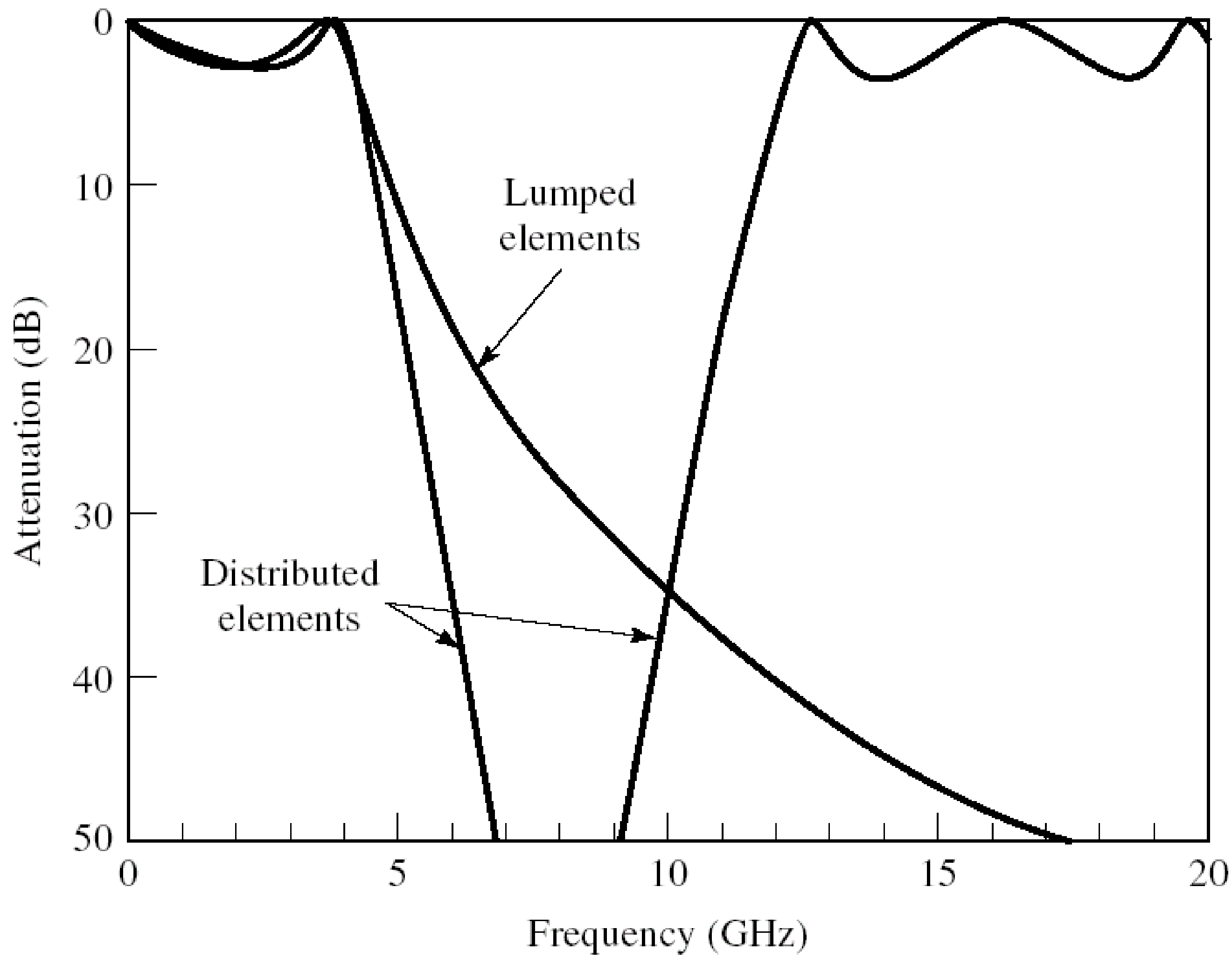


(e)

(*f*) Microstrip fabrication of final filter.



(f)


0
10
20
30
40
50
Attenuation (dB)
Lumped
elements
Distributed
elements
0
5
10
15
20
Frequency (GHz)

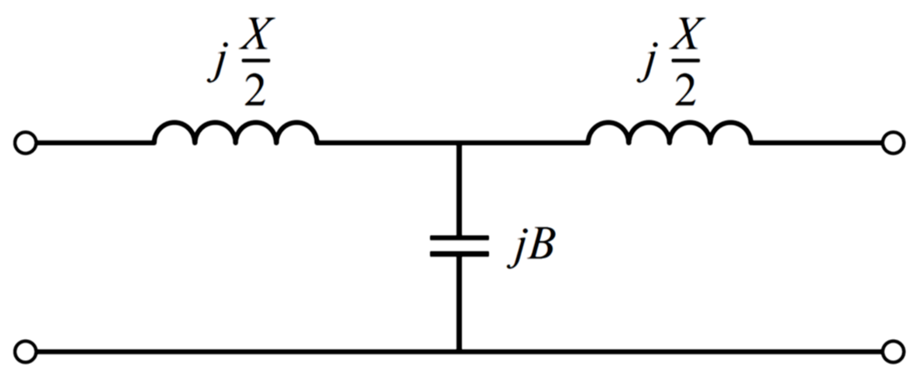
Stepped-Impedance Low-Pass Filters

- Realized with alternating sections of very high and very low characteristics impedance lines.
- Easier to design and take up less space compared to a similar low-pass stub filter.
- Easy implementation results in poorer performance such as slow cutoff.

Consider the T-section equivalent circuit of a short section ($\beta l << \pi/2$) of transmission line, as determined from conversion of the ABCD parameters to Z parameters to identify the individual elements.

$$\frac{X}{2} = Z_0 \tan\left(\frac{\beta l}{2}\right)$$

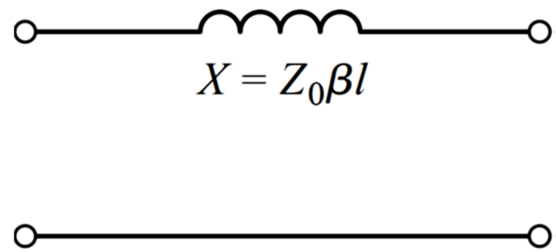
$$B = \frac{1}{Z_0} \sin \beta l$$



For **high Z₀** and small *βl* the equivalent circuit becomes

$$X \cong Z_0 \beta l$$

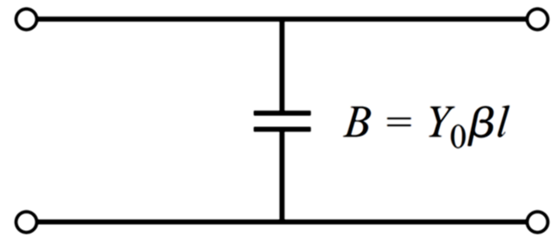
$$B \cong 0$$



For **low Z₀** and small *βl*, the equivalent circuit becomes

$$X \cong 0$$

$$B \cong Y_0 \beta l$$



So the series inductors can be replaced with high-impedance line sections and the shunt capacitors can be replaced with low-impedance line sections. In order to use this approximation, we need to know the highest and lowest feasible transmission line impedances, Z_h and Z_l.

After considering the impedance scaling, we have

$$\beta l = \frac{LR_0}{Z_h} \quad \text{(inductor)} \qquad\qquad \beta l = \frac{CZ_l}{R_0} \quad \text{(capacitor)}$$

The ratio of Z_h/Z_l should be as high as possible, so the actual values of Z_h and Z_l are usually set to the highest and lowest characteristic impedance that can be practically fabricated.

EXAMPLE 8.6 STEPPED-IMPEDANCE FILTER DESIGN

Design a stepped-impedance low-pass filter having a maximally flat response and a cutoff frequency of 2.5 GHz. It is desired to have more than 20 dB insertion loss at 4 GHz. The filter impedance is 50 Ω; the highest practical line impedance is 120 Ω, and the lowest is 20 Ω. Consider the effect of losses when this filter is implemented with a microstrip substrate having $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.02$, and copper conductors of 0.5 mil thickness.

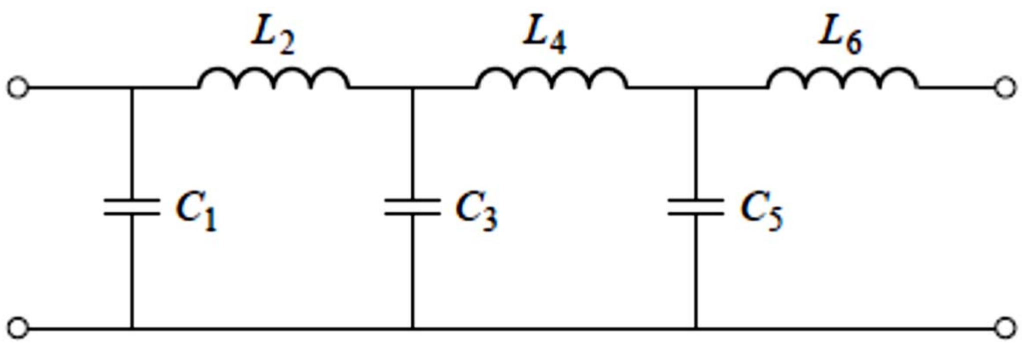
To use Figure 8.26 we calculate

$$\frac{\omega}{\omega_c} - 1 = \frac{4.0}{2.5} - 1 = 0.6;$$

then the figure indicates $N = 6$ should give the required attenuation at 4.0 GHz. Table 8.3 gives the low-pass prototype values as

$$g_1 = 0.517 = C_1,$$
$$g_2 = 1.414 = L_2,$$
$$g_3 = 1.932 = C_3,$$
$$g_4 = 1.932 = L_4,$$
$$g_5 = 1.414 = C_5,$$
$$g_6 = 0.517 = L_6.$$

The low-pass prototype filter is shown in Figure 8.40a.

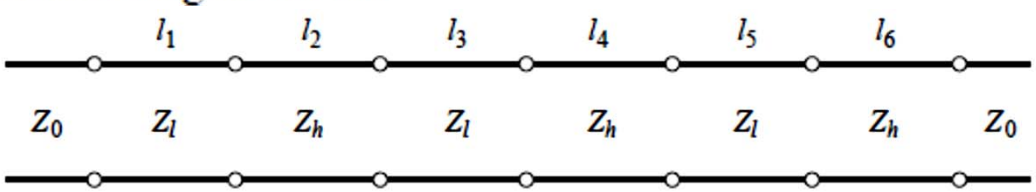


(a)

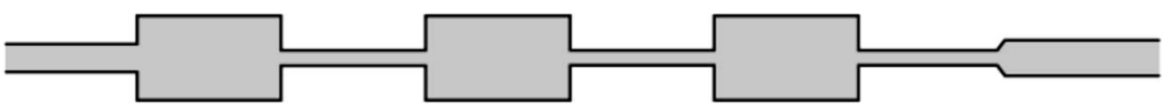
Next, (8.86a) and (8.86b) are used to replace the series inductors and shunt capacitors with sections of low-impedance and high-impedance lines. The required electrical line lengths, $\beta\ell_i$, along with the physical microstrip line widths, W_i, and lengths, ℓ_i, are given in the table below.

Section	$Z_i = Z_\ell$ or $Z_h\,(\Omega)$	$\beta\ell_i$ (deg)	W_i (mm)	ℓ_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41

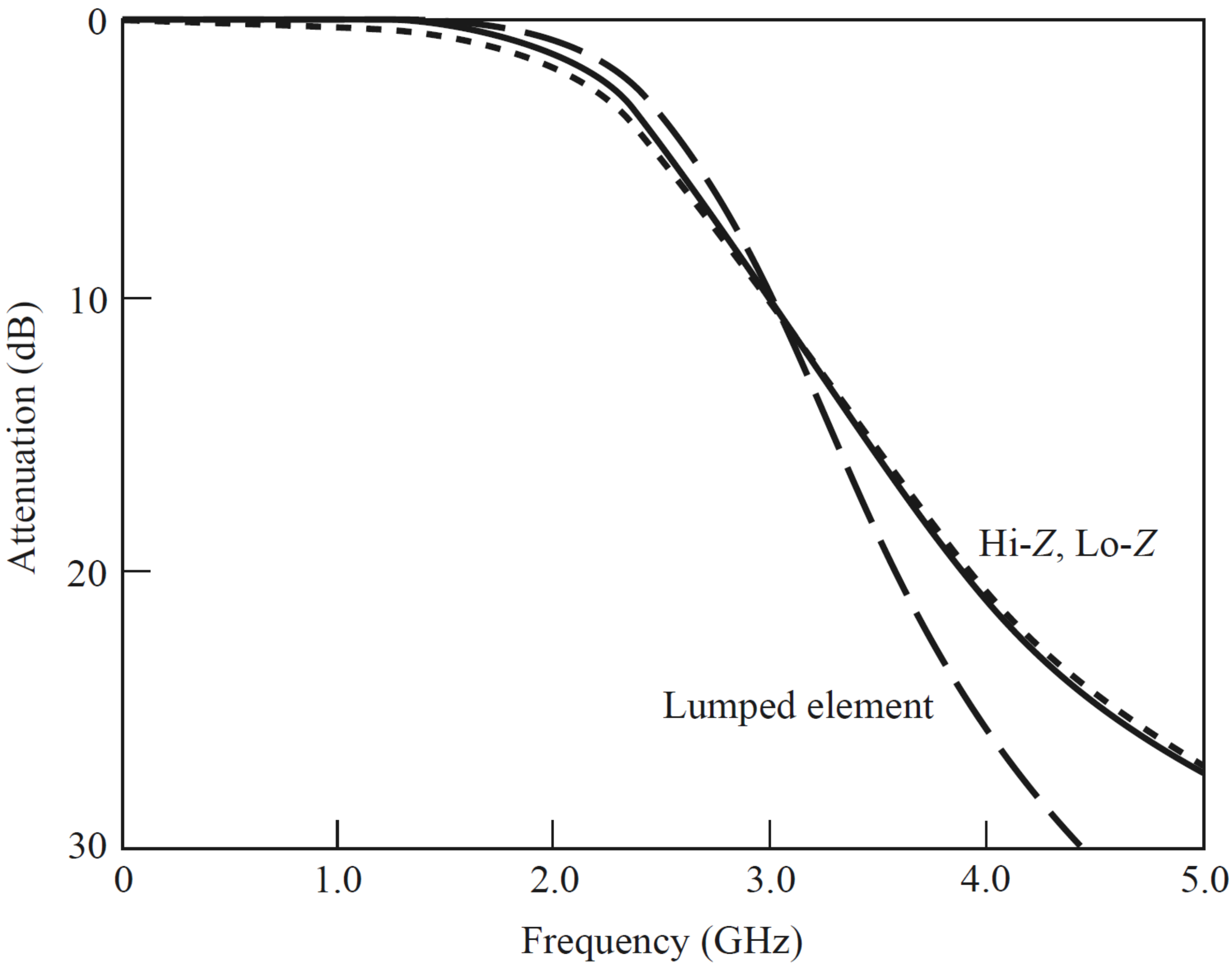
The final filter circuit is shown in Figure 8.40b, with $Z_\ell = 20\ \Omega$ and $Z_h = 120\ \Omega$. Note that $\beta\ell < 45°$ for all but one section. The microstrip layout of the filter is shown in Figure 8.40c.



(b)



(c)

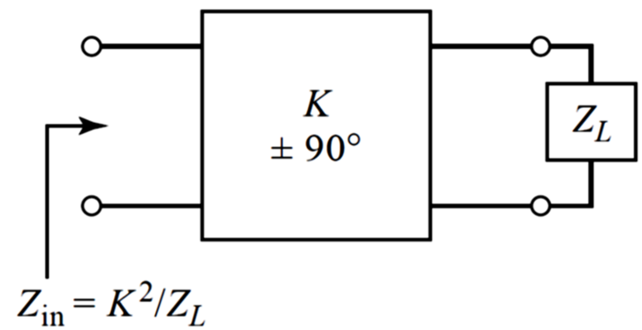

0
10
20
30
Attenuation (dB)
0
1.0
2.0
3.0
4.0
5.0
Frequency (GHz)
Hi-Z, Lo-Z
Lumped element

Impedance and Admittance Inverters

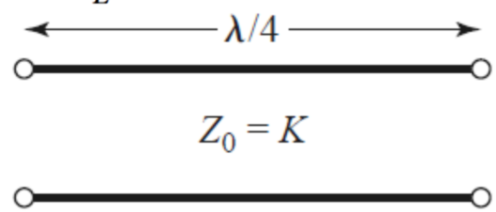
In this process we've uncovered another "magic bullet" comparable to the Kuroda identities, only involving $\lambda/4$ rather than $\lambda/8$ lines. Quarter wave lines can transform series connected element to shunt, and vice versa. Such inverters are especially useful for bandpass or bandstop filters with narrow bandwidths.

For the **impedance (K) inverter**,

$Z_{in} = K^2/Z_L$



For the $\lambda/4$ line, $K = Z_o$

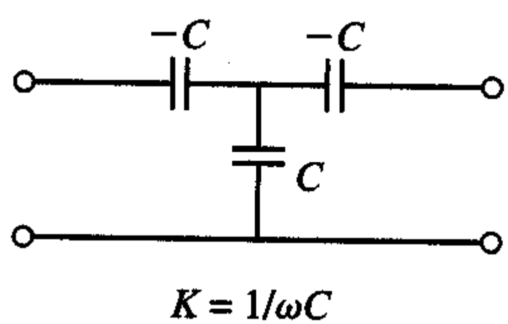


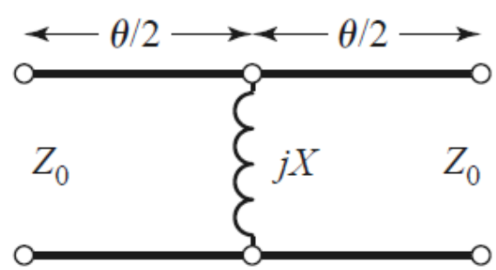
For the lumped element implementation,

$K = Z_o \tan|\theta/2|$,

$$X = \frac{K}{1-(K/Z_o)^2}$$

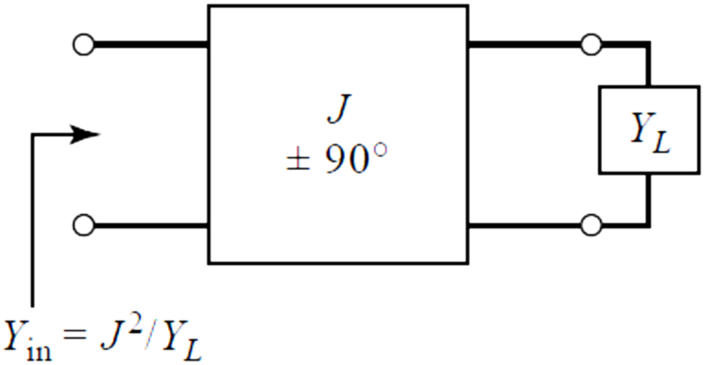
$$\theta = -\tan^{-1}\frac{2X}{Z_o}$$





For the **admittance inverter**, $Y_{in} = J^2/Y_L$

For the $\lambda/4$ line, $J = Y_o$

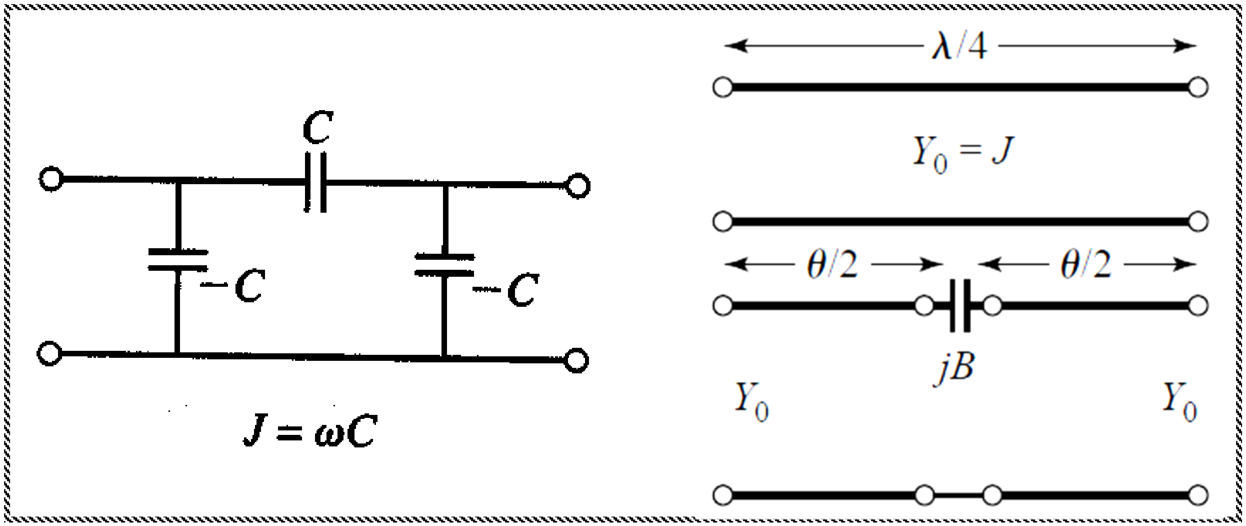


For the lumped element implementation,

$J = Y_o \tan|\theta/2|$,

$$B = \frac{J}{1-(J/Y_o)^2}$$

$$\theta = -\tan^{-1}\frac{2B}{Y_o}$$



Various implementation schemes

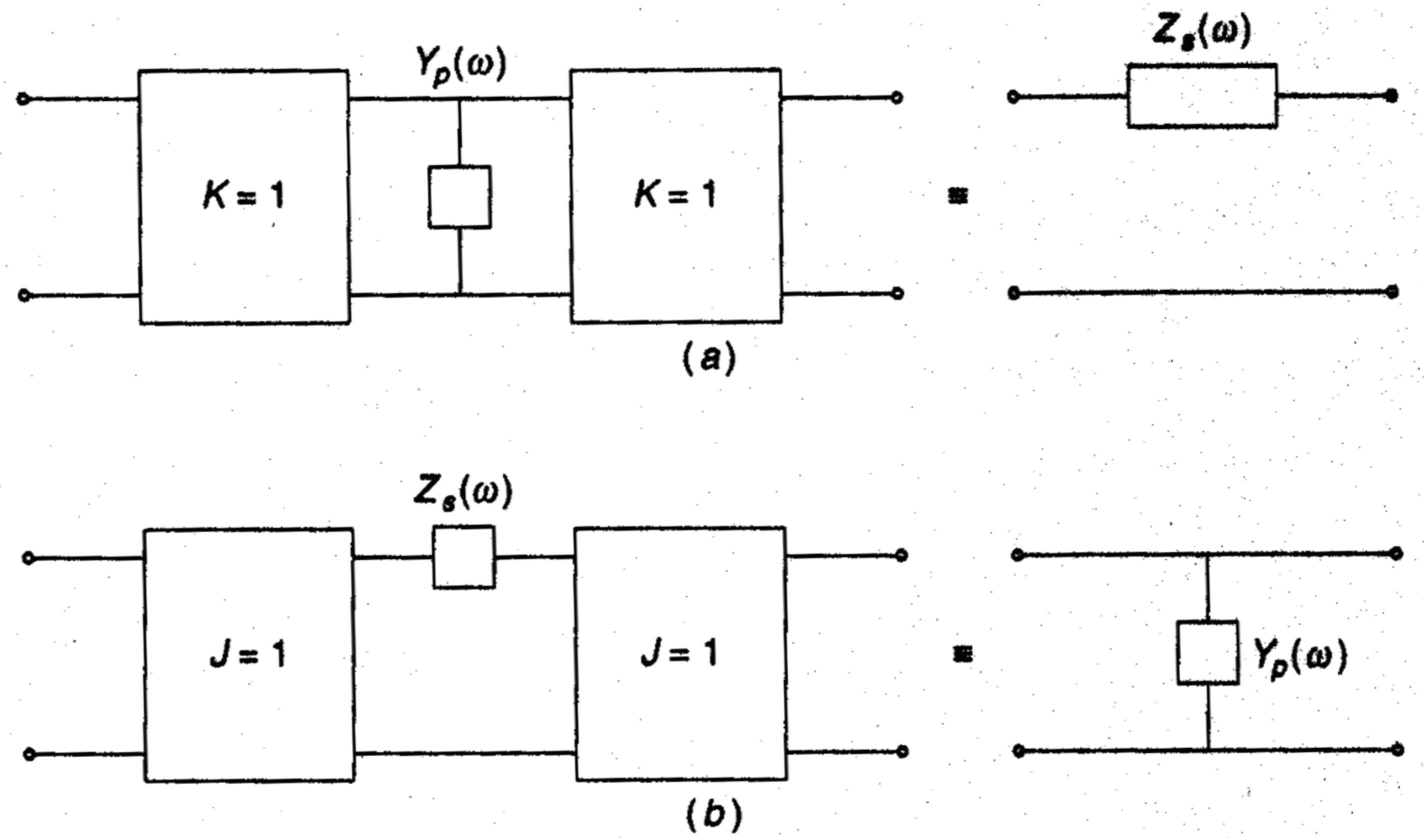
* Negative values of θ, the length of the transmission line sections, poses no problems because they can be absorbed into connecting transmission lines on either side. The same is true for the L and C with negative values

What can be achieved by the impedance and admittance inverter?

• Form the inverse of the load impedance or admittance. Can be used to transform series-connected elements to shunt-connected elements.

• Impedance inverters may be used to convert a bandpass-filter network into a network containing only series tuned circuits.

• Admittance inverters may be used to convert a bandpass-filter network into a network containing only shunt tuned circuits

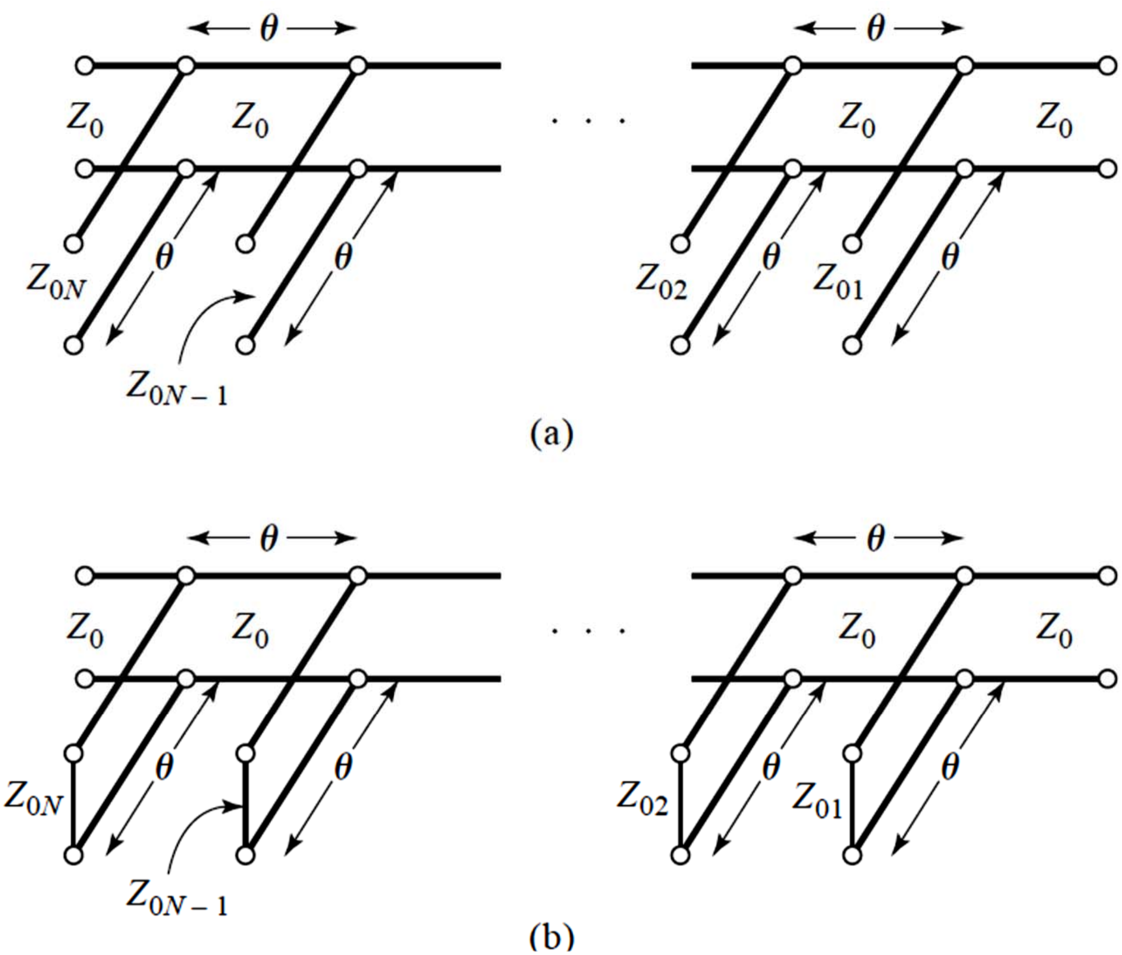


(a) Impedance inverter used to convert a parallel admittance into an equivalent series impedance.

(b) Admittance inverter used to convert a series impedance into an equivalent parallel admittance.

Bandstop and Bandpass Filters Using Quarter-Wave Resonators

Quarter-wave open-circuited or short-circuited transmission line stubs: series or parallel LC resonators, respectively.

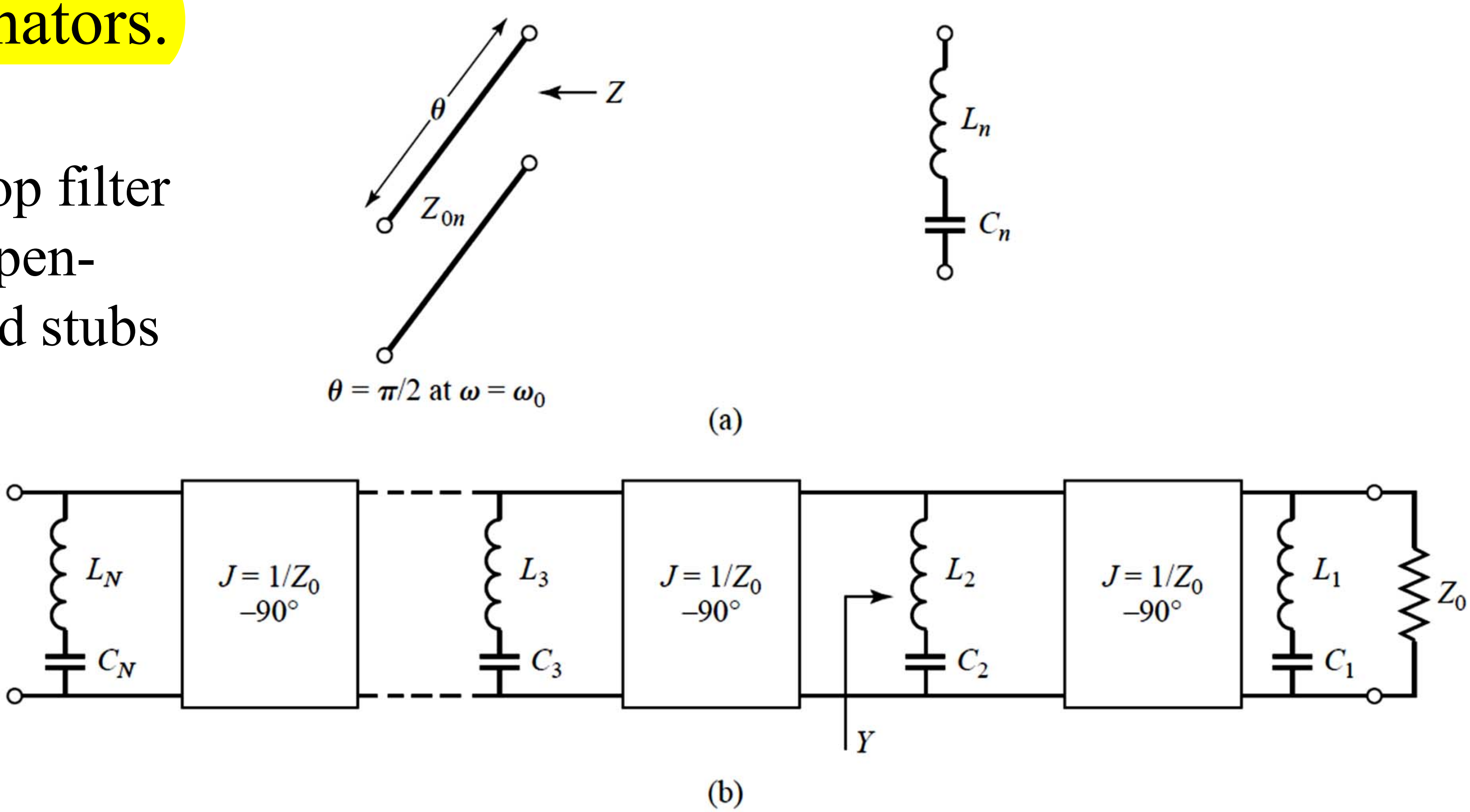


Bandstop and bandpass filters using shunt transmission line resonators ($\theta = \pi/2$ at the center frequency). (a) Bandstop filter. (b) Bandpass filter.

Operating Principle

- λ/4 sections between the stubs act as admittance inverters to effectively convert alternate shunt resonators to series resonators.

Bandstop filter using open-circuited stubs



The input impedance of an open-circuited transmission line of characteristic impedance Z_{0n} is

$$Z = -jZ_{0n} \cot\theta$$

Near resonance,
$$\theta \approx \pi / 2(1 + \Delta\omega / \omega_0)$$

Therefore,
$$Z = jZ_{0n} \tan\frac{\pi\Delta\omega}{2\omega_0} \approx \frac{jZ_{0n}\pi(\omega-\omega_0)}{2\omega_0}$$

The impedance of a series LC circuit is,

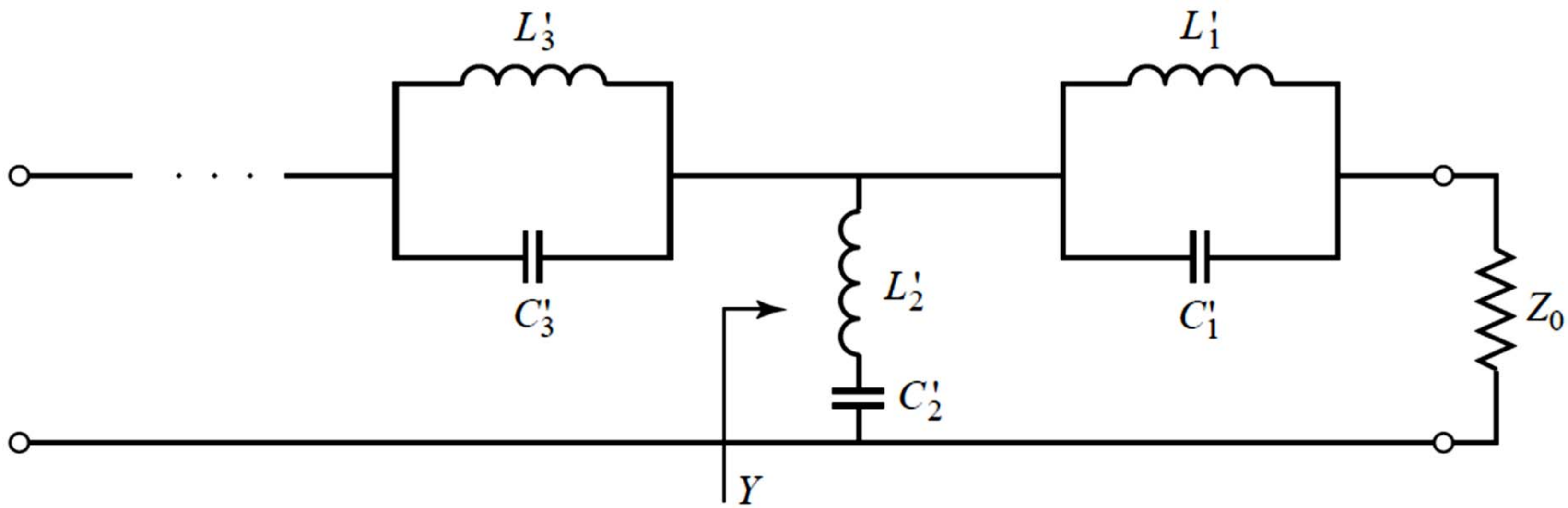
$$Z = j\omega L_n + \frac{1}{j\omega C_n} = j\sqrt{\frac{L_n}{C_n}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$\approx 2j\sqrt{\frac{L_n}{C_n}}\frac{(\omega-\omega_0)}{\omega_0} \approx 2jL_n(\omega-\omega_0)$$

The characteristic impedance of the stub is then given by,

$$Z_{0n} = \frac{4\omega_0 L_n}{\pi}$$

Equivalent lumped-element bandstop filter



(c)

Finding correlation between (b) and (c)

With reference to Figure 8.48b, the admittance Y seen looking toward the L_2C_2 resonator is

$$Y = \frac{1}{j\omega L_2 + (1/j\omega C_2)} + \frac{1}{Z_0^2}\left(\frac{1}{j\omega L_1 + 1/j\omega C_1} + \frac{1}{Z_0}\right)^{-1}$$

$$= \frac{1}{j\sqrt{L_2/C_2}[(\omega/\omega_0) - (\omega_0/\omega)]} + \frac{1}{Z_0}\left\{\frac{1}{j\sqrt{L_1/C_1}[(\omega/\omega_0) - (\omega_0/\omega)]} + \frac{1}{Z_0}\right\}. \tag{8.125}$$

The admittance at the corresponding point in the circuit of Figure 8.48c is

$$Y = \frac{1}{j\omega L_2' + 1/j\omega C_2'} + \left(\frac{1}{j\omega C_1' + 1/j\omega L_1'} + Z_0\right)^{-1}$$

$$= \frac{1}{j\sqrt{L_2'/C_2'}[(\omega/\omega_0) - (\omega_0/\omega)]} + \left\{\frac{1}{j\sqrt{C_1'/L_1'}[(\omega/\omega_0) - (\omega_0/\omega)]} + Z_0\right\}^{-1}. \tag{8.126}$$

These two results will be equivalent if the following conditions are satisfied:

$$\frac{1}{Z_0^2}\sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{C_1'}{L_1'}}, \tag{8.127a}$$

$$\sqrt{\frac{L_2}{C_2}} = \sqrt{\frac{L_2'}{C_2'}}. \tag{8.127b}$$

Since $L_n C_n = L_n' C_n' = 1/\omega_0^2$, these results can be solved for L_n:

$$L_1 = \frac{Z_0^2}{\omega_0^2 L_1'}, \tag{8.128a}$$

$$L_2 = L_2'. \tag{8.128b}$$

Using (8.124) and the impedance-scaled bandstop filter elements from Table 8.6 gives the stub characteristic impedances as

$$Z_{01} = \frac{4Z_0^2}{\pi\omega_0 L_1'} = \frac{4Z_0}{\pi g_1 \Delta}, \tag{8.129a}$$

$$Z_{02} = \frac{4\omega_0 L_2'}{\pi} = \frac{4Z_0}{\pi g_2 \Delta}, \tag{8.129b}$$

where $\Delta = (\omega_2 - \omega_1)/\omega_0$ is the fractional bandwidth of the filter. It is easy to show that the general result for the characteristic impedances of a bandstop filter is

$$Z_{0n} = \frac{4Z_0}{\pi g_n \Delta}. \tag{8.130}$$

For a bandpass filter using short-circuited stub resonators the corresponding result is

$$Z_{0n} = \frac{\pi Z_0 \Delta}{4g_n}. \tag{8.131}$$

These results only apply to filters having input and output impedances of Z_0 and so cannot be used for equal-ripple designs with N even.