8. Ant Colony Optimization

8.1 Introduction

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Instructor: Dr. Masoud Yaghini

Outline

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Introduction

Swarm Intelligence

- **Swarm intelligence** (**SI**)
  - is artificial intelligence based on the collective behavior of decentralized, self-organized systems.
  - The expression was introduced by **Gerardo Beni** and **Jing Wang** in 1989.
  - The natural examples of SI includes the behaviors of certain **ants**, **honeybees**, **wasps**, **beetles**, **caterpillars**, and **termites**

Swarm Intelligence

- **Example of swarm intelligence algorithms:**
  - Ant colony optimization
  - Particle swarm optimization
  - Stochastic diffusion search
  - Swarm robotics

Ant Colony Optimization

- **Ant Colony Optimization (ACO)**
  - is inspired by the foraging behavior of ant colonies
  - ACO algorithms are used for solving discrete optimization problems.
  - ACO is one of the most successful examples of metaheuristic algorithms.

Ant Colony Optimization

- **Examples of ACO algorithms:**
  - Ant System (AS)
  - Elitist Ant System (EAS)
  - Rank-Based Ant System (ASrank)
  - Min-Max Ant System (MMAS)
  - Ant Colony System (ACS)
  - Approximate Nondeterministic Tree Search (ANTS)
  - Hyper-Cube Framework

ACO Brief History

- **1989 & 1990**:
  - Experiments with Argentine ants by **Goss et al.** & **Deneuborg et al.**
  - The ants prefer the shortest path from the nest to the food source
- **1991**:
  - Ant System (AS) was the first ACO algorithm presented for shortest paths by **Dorigo et. al.** (Milan, Italy)
- **1998**:
  - **Ant Colony Optimization** is the name given by **Dorigo**
  - A class of algorithms whose first member was AS.

Ant Colony Optimization

- ACO algorithms can be used to solve both **static** and **dynamic** combinatorial optimization problems.

Ant Colony Optimization

- **Static problems**
  - are those in which the characteristics of the problem are given once and for all when the problem is defined, and do not change while the problem is being solved.
  - An example of such problems is the TSP, in which city locations and their relative distances are part of the problem definition and do not change at run time.

Ant Colony Optimization

- **Dynamic problems**
  - are defined as a function of some quantities whose value is set by the dynamics of an underlying system.
  - The problem instance changes therefore at run time and the optimization algorithm must be capable of adapting online to the changing environment.
  - An example of this situation is **network routing problems** in which the data traffic and the network topology can vary in time.

ACO Applications

Problem type	Problem name	Main references
Routing	Traveling salesman	Dorigo, Maniezzo, & Colorni (1991a,b, 1996) Dorigo (1992) Gambardella & Dorigo (1995) Dorigo & Gambardella (1997a,b) Stützle & Hoos (1997, 2000) Bullnheimer, Hartl, & Strauss (1999c) Cordón, de Viana, Herrera, & Morena (2000)
	Vehicle routing	Bullnheimer, Hartl, & Strauss (1999a,b) Gambardella, Taillard, & Agazzi (1999) Reimann, Stummer, & Doerner (2002)
	Sequential ordering	Gambardella & Dorigo (1997, 2000)

ACO Applications

Problem type	Problem name	Main references
Assignment	Quadratic assignment	Maniezzo, Colorni, & Dorigo (1994) Stützle (1997b) Maniezzo & Colorni (1999) Maniezzo (1999) Stützle & Hoos (2000)
	Graph coloring	Costa & Hertz (1997)
	Generalized assignment	Lourenço & Serra (1998, 2002)
	Frequency assignment	Maniezzo & Carbonaro (2000)
	University course timetabling	Socha, Knowles, & Sampels (2002) Socha, Sampels, & Manfrin (2003)

ACO Applications

Problem type	Problem name	Main references
Scheduling	Job shop	Colorni, Dorigo, Maniezzo, & Trubian (1994)
	Open shop	Pfahringer (1996)
	Flow shop	Stützle (1998a)
	Total tardiness	Bauer, Bullnheimer, Hartl, & Strauss (2000)
	Total weighted tardiness	den Besten, Stützle, & Dorigo (2000) Merkle & Middendorf (2000, 2003a) Gagné, Price, & Gravel (2002)
	Project scheduling	Merkle, Middendorf, & Schmeck (2000a, 2002)
	Group shop	Blum (2002a, 2003a)

ACO Applications

Problem type	Problem name	Main references
Subset	Multiple knapsack	Leguizamón & Michalewicz (1999)
	Max independent set	Leguizamón & Michalewicz (2000)
	Redundancy allocation	Liang & Smith (1999)
	Set covering	Leguizamón & Michalewicz (2000) Hadji, Rahoual, Talbi, & Bachelet (2000)
	Weight constrained graph tree partition	Cordone & Maffioli (2001)
	Arc-weighted *l*-cardinality tree	Blum & Blesa (2003)
	Maximum clique	Fenet & Solnon (2003)

ACO Applications

Problem type	Problem name	Main references
Machine learning	Classification rules	Parpinelli, Lopes, & Freitas (2002b)
	Bayesian networks	de Campos, Gámez, & Puerta (2002b)
	Fuzzy systems	Casillas, Cordón, & Herrera (2000)
Network routing	Connection-oriented network routing	Schoonderwoerd, Holland, Bruten, & Rothkrantz (1996) Schoonderwoerd, Holland, & Bruten (1997) White, Pagurek, & Oppacher (1998) Di Caro & Dorigo (1998d) Bonabeau, Henavy, Guérin, Snyers, Kuntz, & Theraulaz (1998)
	Connectionless network routing	Di Caro & Dorigo (1997, 1998c,f) Subramanian, Druschel, & Chen (1997) Heusse, Snyers, Guérin, & Kuntz (1998) van der Put (1998)
	Optical network routing	Navarro Varela, & Sinclair (1999)

Real Ants

Stigmergy

- Ant colonies, in spite of the simplicity of their individuals, present a highly structured social organization.
- As a result of this organization, ant colonies can accomplish complex.
- Ants coordinate their activities via **stigmergy**

Stigmergy

- **Stigmergy**
  - is a form of **indirect communication** mediated by modifications of the environment.
  - an individual modifies the environment
  - other individuals respond to that change at a later time
  - The environment mediates the communication among individuals
  - A foraging ant deposits a chemical on the ground which increases the probability that other ants will follow the same path.

Pheromones

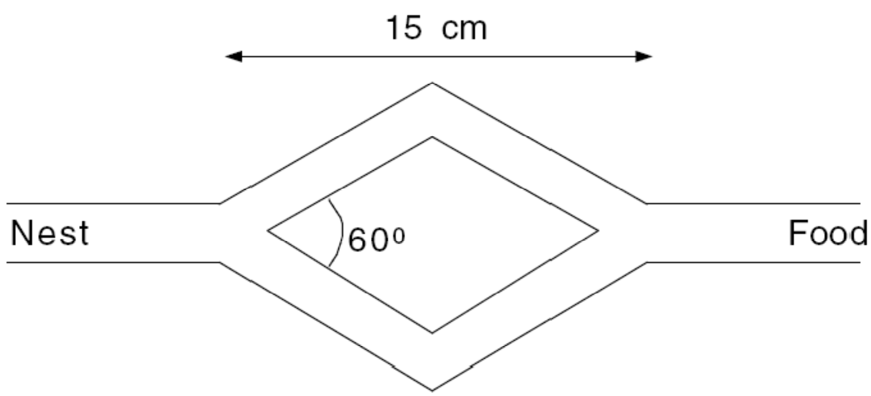
- **Pheromones**
  - The communication among individuals, or between individuals and the environment, is based on the use of chemicals produced by the ants.
  - These chemicals are called **pheromones**.
  - **Trail pheromone** is a specific type of pheromone that some ants use for marking paths on the ground, for example, paths from food sources to the nest.

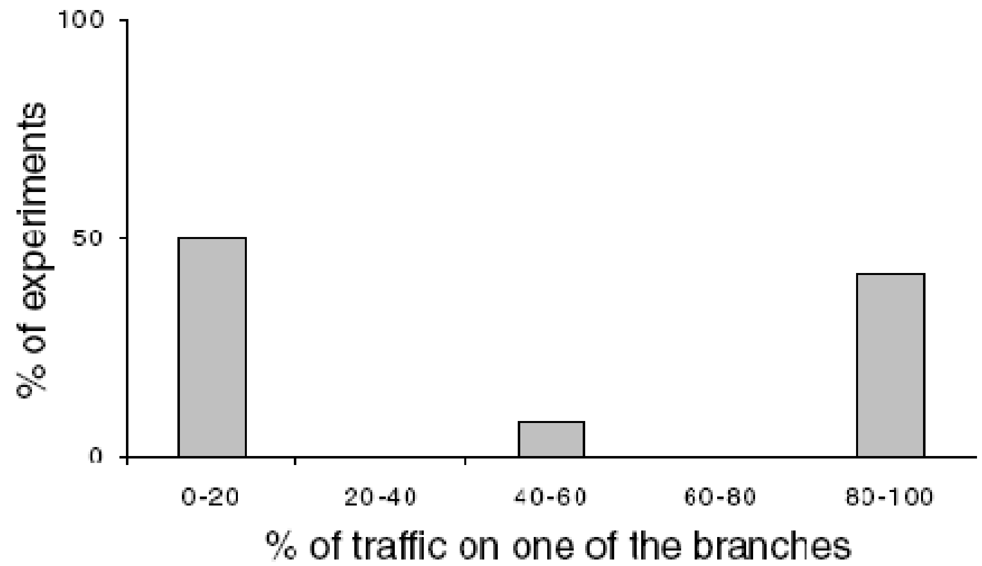
Double Bridge Experiments

- **Double Bridge Experiments**
  - **Deneubourg** and colleagues have shown that foraging ants can find the shortest path between their nest and a food source
  - They used a double bridge connecting a nest of ants and a food source.
  - They ran experiments varying the length of the two branches of the double bridge.

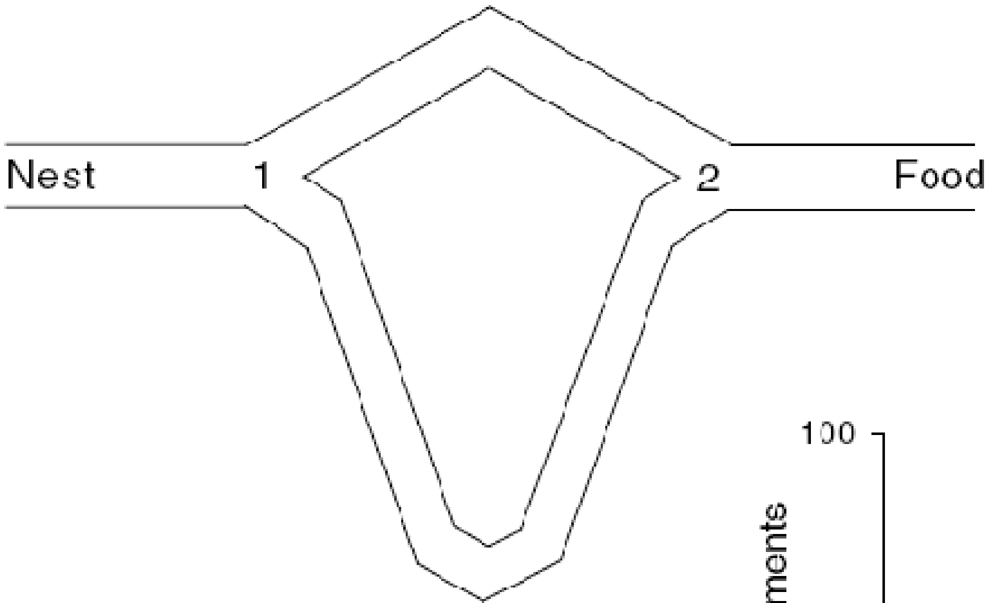
Double Bridge Experiments

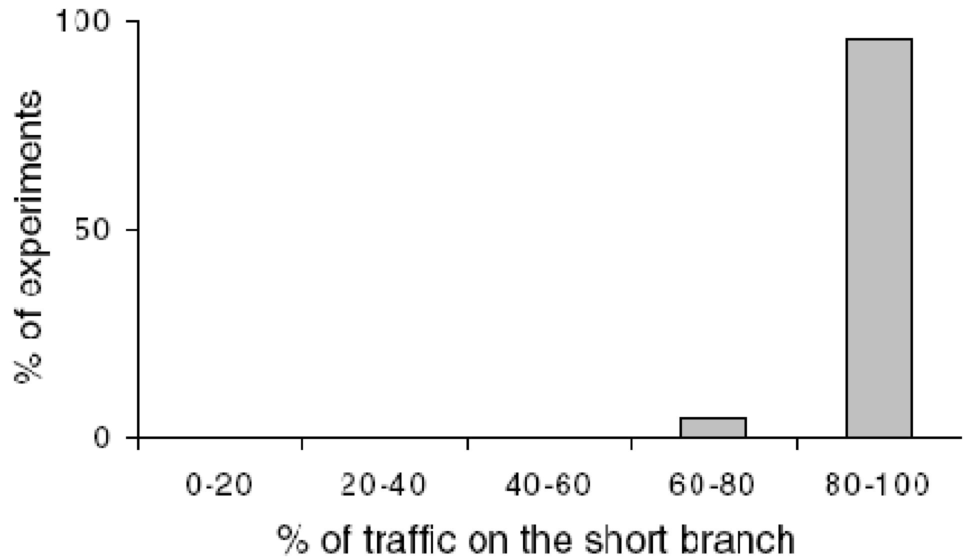
First Experiment



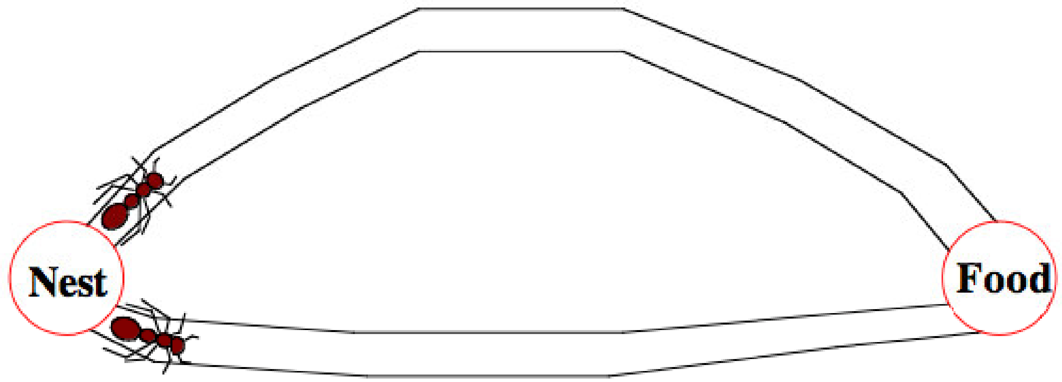


Second Experiment



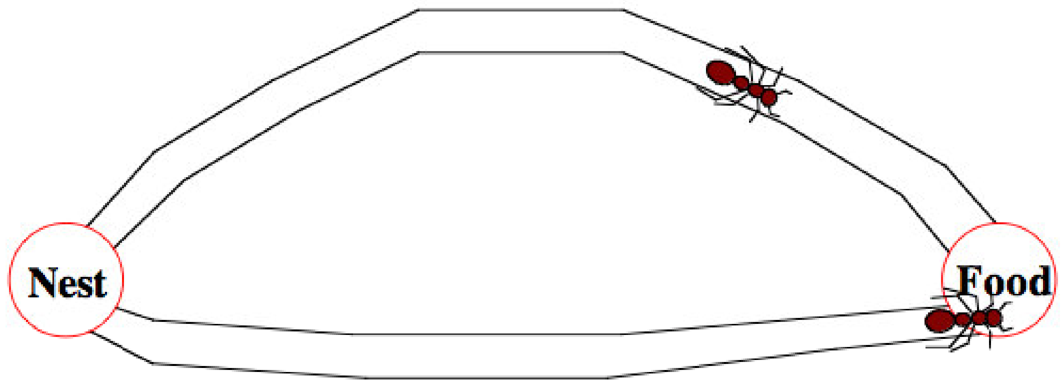


Foraging behavior of Ants



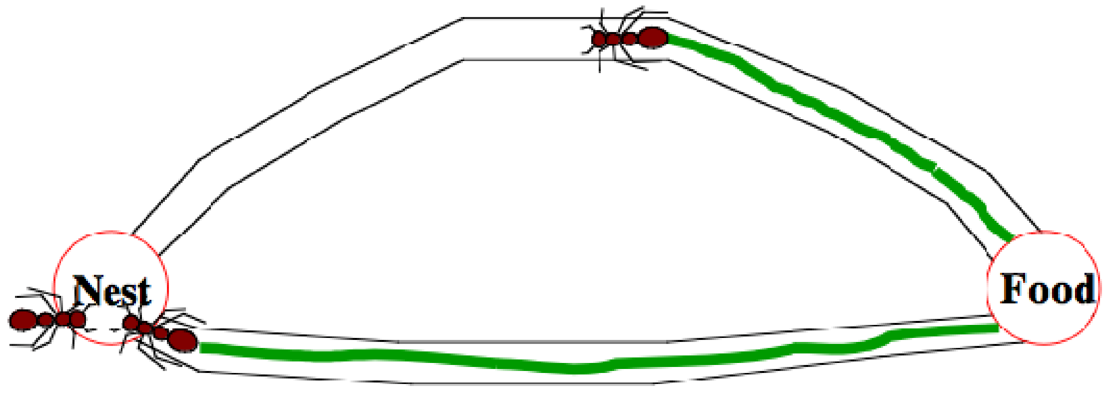
- 2 ants start with equal probability of going on either path.

Foraging behavior of Ants



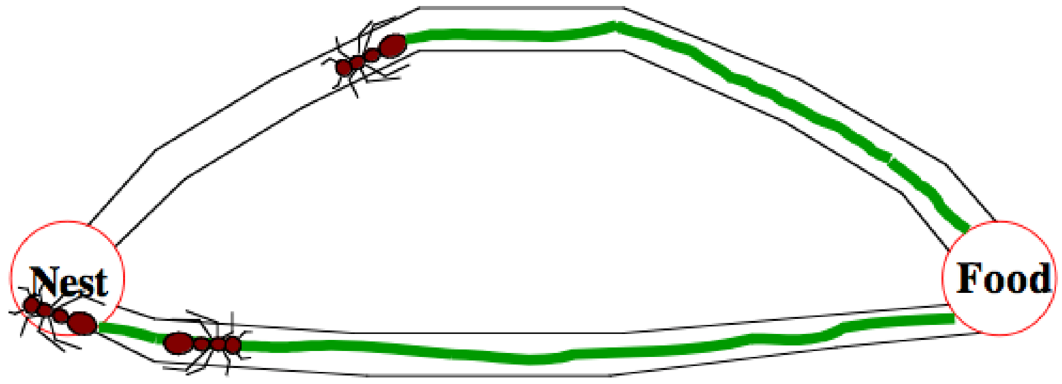
- The ant on shorter path has a shorter to-and-fro time from it's nest to the food.

Foraging behavior of Ants



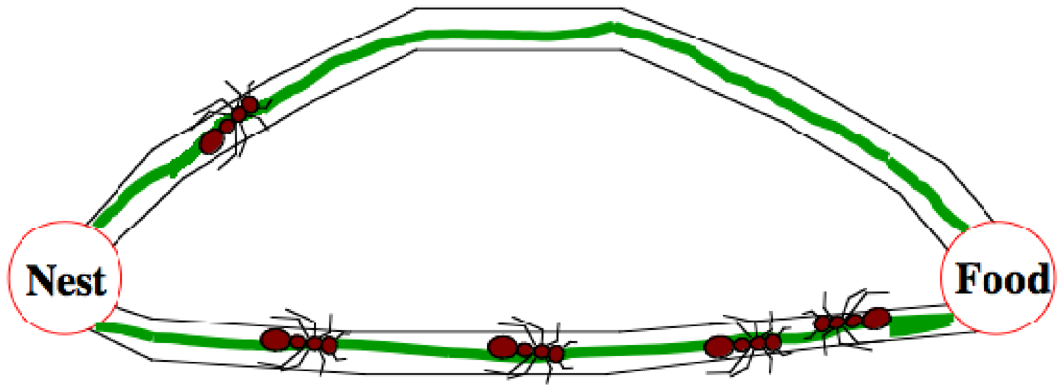
- The density of pheromone on the shorter path is higher because of 2 passes by the ant (as compared to 1 by the other).

Foraging behavior of Ants



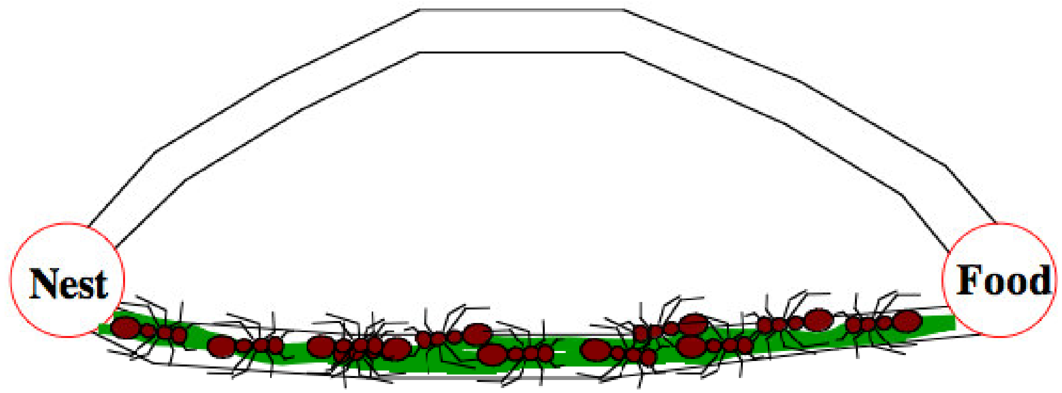
- The next ant takes the shorter route.

Foraging behavior of Ants



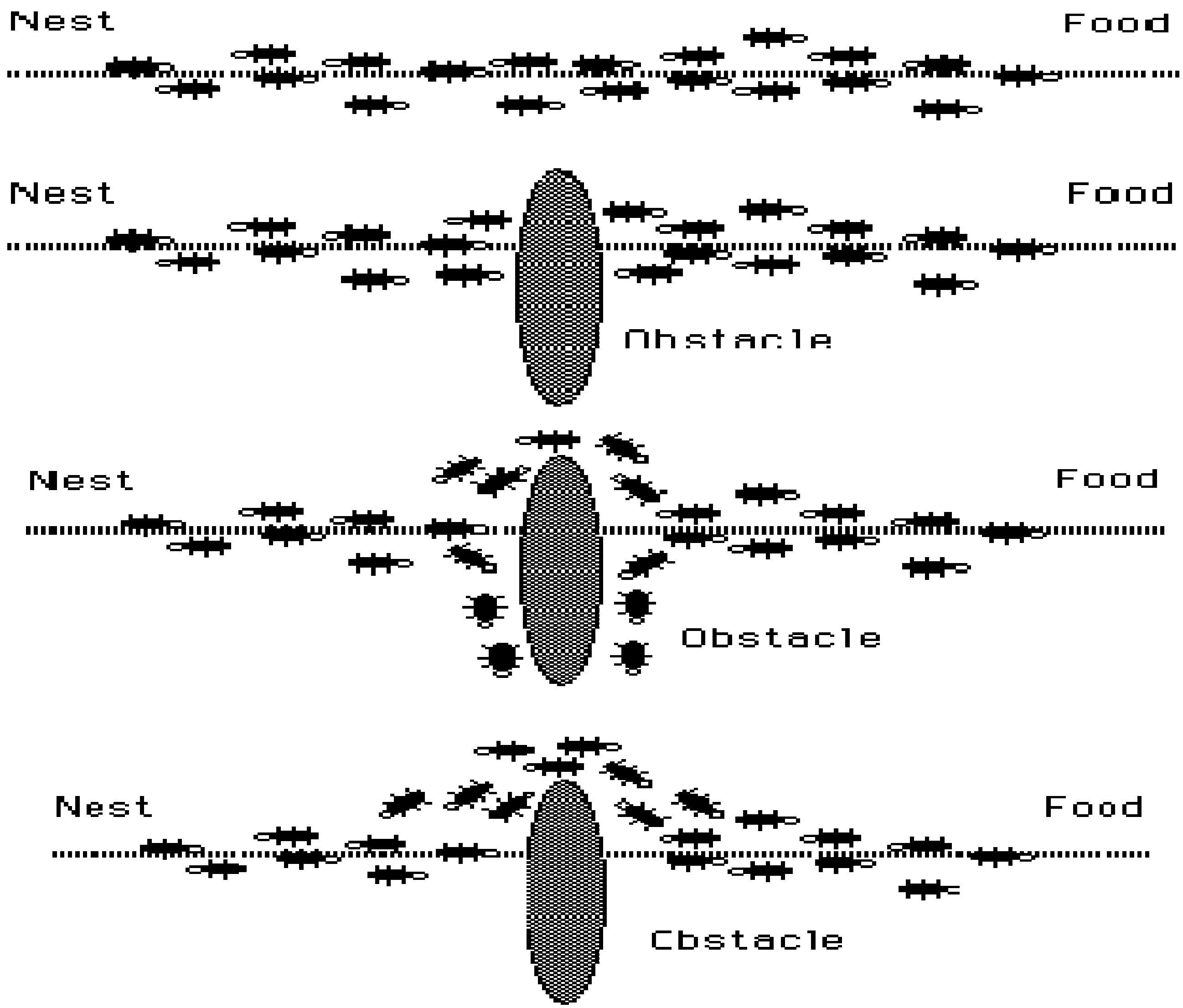
- Over many iterations, more ants begin using the path with higher pheromone, thereby further reinforcing it.

Foraging behavior of Ants



- After some time, the shorter path is almost exclusively used.

Foraging behavior of Ants



Inspiring Source of ACO

- This collective trail-laying and trail-following behavior whereby an ant is influenced by a chemical trail left by other ants is the inspiring source of ACO.

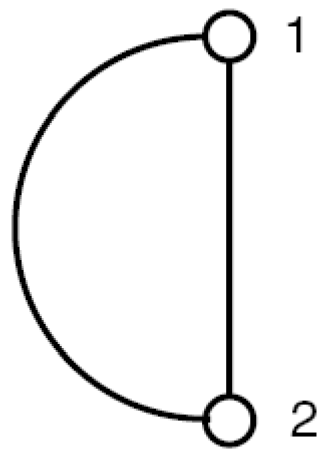
Artificial Ants

Artificial Ants

- The **double bridge experiments** show clearly that ant colonies have a built-in optimization capability
- By the use of probabilistic rules based on local information they can find the shortest path between two points in their environment.
- It is possible to design **artificial ants** that, by moving on a graph modeling the double bridge, find the shortest path between the two nodes corresponding to the nest and to the food source.

Artificial Ants

- Consider this graph



- The graph consists of two nodes (1 and 2, representing the nest and the food respectively)

Artificial Ants

- The nodes are connected by a **short** and a **long** arc
- In the example the long arc is **r times** longer than the short arc, where r is an integer number.
- We assume the time to be discrete (t = 1, 2, …) and that at each time step each ant moves toward a neighbor node at constant **speed of one unit of length per time unit**.

Artificial Ants

- Ants add **one unit of pheromone** to the arcs they use.
- Ants move on the graph by choosing the path probabilistically:
  - $P_{is}(t)$ is the probability for an ant located in node i at time t to choose the short path, and
  - $P_{il}(t)$ the probability to choose the long path.
- These probabilities are a function of the pheromone trails φ_{ia} that ants in node i

Artificial Ants

- The probabilities

$$p_{is}(t) = \frac{[\varphi_{is}(t)]^{\alpha}}{[\varphi_{is}(t)]^{\alpha} + [\varphi_{il}(t)]^{\alpha}}$$

$$p_{il}(t) = \frac{[\varphi_{il}(t)]^{\alpha}}{[\varphi_{is}(t)]^{\alpha} + [\varphi_{il}(t)]^{\alpha}}$$

Artificial Ants

- Trail update on the two branches is performed as follows:

$$\varphi_{is}(t) = \varphi_{is}(t-1) + p_{is}(t-1)m_i(t-1) + p_{js}(t-1)m_j(t-1),$$
$$(i = 1, j = 2; i = 2, j = 1),$$

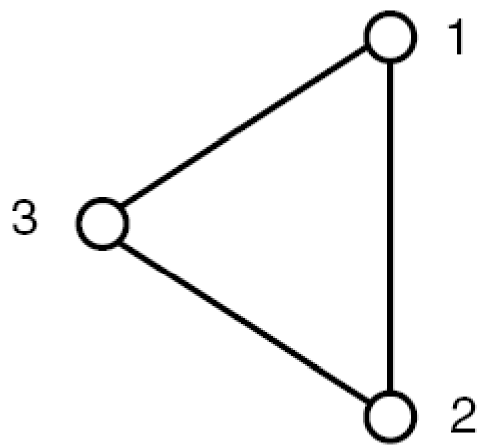
$$\varphi_{il}(t) = \varphi_{il}(t-1) + p_{il}(t-1)m_i(t-1) + p_{jl}(t-r)m_j(t-r),$$
$$(i = 1, j = 2; i = 2, j = 1),$$

  - Where $m_i(t)$ the number of ants on node i at time t, is given by

$$m_i(t) = p_{js}(t-1)m_j(t-1) + p_{jl}(t-r)m_j(t-r),$$
$$(i = 1, j = 2; i = 2, j = 1).$$

Artificial Ants

- Another way of modeling:

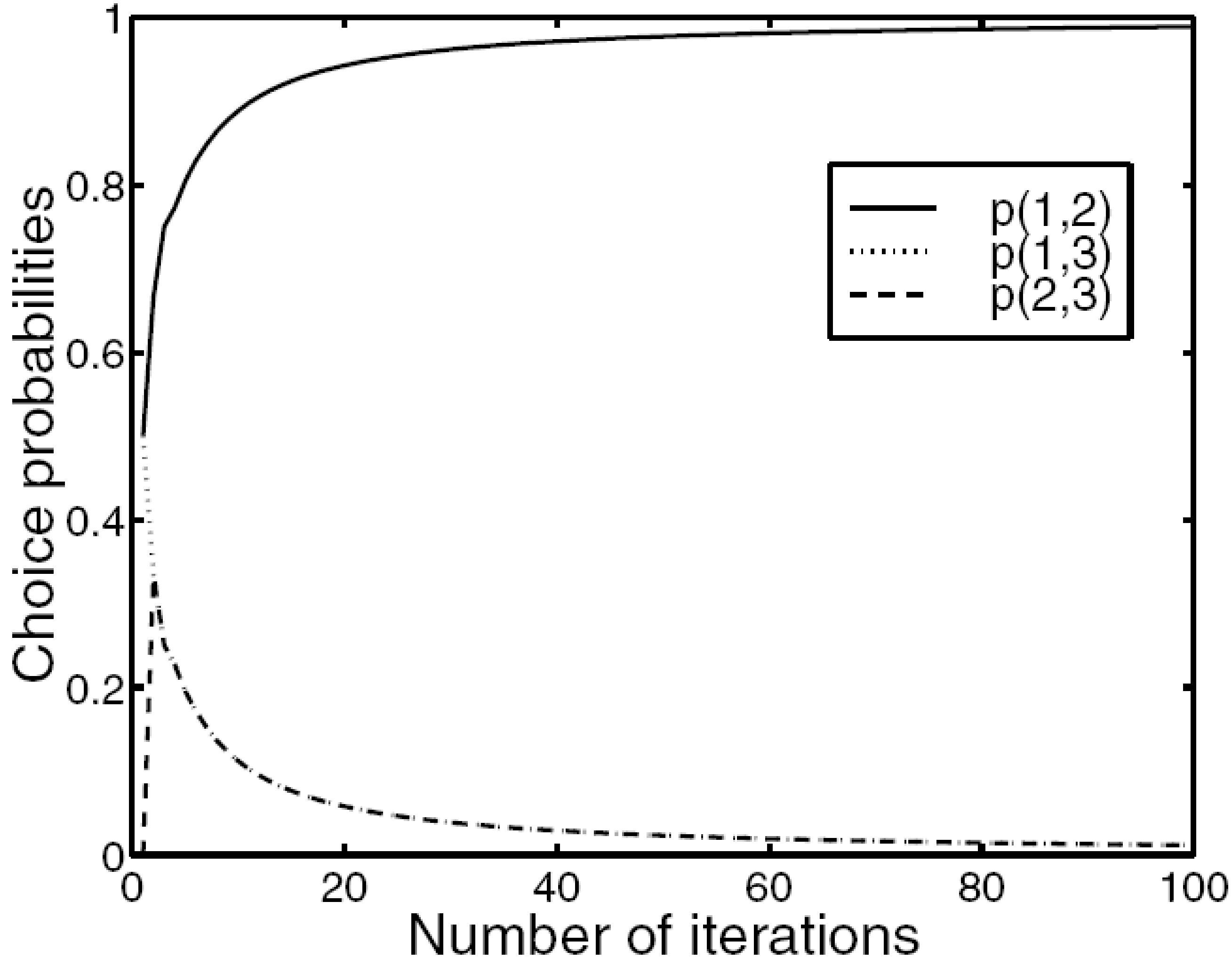


- In this model each arc of the graph has the same length, and a longer branch is represented by a sequence of arcs.
- In the figure, for example, the long branch is twice as long as the short branch.

Artificial Ants

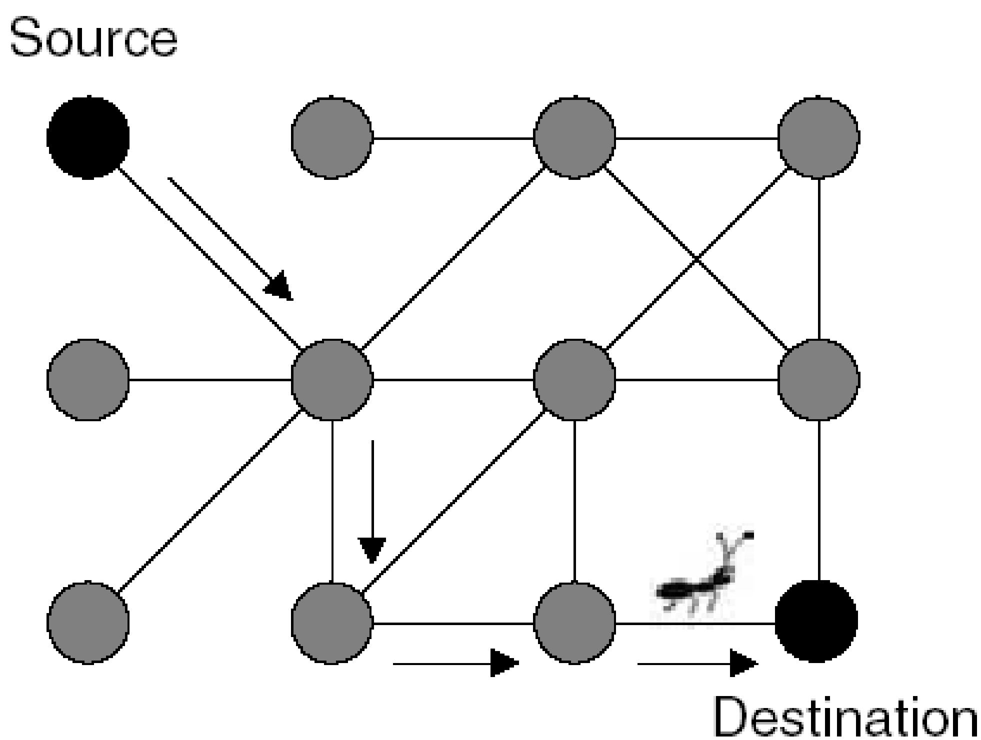
- Pheromone updates are done with one time unit delay on each arc.
- The two models are equivalent from a computational point of view, yet the second model permits an easier algorithmic implementation when considering graphs with many nodes.
- By setting the **number of ants to 20**, the **branch length ratio** to r=2, and the **parameter α** to 2, and t=100, the system converges rapidly toward the use of the short branch.

Artificial Ants



Artificial Ants

- Let us consider a static, connected graph $G = (N, A)$, where N is the set of nodes and A is the set of undirected arcs connecting them.



Artificial Ants

- **Artificial ants** whose behavior is a straightforward extension of the behavior of the real ants, while building a solution, may **generate loops**.
- As a consequence of the forward pheromone trail updating mechanism, loops tend to become more and more attractive and ants can get **trapped** in them.

Artificial Ants

- Artificial ants are given a **limited form of memory** in which they can store:
  - The **paths** they have followed so far, and
  - The **cost** of the links they have traversed.
- Via the use of memory, the ants can implement a number of useful behaviors

Artificial Ants

- The artificial ants have these behaviors:
  1. Probabilistic solution construction biased by **pheromone trails**, without forward pheromone updating
  2. Deterministic backward path with **loop elimination** and with pheromone updating
  3. **Evaluation of the quality** of the solutions generated and use of the solution quality in determining the quantity of pheromone to deposit

References

References

- M. Dorigo and T. Stützle. **Ant Colony Optimization**, MIT Press, Cambridge, 2004.

The End