In the name of God

A Genetic Algorithm Approach for Solving the Train Formation Problem

Martinelli and Teng (1995)

Spring 2009

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Outlines

- Introduction
- Three Formulations for the TFP
- GA Formulations to the TFP
- Validation of the GA Model
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Introduction

Railroad System Operating Plans

- Railroad system operating plans are developed to perform the sequential decision process of:
  - **Car block decisions**
  - **Train formation decisions**
  - **Train schedule decisions**
  - **Empty car distribution decisions**

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Railroad System Operating Plans

- **Car block decisions**
  - determine which blocks the cars will be assigned to, or which demand each block will carry.
- **Train formation decisions**
  - determine which train the blocks will be assigned to, or which b
- **Train schedule decisions**
  - determine when trains will be released from their origin station and arrive at their destination station. lock each train will carry.
- **Empty car distribution decisions**
  - determine where the empty cars will be sent.

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Aim

- In this study, we present the development of a **genetic algorithm (GA)** as a possible technique for **train formation problem (TFP)**.

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Why Genetic Algorithm?

- Although mathematical programming formulations and algorithms are available for solving the **train formation problem**, the computational time required for their convergence is usually excessive.
- New approaches such as artificial intelligence are necessary and may prove quite fruitful if shorter implementation times can be achieved without a substantial loss in solution integrity.
- One such artificial intelligence technique is **genetic algorithms (GA)**.

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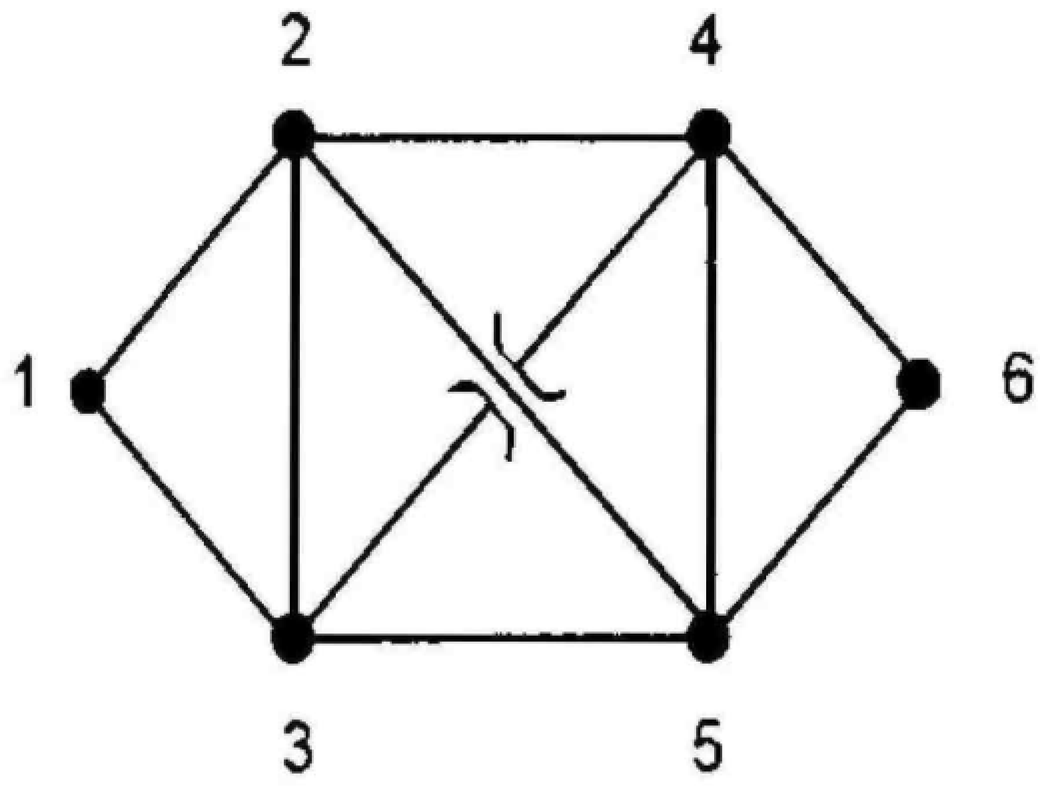
Three Formulations for the TFP

Formulation

- An example railroad network having 6 **nodes** (representing **yards**) and 10 **links** (representing **line segments**)



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Formulation

- **Trains**:
  - **Short distance service**
    - The short distance trains are those whose origin and destination yards are adjacent
    - Short distance trains are always provided for each link
  - **Long distance service**
    - long distance trains are those whose origin and destination yards are not adjacent.
    - The existence of long distance trains is determined by the train formation plan.

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Formulation

- **Demands**:
  - **Short distance demands**
    - Those demands whose origin and destination are connected directly by one link.
    - Short distance demands are carried by short distance trains
  - **Long distance demands**
    - Those demands whose origin and destination are not directly connected.
    - Long distance demands are carried by a combination of short and long distance trains.

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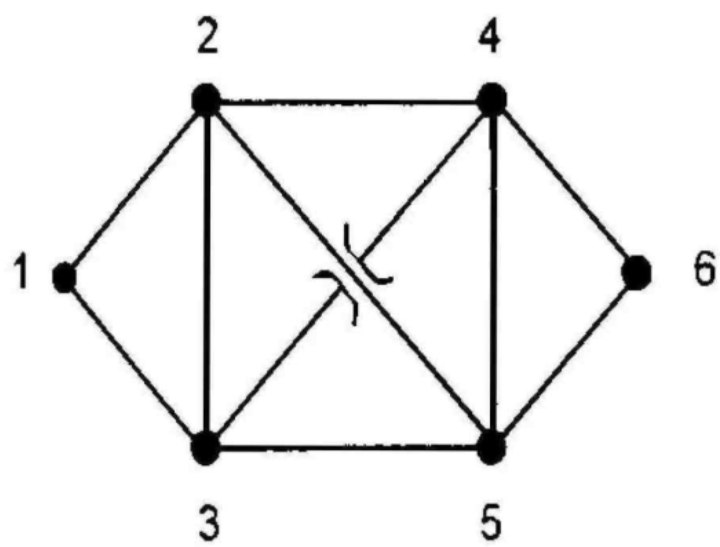
Formulation

- **Routes**:
  - There are a number of different physical routes available for a given demand.
  - For example. for demand from Yard 1 to Yard 6, there might be four different physical routes possible: (1,2,4,6), (l, 2, 5. 6), (1, 3, 4, 6), and (1, 3, 5, 6).



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Formulation

- **Itineraries:**
  - On a certain route, there are always a high number of possible **itineraries** (or **assignments**).
  - These itineraries are distinguished from each other by the number and types of trains.

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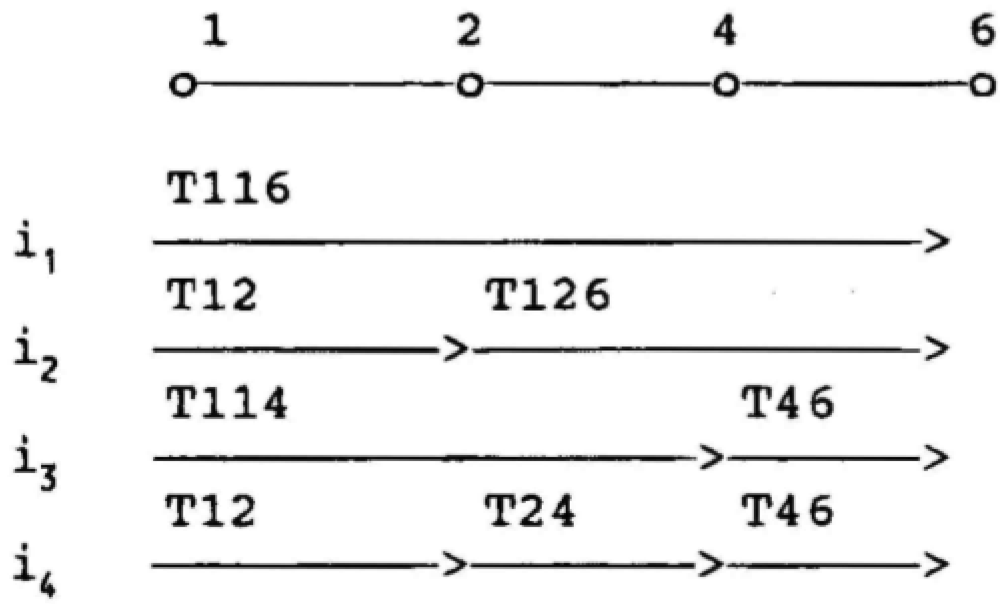
Formulation

- For example, along physical route (1,2,4,6), there might be four itineraries possible.
- Referring to Itinerary i_2, the demand for Yard 1 to 6 will be relayed from Yard 1 to Yard 2, and then to Yard 6.



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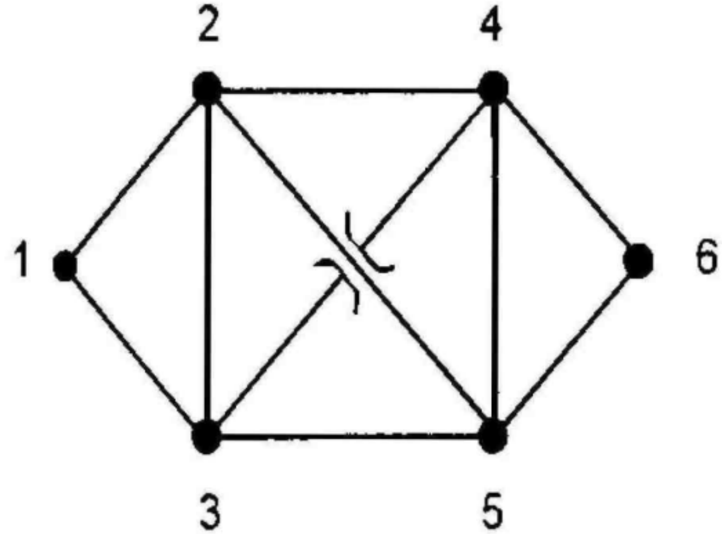
Formulation

- The label of **long distance trains** and the route they follow are:



Yard	Train	Train Route
1	T116	(1,2,4,6)
	T216	(1,2,5,6)
	T316	(1,3,4,6)
	T416	(1,3,5,6)
	T114	(1,2,4)
	T214	(1,3,4)
	T115	(1,2,5)
	T215	(1.3,5)
2	T126	(2,4,6)
	T226	(2,5,6)
3	T136	(3,4,6)
	T236	(3,5,6)
4	T141	(4,2,1)
	T241	(4,3,1)
5	T151	(5,2,1)
	T251	(5,3,1)
6	T161	(6,4,2,1)
	T261	(6,4.3,1)
	T361	(6,5,2,1)
	T461	(6,5,3,1)
	T162	(6,4,2)
	T261	(6,5,2)
	T163	(6,4,3)
	T263	(6,5,3)

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Formulation

- The **short distance trains** are denoted such as **T12**, where 1 and 2 are the train's origin and destination, respectively.
- Whereas the **long distance trains** are in the form of T216, where 1 (second number) and 6 are the train's origin and destination, respectively, 2 is the sequence of the possible roads the train can follow between 1 and 6.

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Formulation

- The demand matrix:

	1	2	3	4	5	6
1		64	94	121	150	150
2	78		87	27	54	107
3	72	95		4	14	150
4	150	61	19		10	34
5	136	38	89	87		99
6	150	150	140	67	26	

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Formulation

- It is a common practice for the sake of convenience that, when managing the traffic flow on the railroad network, **each demand** is usually confined to **only one itinerary**.
- If for each demand, a set of 0-1 variables are defined for the choice of itinerary, the TFP could be formulated as a **0-1 integer program**.
- If the objective is minimizing the **delay times** including the **travel times** of the cars incurred in the railroad system, then the TFP can be formulated as follows.

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Formulation

i	the index of Demands
j	the index of Trains
l	the index of Links
r_i	the amount of traffic of Demand i
t_l	the average travel time on Link l
v_j	Train j's operating time at its destination yard
L	the total number of links
M	the total number of demands
N	the total number of trains possible provided

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Formulation

R_i	the set of itineraries by which Demand i was supposed to be carried
S_j	the set of itineraries which include Train j as one part of their line haul
P_l	the set of trains which pass through Link l
X_j	the volume of cars in Train j
Y_l	the volume on Link l
$x_{i,k}$	a binary integer variable representing the demand-itinerary choice, it will be 1 if demand i is carried by itinerary k, otherwise zero.

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First Case

$$MIN \sum_{l=1}^{L} t_l Y_l + \sum_{j=1}^{N} v_j X_j \tag{1}$$

Subject to:

$$\sum_{k \in R_i} x_{i,k} = 1 \tag{2}$$

where:

$$X_j = \sum_{i=1}^{M} \sum_{k \in S_j} r_i x_{i,k}$$

$$Y_l = \sum_{j \in p_l} X_j$$

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First Case

- In the objective function,
  - the first summation is for the travel times incurred on **line segments**, and
  - the second summation is for the times incurred at yards.
- The second Equation is the demand-route restrictions.
- The demand flow conservation and balance constraints usually appear in transportation network models, but are automatically satisfied by this formulation.
- This is the **first case** we will investigate in this study.

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Second Case

- In first case formulation, it is assumed that the times in which the traffic is incurred at yards and on line segments are **independent** of the traffic volume.
- However, in reality, the times are always dependent on the volume.
- The relationship between times and the traffic volume is **nonlinear**.
- Modifying the objective function accordingly, we have:

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Second Case

$$MIN \quad \sum_{l=1}^{L} t_l(Y_l)Y_l + \sum_{j=1}^{N} v_j(X_j)X_j$$

Subject to:

$$\sum_{k \in R_i} x_{i,k} = 1$$

where:

$$x_j = \sum_{i=1}^{M} \sum_{k \in S_j} r_i x_{i,k}$$

$$Y_l = \sum_{j \in p_l} X_j$$

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Third Case

- Furthermore, in practice, it is likely to impose constraints on some variables such as link flow and train load.
- These constraints can be formulated as:

$$Y_l \leq b_l \quad \text{for} \quad l=1,2, \ldots, L$$

$$X_j \geq b_2 \quad \text{for} \quad j=1,2, \ldots, N$$

- The first indicates that the traffic volumes on links should be less than b_l. The second indicates that the trains can be provided only when the loads on them are larger than b_2

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Third Case

$$MIN \quad \sum_{l=1}^{L} t_l(Y_l)Y_l + \sum_{j=1}^{N} v_j(X_j)X_j$$

Subject to:

$$\sum_{k \in R_i} x_{i,k} = 1$$

$$Y_l \leq b_1 \quad \text{for} \quad l=1,2,\ \ldots,\ L$$

$$X_j \geq b_2 \quad \text{for} \quad j=1,2,\ \ldots,\ N$$

where:

$$x_j = \sum_{i=1}^{M} \sum_{k \in S_j} r_i x_{i,k}$$

$$Y_l = \sum_{j \in p_l} X_j$$

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Data

- In this study, L = 10 (links), M = 30 (demands) and N = 44 (trains).
- All the t_l have values of 10 hr/car .
- All the v_j take values around 13-15 hr/car.

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Data

- There are 10 long distance demands.
- For demand from 1 to 6 and from 6 to 1, each is assumed to have 16 possible itineraries.
- For the remaining 8 long distance demands, each is assumed to have 4 possible itineraries.
- All of the formulations in these three cases are binary integer programs.

Data

- For Case 1, some exact algorithms have been proved to be effective conventionally.
- The common point of these algorithms might-be the use of the linear characteristics of the objective function.
- In each operation of "branch," for example, relaxed linear programming can be efficiently solved.
- However, in Cases 2 and 3, the objective functions are not linear.

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GA Formulations to the TFP

GA Formulations to the TFP

- Train formation decisions are represented by 0-1 strings

Demand i				Demand i+1			
Route 1		Route 2		Route 1		Route 2	
itinerary k	itinerary k+1	itinerary k+2	itinerary k+3	itinerary k+4	itinerary k+5	itinerary k+6	itinerary k+7
$x_{i,k}$	$x_{i,k+1}$	$x_{i,k+2}$	$x_{i,k+3}$	$x_{i+1,k+4}$	$x_{i+1,k+5}$	$x_{i+1,k+6}$	$x_{i+1,k+7}$

- In this figure $x_{i,k}$ the decision variable where each route for each demand has two itineraries.

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GA Formulations to the TFP

- The evaluation functions derived for the three cases are the following:

$$BM - \left[\sum_{l=1}^{L} t_l Y_l + \sum_{j=1}^{N} v_j X_j \right]$$

$$BM - \left[\sum_{l=1}^{L} t_l (Y_l) Y_l + \sum_{j=1}^{N} v_j (X_j) X_j \right]$$

$$BM - \left[\sum_{l=1}^{L} t_l (Y_l) Y_l + \sum_{j=1}^{N} v_j (X_j) X_j + \sum_{l=1}^{L} f(Y_l - b_1) + \sum_{j=1}^{N} g(X_j - b_2) \right]$$

- Where BM is used to convert the minimizing objective functions to maximizing. The variables f and g are penalty functions.

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GA Formulations to the TFP

- The GA operations are designed as follows:

1) The initial set of solutions are generated randomly and

2) The operations of reproduction and mutation are done with regard to the constraints represented in Equation 2, the mate sites in the crossover operation are selected uniformly at random from a specific set of positions. instead of from a set of consecutive numbers like that in the general GA. In this way, the constraint in Equation 2 can be guaranteed automatically in the genetic algorithm operation.

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Calibration of the GA Model

Calibration of the GA Model

- The **calibration process** for the GA model is to find the **appropriate parameters** by which the best solutions of the GA model can be obtained.
- These parameters include:
  - the size of the population
  - the number of generations
  - the crossover probability
  - the mutation probability
- In order to quicken the calibration, it is decomposed into two steps.

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Calibration of the GA Model

- **First Step:**
  - The **first step** considers only the size of the population and the number of generations.
  - When generating the schemes, only these two parameters vary within certain ranges, whereas the crossover probability and the mutation probability are fixed at 0.9 and 0.03. respectively.
  - From this step, the optimal number of generations and population size are determined.

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Calibration of the GA Model

- **Second Step:**
  - Given the population and the number of generations values, the **second step** generates schemes by varying the crossover and mutation probability in a certain range.
  - From this step, the optimal values for these parameters are obtained.

- The calibration process is conducted for **all three cases**.

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First Step

- In the first step, the generations are set at 100, 200, …, 1000, respectively.
- The population sizes are set at 10, 20, ... , 100. respectively.
- Then, for each case, 100 schemes will need to be generated and evaluated.
- After a rough scanning of all the results. it is determined that **the generation of 1000** is the most appropriate to evaluate the performance of the GA model.

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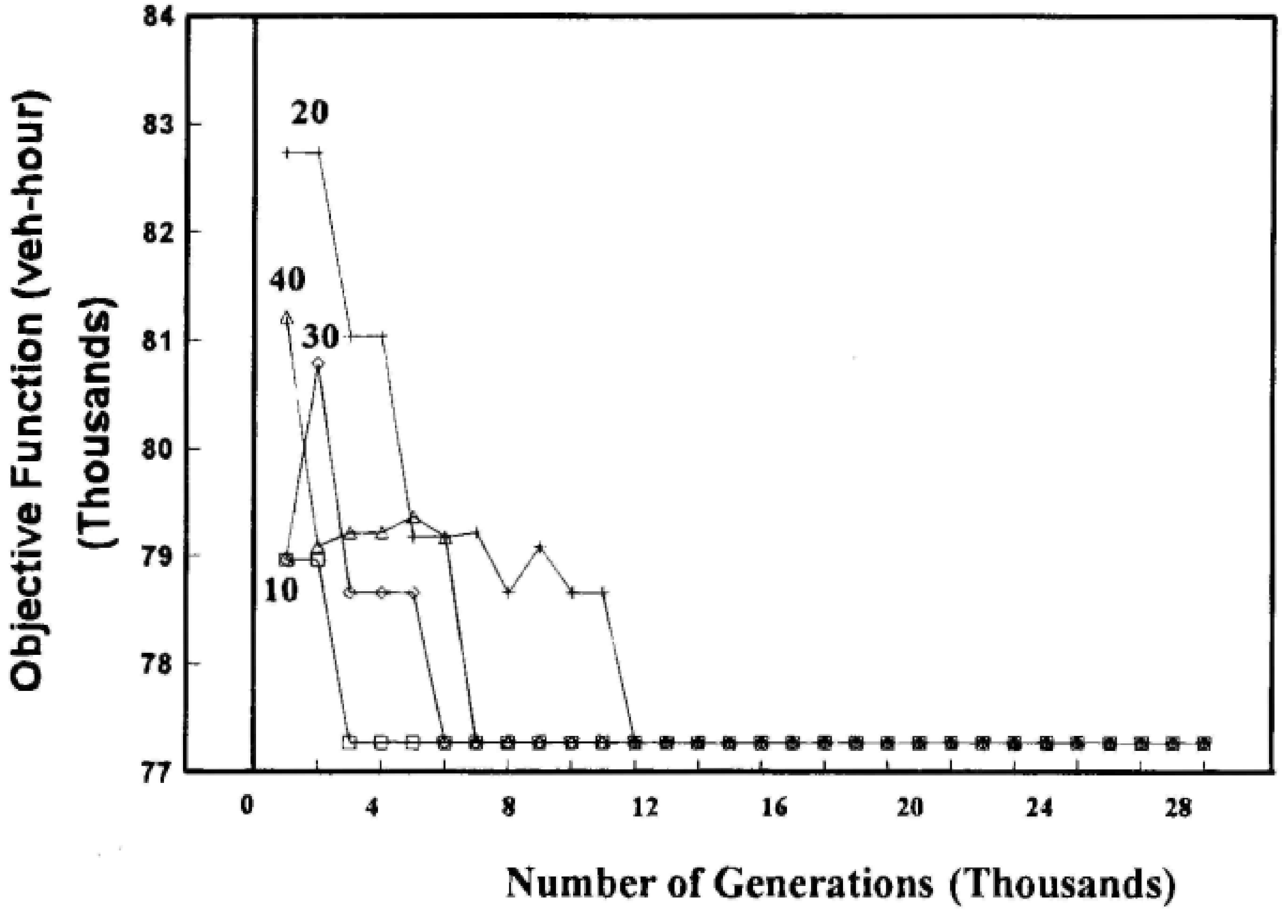
Figure 4 Determination of population size for Case 1

- A sample plot to investigate the influence of the **population size** on the search process case 1:



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First Step

- The population sizes are determined by two criteria:
  - the time the GA model uses to decrease the objective function values to the best solution and
  - the stability after the best solutions have been achieved.
- The optimal population size is found to be:
  - 10 for Case 1
  - 100 for Case 2
  - 70 for Case 3

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Second Step

- Following the procedures for Step 2, the crossover probabilities are set at 0.6, 0.7, 0.8, 0.9, and 1.0 respectively.
- The mutation probabilities are set at 0.01, 0.02, 0.03, 0.04, and 0.05, respectively.

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Second Step

- For ease of analysis, the solution searching processes are classified into **four patterns**.
- **Pattern 1**
  - the searching processes are stable after the smallest values are found.
  - The generation at which the smallest values are found is called the **stable generation**.
  - This pattern is viewed to have the best performance.

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Second Step

- **Pattern 2 and Pattern 3**
  - are similar in the solution search processes. Both patterns indicate that the search processes will fluctuate after the smallest objective function value is achieved.
  - However, in Pattern 2, the search process stays at the convergence status for a longer time than that in Pattern 3.

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Figure 6 Search process: Pattern 2

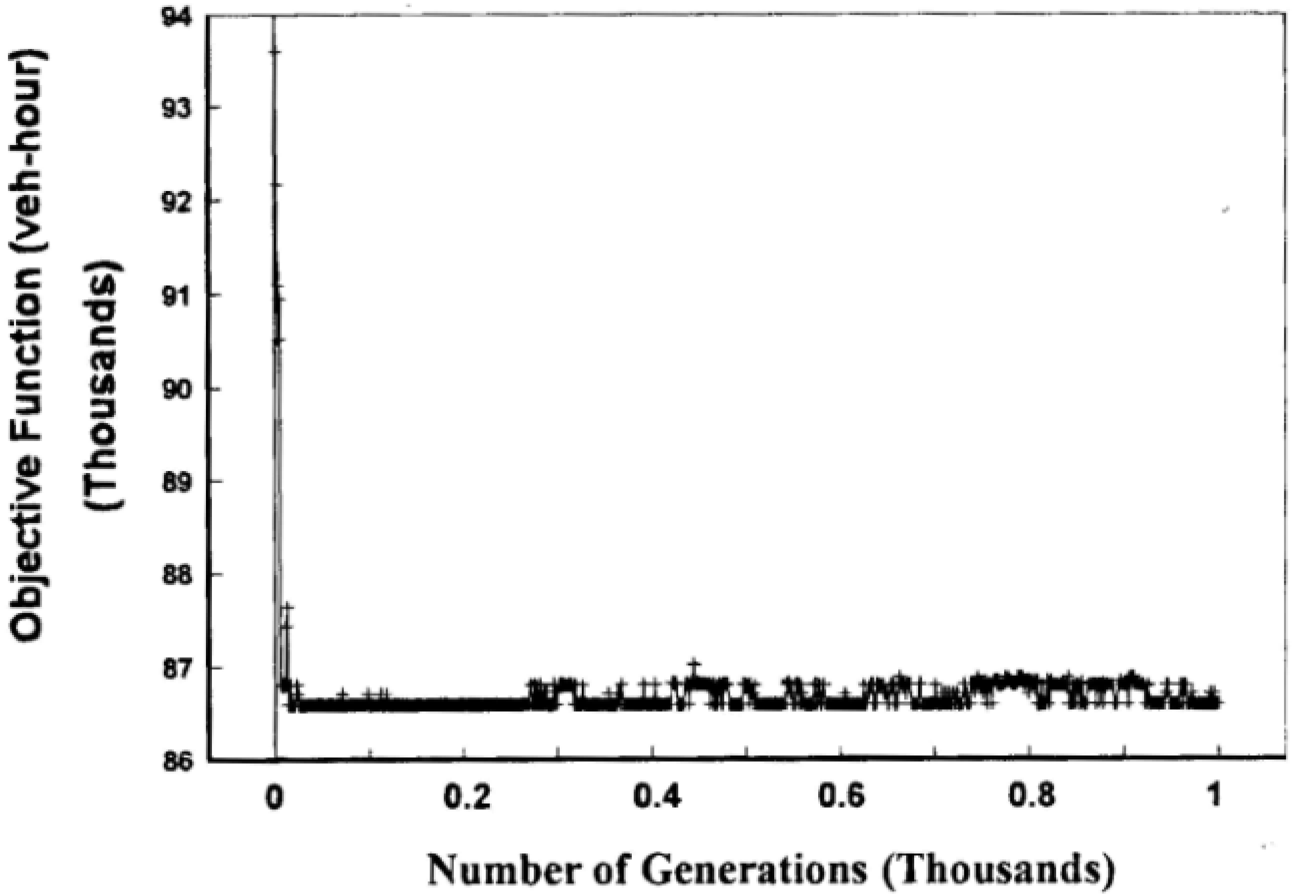

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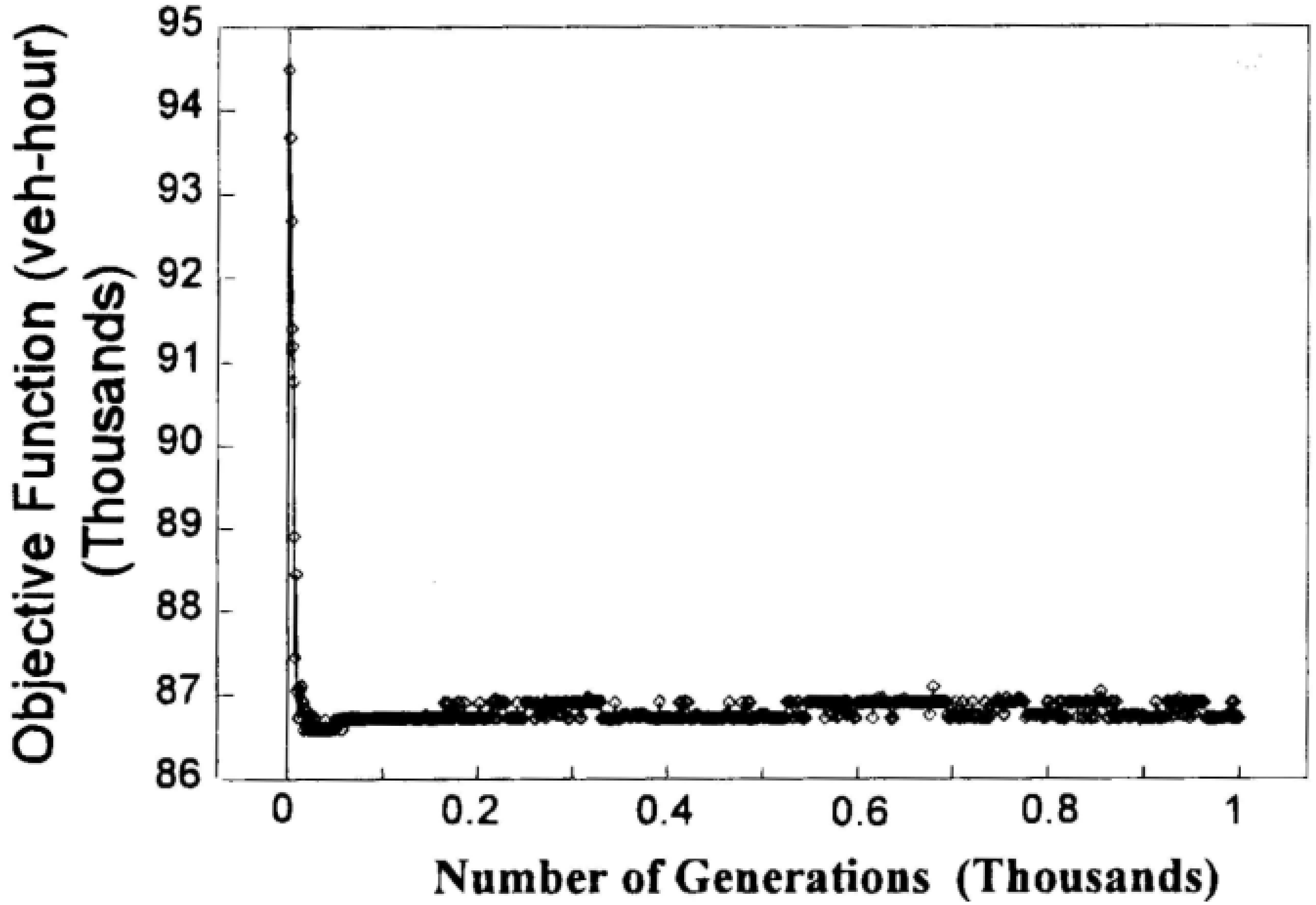
Figure 6 Search process: Pattern 3



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Second Step

- **Pattern 4:**
  - the extent of fluctuation in Pattern 2 and 3 is smaller than that in Pattern 4.
- Among these four patterns.
  - **Pattern 1** shows a strong ability to keep the smallest values they achieved.
  - **Pattern 4** is the worst condition.

Figure 7 Search process: Pattern 4

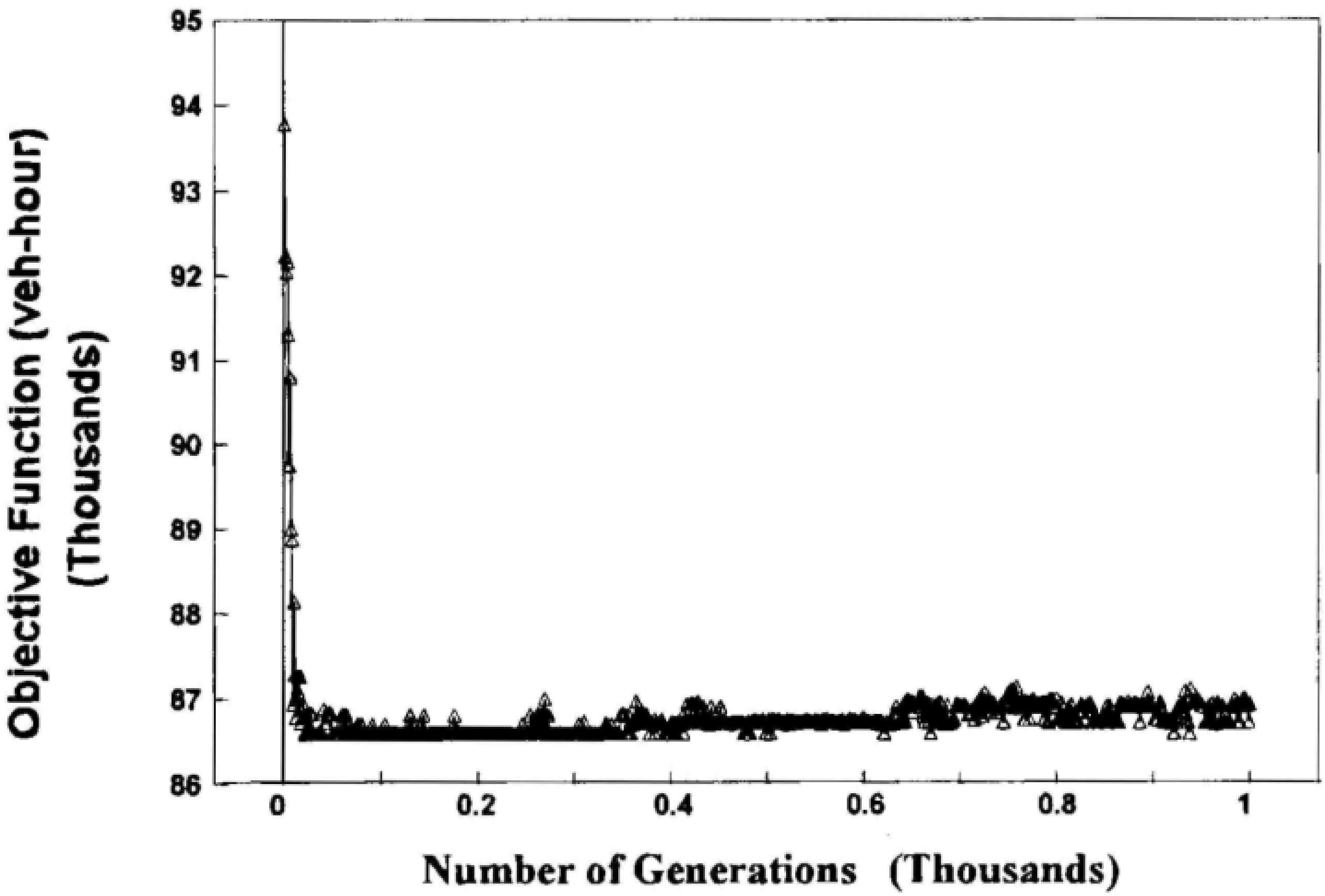

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Table 6: Calibration for Crossover and Mutation Probability

Mutation Probability	Crossover Probability Case 1				
	0.6	0.7	0.8	0.9	1.0
0.01	3	2	2	2	2
0.02	3	2	2	2	2
0.03	4	3	3	3	3
0.04	4	2	3	3	3
0.05	8	2	4	4	10

Mutation Probability	Crossover Probability Case 2				
	0.6	0.7	0.8	0.9	1.0
0.01	19	10	16	16	(3)
0.02	46	(3)	186	(2)	19
0.03	(2)	160	73	(2)	(3)
0.04	853	117	73	48	81
0.05	(3)	(4)	(2)	(3)	(4)

Mutation Probability	Crossover Probability Case 3				
	0.6	0.7	0.8	0.9	1.0
0.01	(3)	(2)	46	(3)	(3)
0.02	(4)	92	32	(3)	(3)
0.03	72	43	277	529	834
0.04	(2)	121	(2)	123	(2)
0.05	(3)	(4)	(2)	(2)	(2)

Second Step

- The number outside parentheses represents the stable generation in Pattern 1, and the numbers in parentheses represent the designation of patterns.
- The crossover and mutation probabilities are determined by the corresponding row and column values of the cell which have the **smallest stable generation**.

Second Step

- The crossover and mutation probabilities are:
  - **For Case 1**, corresponding the stable generation of **2**, they are determined to be **0.7** and **0.01**, respectively.
  - **For Case 2**, corresponding the stable generation of **10**, they are determined to be **0.7** and **0.01**, respectively.
  - **For Case 3**, corresponding to the stable generation of **32**, they are determined to be **0.8** and **0.02**, respectively.

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Second Step

- Referring to result Table, it can be seen that the linear case (Case 1) involves fewer generations to obtain the optimal solution than the nonlinear cases (Case 2 and Case 3).
  - In Case 1, the GA model always obtains the **optimal solutions**.
  - In Case 2, there are 11 schemes which cannot yield the optimal solution.
  - In Case 3, there are 15 schemes which cannot yield the optimal solution.

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Second Step

- Furthermore. from the Table, it can be seen that **additional constraints in Case 3** make the **solution searching process longer**.
- In Case 2, there are 14 stable schemes. and the average stable generation is 123, whereas in Case 3, there are 10 such schemes with an average of 217.

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Validation of the GA Model

Validation of the GA Model

- The task of the validation process of the genetic algorithm model is to test whether the obtained solutions are optimal.
- In this study different validation methods are employed for different cases.
- **For Case 1**,
  - because the optimal solutions can be obtained from available software developed by conventional algorithms, the solutions from the GA model are compared with them.

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Validation of the GA Model

- **For Cases 2 and 3,**
  - The optimality of the solutions from the GA model are evaluated by observing whether the solutions have obeyed the constraints imposed by the nonlinear delay time functions and imposed by some practical considerations.
  - After the feasibility of the solutions is determined, a variety of combinations around the obtained solutions are tested to see whether smaller objective values can be achieved.

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Validation of Case 1

- In Case 1, the optimal solution is obtained by using **Quant Systems** (Version 2.1).
- This solution is compared with that obtained from the GA model.
- The solutions are the same except that GA models can also produce other optimal solutions when multiple solutions exist.
- In each case, all the solutions are optimal. whereas the itinerary choices are different. The train loads and link volumes from one of the optimal solutions are listed in Tables 7 and 8.

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Table 7 Train Loads of the Three Cases for Validation

	The Load Assigned for the Long Distance Train											
	T114	T214	T115	T215	T116	T216	T316	T416	T126	T226	T136	T236
Case 1	121	0	0	150	150	0	0	0	0	107	150	0
Case 2	121	0	150	0	0	0	150	0	107	0	0	150
Case 3	0	0	0	0	0	0	0	0	0	0	0	0
	The Load Assigned for the Long Distance Train											
	T141	T241	T151	T251	T161	T261	T361	T461	T162	T262	T163	T263
Case 1	0	150	0	136	0	0	0	150	150	0	140	0
Case 2	0	150	136	0	150	0	0	0	0	150	0	140
Case 3	0	0	0	0	0	0	0	0	0	0	0	0

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Table 8 Link Volumes of the Three Cases for Validation

	Traffic Volume on Each Link of Each Direction (car)									
	link 12	link 21	link 13	link 31	link 24	link 42	link 35	link 53	link 25	link 52
Case 1	335	78	244	508	298	211	164	375	161	38
Case 2	335	364	244	222	255	211	164	229	204	324
Case 3	335	364	244	222	255	211	164	229	204	324
	Traffic Volume on Each Link of Each Direction (car)									
	link 34	link 43	link 46	link 64	link 56	link 65	link 23	link 32	link 45	link 54
Case 1	154	319	334	357	206	176	87	95	10	82
Case 2	154	169	291	217	249	316	87	95	10	82
Case 3	154	169	291	217	249	316	87	95	10	82

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Validation of Case 2

- In Case 2, the **link volumes** should be as small as possible, because it is assumed that the times in which the traffic is incurred on line segments are **dependent** of the traffic volume.
- From the point of view of networks, however, the link volumes should not fluctuate dramatically.
- Observe Table 8, comparing with the solution of Case 1, the link volumes in Case 2 are indeed evenly distributed.
- The largest value in Case 1, 508, disappeared in Case 2.

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Validation of Case 2

- From this observation, it is apparent that the nonlinear functions are effective.
- Following this analysis, the solutions are analyzed by providing **all possible and comparable schemes**.
- It is concluded that the solutions are truly optimal in terms of minimizing the total delay in the rail· road system.

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Validation of Case 3

- In Case 3. beside the nonlinear objective function, the remaining two constraints are added, that is,
  - the volume on each link should be no more than 300 cars (which is realized by setting b_1 equals 300)
  - the load on each long distance train should be more than 200 cars (which is realized by setting b_2 equals 200)
- These two constraints establish that the possible load on each of the long distance trains are not sufficiently large to justify the provision.

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Validation of Case 3

- In Table 7, the loads are really zero, whereas in Table 8, it appears that the link volume constraints are not effective.
- However, after careful calculation, the overall demand through those links where the volumes exceed 300, it is found that there is no way to distribute these volumes without avoiding the penalty of the violation of the constraints.
- Thus, the solution is truly the optimal.

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GA Model Computational Performance

GA Model Computational Performance

- **In Case 1**
  - the Quant Systems consumes 1.17 sec of CPU to produce the optimal solution. However, the GA model uses **less time**.
- **In Cases 2 and 3**
  - for the number of generations equal to 1000, the GA model requires approximately 10 min of CPU.
  - Because both cases can obtain the optimal solutions in less than 40 generations, the computation time should be about 20 sec. Comparing with the size of the problem, this computation time is quite satisfactory.

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GA Model Computational Performance

- Using the calibrated parameters, the GA model is used for varieties of demand patterns.
- In Table 2, the long distance demands are varied in the range of 100 to 150 cars; there are almost no computation time variations.
- For all the demand patterns, the GA model produces the optimal solutions within 40 generations.

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Conclusions

Conclusions

- Several conclusions can be derived from this study.
- **First**
  - A GA model is able to produce optimal solutions for the formulations which might be difficult conventionally.
  - Also, the computation time is satisfactory.
- **Second**
  - A GA model is not as sensitive to the input patterns.

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Conclusions

- **Third**,
  - the implementation process for a GA model is straight forward.
  - In all three cases, the implementation simply involves the adjustment of the objective function formulations.
  - There is no need to give the structure of the formulation a special consideration.
  - The calibration and validation process are also straight forward.

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Conclusions

- **Fourth**
  - The binary representation for the binary integer program (BIP) is especially effective.
- Based on the principle introduced in this study, GA models can likely be effective when applied to large railroad networks.

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Future Research

- The patterns recognition of the solution searching process needs to be analyzed quantitatively instead of qualitatively.
- To this end, some **statistical model** might need to be developed.

References

References

- D. Martinelli and H. Teng, **A Genetic Algorithm Approach For Solving The Train Formation Problem**, Transportation Research Record, No. 1497, p. 62-69, 1995.

The end