Data Mining

2.7 Data Discretization and Concept Hierarchy Generation

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  - Entropy-Based Discretization
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Discretization and Concept Hierarchy Generation for Numerical Data
Data Discretization

- Data Discretization:
  - Dividing the range of a continuous attribute into intervals
  - Interval labels can then be used to replace actual data values
  - Reduce the number of values for a given continuous attribute
  - Some classification algorithms only accept categorical attributes.
  - This leads to a concise, easy-to-use, knowledge-level representation of mining results.
Data Discretization

Discretization techniques can be categorized based on whether it uses class information, as:

- **Supervised discretization**
  - the discretization process uses class information
- **Unsupervised discretization**
  - the discretization process does not use class information
Data Discretization

Discretization techniques can be categorized based on which direction it proceeds, as:

- **Top-down**
  - If the process starts by first finding one or a few points (called split points or cut points) to split the entire attribute range, and then repeats this recursively on the resulting intervals.

- **Bottom-up**
  - Starts by considering all of the continuous values as potential split-points,
  - Removes some by merging neighborhood values to form intervals, and
  - Then recursively applies this process to the resulting intervals.
Data Discretization

- **Typical methods:**
  - Binning
  - Entropy-based discretization
  - Interval merging by \( \chi^2 \) Analysis
  - Clustering analysis

- All the methods can be applied recursively
- Each method assumes that the values to be discretized are sorted in ascending order.
Binning
Binning

- The sorted values are distributed into a number of buckets, or bins, and then replacing each bin value by the bin mean or median

- **Binning** is:
  - a top-down splitting technique based on a specified number of bins.
  - an unsupervised discretization technique, because it does not use class information

- **Binning methods:**
  - Equal-width (distance) partitioning
  - Equal-depth (frequency) partitioning
Equal-width (distance) partitioning

- Divides the range into N intervals of equal size: uniform grid
- if A and B are the lowest and highest values of the attribute, the width of intervals will be: \( W = (B - A)/N \).
- The most straightforward, but outliers may dominate presentation
- Skewed data is not handled well
Equal-width (distance) partitioning

- Sorted data for price (in dollars):
  - 4, 8, 15, 21, 21, 24, 25, 28, 34
- \( W = (B - A)/N = (34 - 4) / 3 = 10 \)
  - Bin 1: 4-14, Bin2: 15-24, Bin 3: 25-34
- Equal-width (distance) partitioning:
  - Bin 1: 4, 8
  - Bin 2: 15, 21, 21, 24
  - Bin 3: 25, 28, 34
Equal-depth (frequency) partitioning

- Divides the range into N intervals, each containing approximately same number of samples
- Good data scaling
- Managing categorical attributes can be tricky
Equal-depth (frequency) partitioning

- Sorted data for price (in dollars):
  - 4, 8, 15, 21, 21, 24, 25, 28, 34

- Equal-depth (frequency) partitioning:
  - Bin 1: 4, 8, 15
  - Bin 2: 21, 21, 24
  - Bin 3: 25, 28, 34
Entropy-Based Discretization
Entropy-Based Discretization

- Entropy-based discretization is a **supervised, top-down splitting** technique.
- It explores class distribution information in its calculation and determination of split-points.
- Let D consist of data instances defined by a set of attributes and a class-label attribute.
- The class-label attribute provides the class information per instance.
The basic method for entropy-based discretization of an attribute $A$ within the set is as follows:

1) Each value of $A$ can be considered as a potential interval boundary or split-point (denoted split point) to partition the range of $A$.

   - That is, a split-point for $A$ can partition the instances in $D$ into two subsets satisfying the conditions $A \leq split\_point$ and $A > split\_point$, respectively,
   - thereby creating a binary discretization.
Entropy-Based Discretization

2) the information gain after partitioning is

\[
Info_A(D) = \frac{|D_1|}{|D|} \text{Entropy}(D_1) + \frac{|D_2|}{|D|} \text{Entropy}(D_2)
\]

where \( D_1 \) and \( D_2 \) correspond to the instances in \( D \)

\(|D|\) is the number of instances in \( D \), and so on.

The entropy function for a given set is calculated based on the class distribution of the tuples in the set.

For example, given \( m \) classes, \( C_1, C_2, \ldots, C_m \), the entropy of \( D_1 \) is:

\[
\text{Entropy}(D_1) = - \sum_{i=1}^{m} p_i \log_2(p_i)
\]
Entropy-Based Discretization

- where \( p_i \) is the probability of class \( C_i \) in \( D_1 \), determined by dividing the number of tuples of class \( C_i \) in \( D_1 \) by \( |D_1| \), the total number of tuples in \( D_1 \).
- Therefore, when selecting a split-point for attribute \( A \), we want to pick the attribute value that gives the minimum expected information requirement (i.e., \( \min(\text{Info}_A(D)) \)).

3) The process of determining a split-point is recursively applied to each partition obtained, until some stopping criterion is met, such as:

- when the minimum information requirement on all candidate split-points is less than a small threshold, \( e \),
- or when the number of intervals is greater than a threshold, \( \text{max\_interval} \).
Entropy-Based Discretization

- The interval boundaries (split-points) are defined may help improve classification accuracy.
- The entropy and information gain measures described here are also used for decision tree induction.
Interval Merge by $\chi^2$ Analysis
Interval Merge by $\chi^2$ Analysis

- **ChiMerge:**
  - It is a bottom-up method
  - Find the best neighboring intervals and merge them to form larger intervals recursively
  - The method is supervised in that it uses class information.
  - The basic notion is that for accurate discretization, the relative class frequencies should be fairly consistent within an interval.
  - Therefore, if two adjacent intervals have a very similar distribution of classes, then the intervals can be merged. Otherwise, they should remain separate.
  - ChiMerge treats intervals as discrete categories
Interval Merge by $\chi^2$ Analysis

- The ChiMerge method:
  - Initially, each distinct value of a numerical attribute $A$ is considered to be one interval
  - $\chi^2$ tests are performed for every pair of adjacent intervals
  - Adjacent intervals with the least $\chi^2$ values are merged together, since low $\chi^2$ values for a pair indicate similar class distributions
  - This merge process proceeds recursively until a predefined stopping criterion is met (such as significance level, max-interval, max inconsistency, etc.)
Cluster Analysis
Cluster Analysis

- Cluster analysis is a popular data discretization method.
- A clustering algorithm can be applied to discretize a numerical attribute, A, by partitioning the values of A into clusters or groups.
- Clustering takes the distribution of A into consideration, as well as the closeness of data points, and therefore is able to produce high-quality discretization results.
Cluster Analysis

- Clustering can be used to generate a concept hierarchy for A by following either a top-down splitting strategy or a bottom-up merging strategy, where each cluster forms a node of the concept hierarchy.

- In the former, each initial cluster or partition may be further decomposed into several subclusters, forming a lower level of the hierarchy.

- In the latter, clusters are formed by repeatedly grouping neighboring clusters in order to form higher-level concepts.
Concept Hierarchy Generation for Categorical Data
Generalization is the generation of concept hierarchies for categorical data.

Categorical attributes have a finite (but possibly large) number of distinct values, with no ordering among the values.

Examples include
- geographic location,
- job category, and
- itemtype.
There are several methods for the generation of concept hierarchies for categorical data:

- Specification of a partial ordering of attributes explicitly at the schema level by users or experts
- Specification of a portion of a hierarchy by explicit data grouping
- Specification of a set of attributes, but not of their partial ordering
Concept Hierarchy Generation for Categorical Data

- **Specification of a partial ordering of attributes explicitly at the schema level by users or experts**
  - Example: a relational database or a dimension location of a data warehouse may contain the following group of attributes: street, city, province or state, and country.
  - A user or expert can easily define a concept hierarchy by specifying ordering of the attributes at the schema level.
  - A hierarchy can be defined by specifying the total ordering among these attributes at the schema level, such as:
    - street < city < province or state < country
Concept Hierarchy Generation for Categorical Data

- Specification of a portion of a hierarchy by explicit data grouping
  - we can easily specify explicit groupings for a small portion of intermediate-level data.
  - For example, after specifying that province and country form a hierarchy at the schema level, a user could define some intermediate levels manually, such as:
    - {Urbana, Champaign, Chicago} < Illinois
Concept Hierarchy Generation for Categorical Data

- **Specification of a set of attributes, but not of their partial ordering**
  - A user may specify a set of attributes forming a concept hierarchy, but omit to explicitly state their partial ordering.
  - The system can then try to automatically generate the attribute ordering so as to construct a meaningful concept hierarchy.
  - Example: Suppose a user selects a set of location-oriented attributes, *street*, *country*, *province_or_state*, and *city*, from the *AllElectronics* database, but does not specify the hierarchical ordering among the attributes.
Concept Hierarchy Generation for Categorical Data

- Automatic generation of a schema concept hierarchy based on the number of distinct attribute values.
- The attribute with the most distinct values is placed at the lowest level of the hierarchy.
- Exceptions, e.g., weekday, month, quarter, year.

```
street                    674,339 distinct values
  city                     3,567 distinct values
    province_or_state     365 distinct values
      country             15 distinct values
```
References
References

- J. Han, M. Kamber, *Data Mining: Concepts and Techniques*, Elsevier Inc. (2006). (Chapter 2)
The end
● Concept hierarchy formation
  – Recursively reduce the data by collecting and replacing low level concepts (such as numeric values for age) by higher level concepts (such as young, middle-aged, or senior)