Data Mining

4. Cluster Analysis

4.2 Types of Data in Cluster Analysis

Spring 2010

Instructor: Dr. Masoud Yaghini

Outline

- Data Structures
- Interval-Valued (Numeric) Variables
- Binary Variables
- Categorical Variables
- Ordinal Variables
- Variables of Mixed Types
- References

Data Structures

Data Structures

- Clustering algorithms typically operate on either of the following two data structures:
  - Data matrix
  - Dissimilarity matrix

Data matrix

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

- This represents *n* objects, such as persons, with *p* variables (measurements or attributes), such as age, height, weight, gender, and so on.
- The structure is in the form of a relational table, or *n*-by-*p* matrix (*n* objects *p* variables)

Dissimilarity matrix

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \cdots & \cdots & 0 \end{bmatrix}$$

- It is often represented by an *n*-by-*n* where *d(i, j)* is the measured difference or dissimilarity between objects *i* and *j*.
- In general, *d(i, j)* is a nonnegative number that is
  - close to *0* when objects *i* and *j* are highly similar or "near" each other
  - becomes larger the more they differ
- Where *d(i, j)=d(j, i),* and *d(i, i)=0*

Type of data in clustering analysis

- Dissimilarity can be computed for
  - Interval-scaled (numeric) variables
  - Binary variables
  - Categorical (nominal) variables
  - Ordinal variables
  - Ratio variables
  - Mixed types variables

Interval-Valued (Numeric) Variables

Interval-valued variables

- Interval-scaled (numeric) variables are continuous measurements of a roughly linear scale.
- Examples
  - weight and height, latitude and longitude coordinates (e.g., when clustering houses), and weather temperature.
- The measurement unit used can affect the clustering analysis
  - For example, changing measurement units from meters to inches for height, or from kilograms to pounds for weight, may lead to a very different clustering structure.

Data Standardization

- Expressing a variable in smaller units will lead to a larger range for that variable, and thus a larger effect on the resulting clustering structure.
- To help avoid dependence on the choice of measurement units, the data should be standardized.
- Standardizing measurements attempts to give all variables an equal weight.
- To standardize measurements, one choice is to convert the original measurements to unitless variables.

Data Standardization

- Standardize data
  - Calculate the mean absolute deviation:

$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + \ldots + |x_{nf} - m_f|)$$

  - where $m_f = \frac{1}{n}(x_{1f} + x_{2f} + \ldots + x_{nf})$.
  - Calculate the standardized measurement (z-score)

$$z_{if} = \frac{x_{if} - m_f}{s_f}$$

Data Standardization

- Using mean absolute deviation is more robust to outliers than using standard deviation
- When computing the mean absolute deviation, the deviations from the mean are not squared; hence, the effect of outliers is somewhat reduced.
- Standardization may or may not be useful in a particular application.
  - Thus the choice of whether and how to perform standardization should be left to the user.
- Methods of standardization are also discussed under normalization techniques for data preprocessing.

Dissimilarity Between Objects

- Distances are normally used to measure the similarity or dissimilarity between two data objects described by interval-scaled variables

Dissimilarity Between Objects

- Euclidean distance: the most popular distance measure

$$d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + ... + |x_{i_p} - x_{j_p}|^2)}$$

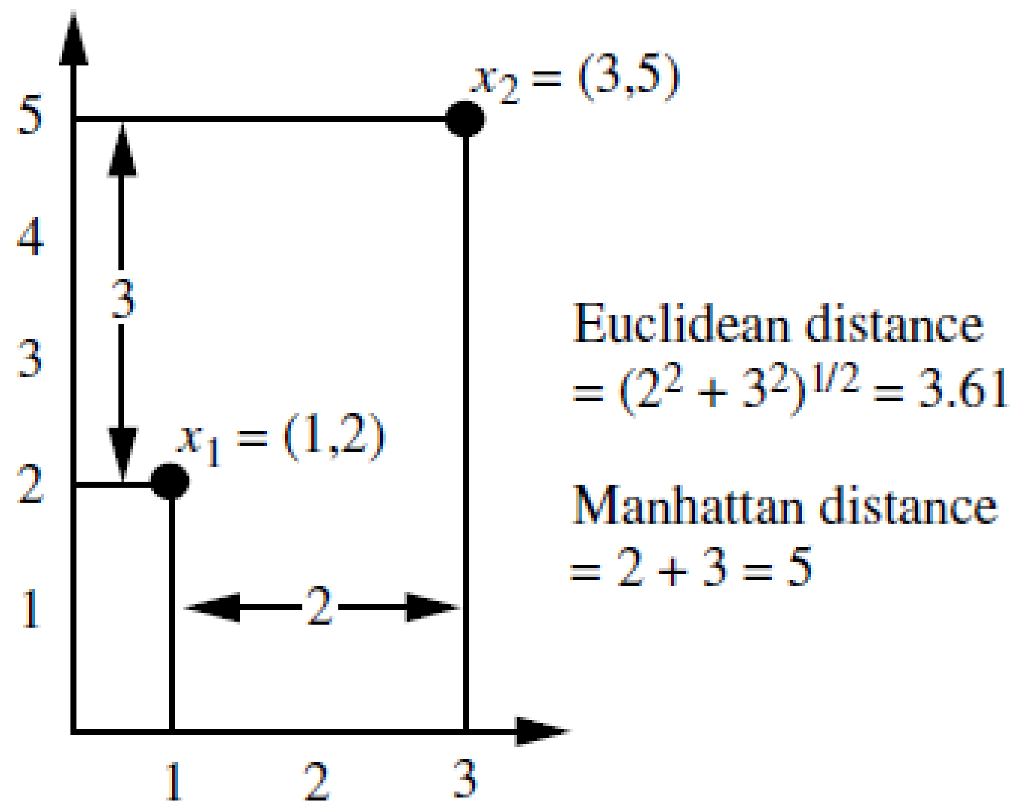
  - where $i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ are two p-dimensional data objects

- Manhattan (city block) distance: another well-known metric

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + ... + |x_{i_p} - x_{j_p}|$$

Dissimilarity Between Objects

- **Example**: Let x1 = (1, 2) and x2 = (3, 5) represent two objects



Dissimilarity Between Objects

- Properties of Euclidean and Manhattan distances:
  - $d(i,j) \geq 0$: Distance is a nonnegative number.
  - $d(i,i) = 0$: The distance of an object to itself is 0.
  - $d(i,j) = d(j,i)$: Distance is a symmetric function.
  - $d(i,j) \leq d(i,k) + d(k,j)$: Going directly from object *i* to object *j* in space is no more than making a detour over any other object *h (triangular inequality).*

Dissimilarity Between Objects

- Minkowski distance: a generalization of both Euclidean distance and Manhattan distance

$$d(i,j)=\sqrt[q]{(|x_{i_1}-x_{j_1}|^q+|x_{i_2}-x_{j_2}|^q+\ldots+|x_{i_p}-x_{j_p}|^q)}$$

  - Where q is a positive integer
  - It represents the Manhattan distance when q = *1 and Euclidean distance* when q = *2*

Binary Variables

Binary Variables

- A binary variable has only two states: 0 or 1, where 0 means that the variable is absent, and 1 means that it is present.
- Given the variable **smoker** describing a patient,
  - 1 indicates that the patient smokes
  - 0 indicates that the patient does not.
- Treating binary variables as if they are interval-scaled can lead to misleading clustering results.
- Therefore, methods specific to binary data are necessary for computing dissimilarities.

Binary Variables

- One approach involves computing a dissimilarity matrix from the given binary data.
- If all binary variables are thought of as having the same weight, we have the 2-by-2 contingency table

Contingency Table

		object j		
		1	0	sum
	1	q	r	$q+r$
object i	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- where
  - q is the number of variables that equal 1 for both objects i and j,
  - r is the number of variables that equal 1 for object i but that are 0 for object j,
  - s is the number of variables that equal 0 for object i but equal 1 for object j, and
  - t is the number of variables that equal 0 for both objects i and j.
  - *p is t*he total number of variables, $p = q+r+s+t$.

Symmetric Binary Dissimilarity

- A binary variable is symmetric if both of its states are equally valuable and carry the same weight
  - Example: the attribute gender having the states male and female.
- Dissimilarity that is based on symmetric binary variables is called symmetric binary dissimilarity.
- The dissimilarity between objects i and j:

$$d(i, j) = \frac{r+s}{q+r+s+t}$$

Asymmetric Binary Dissimilarity

- A binary variable is asymmetric if the outcomes of the states are not equally important,
  - Example: the positive and negative outcomes of a HIV test.
  - we shall code the most important outcome, which is usually the rarest one, by 1 (HIV positive)
- Given two asymmetric binary variables, the agreement of two 1s (a positive match) is then considered more significant than that of two 0s (a negative match).
- Therefore, such binary variables are often considered “monary” (as if having one state).
- The dissimilarity based on such variables is called asymmetric binary dissimilarity

Asymmetric Binary Dissimilarity

- In asymmetric binary dissimilarity the number of negative matches, t, is considered unimportant and thus is ignored in the computation:

$$d(i, j) = \frac{r + s}{q + r + s}$$

Asymmetric Binary Similarity

- The **asymmetric binary similarity** between the objects i and j, or sim(i, j), can be computed as

$$sim(i, j) = \frac{q}{q + r + s} = 1 - d(i, j)$$

- The coefficient sim(i, j) is called the Jaccard coefficient
- When both symmetric and asymmetric binary variables occur in the same data set, the mixed variables approach can be applied (described later)

Example: Dissimilarity Between Binary Variables

- Suppose that a patient record table contains the attributes :
  - name: an object identifier
  - gender: a symmetric attribute
  - fever, cough, test-1, test-2, test-3, test-4: the asymmetric attributes

name	*gender*	*fever*	*cough*	*test-1*	*test-2*	*test-3*	*test-4*
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	Y	N	N	N	N
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Example: Dissimilarity Between Binary Variables

- For asymmetric attribute values
  - let the values Y (yes) and P (positive) be set to1, and
  - the value N (no or negative) be set to 0.
- Suppose that the distance between objects (patients) is computed based only on the asymmetric variables.
- The distance between each pair of the three patients, Jack, Mary, and Jim, is

$$d(i, j) = \frac{r + s}{q + r + s}$$

$$\begin{aligned} d(Jack, Mary) &= \frac{0+1}{2+0+1} = 0.33 \\ d(Jack, Jim) &= \frac{1+1}{1+1+1} = 0.67 \\ d(Mary, Jim) &= \frac{1+2}{1+1+2} = 0.75 \end{aligned}$$

Example: Dissimilarity Between Binary Variables

- These measurements suggest that
  - Mary and Jim are unlikely to have a similar disease because they have the highest dissimilarity value among the three pairs.
  - Of the three patients, Jack and Mary are the most likely to have a similar disease.

Categorical Variables

Categorical Variables

- A categorical (nominal) variable is a generalization of the binary variable in that it can take on more than two states.
  - Example: map_color is a categorical variable that may have five states: red, yellow, green, pink, and blue.
- The states can be denoted by letters, symbols, or a set of integers.

Dissimilarity between categorical variables

- Method 1: Simple matching
  - The dissimilarity between two objects i and j can be computed based on the ratio of mismatches:

$$d(i, j) = \frac{p - m}{p}$$

  - m is the number of matches (i.e., the number of variables for which i and j are in the same state)
  - p is the total number of variables.
- Weights can be assigned to increase the effect of m or to assign greater weight to the matches in variables having a larger number of states.

Example: Dissimilarity between categorical variables

- Suppose that we have the sample data
  - where test-1 is categorical.

object identifier	*test-1 (categorical)*
1	code-A
2	code-B
3	code-C
4	code-A

Categorical Variables

- Let's compute the dissimilarity the matrix

$$\begin{bmatrix} 0 & & & \\ d(2,1) & 0 & & \\ d(3,1) & d(3,2) & 0 & \\ d(4,1) & d(4,2) & d(4,3) & 0 \end{bmatrix}$$

- Since here we have one categorical variable, test-1, we set $p = 1$ in

$$d(i, j) = \frac{p - m}{p}$$

Categorical Variables

- So that d(i, j) evaluates to 0 if objects i and j match, and 1 if the objects differ. Thus,

$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Categorical Variables

- Method 2: use a large number of binary variables
  - creating a new asymmetric binary variable for each of the nominal states
  - For an object with a given state value, the binary variable representing that state is set to 1, while the remaining binary variables are set to 0.
  - For example, to encode the categorical variable map _color, a binary variable can be created for each of the five colors listed above.
  - For an object having the color yellow, the yellow variable is set to 1, while the remaining four variables are set to 0.

Ordinal Variables

Ordinal Variables

- A discrete ordinal variable resembles a categorical variable, except that the M states of the ordinal value are ordered in a meaningful sequence.
  - Example: professional ranks are often enumerated in a sequential order, such as assistant, associate, and full for professors.
- Ordinal variables may also be obtained from the discretization of interval-scaled quantities by splitting the value range into a finite number of classes.
- The values of an ordinal variable can be mapped to ranks.
  - Example: suppose that an ordinal variable f has M_f states.
  - These ordered states define the ranking $1, \ldots, M_f$.

Ordinal Variables

- Suppose that f is a variable from a set of ordinal variables describing n objects.
- The dissimilarity computation with respect to f involves the following steps:
- Step 1:
  - The value of f for the ith object is x_{if} , and f has M_f ordered states, representing the ranking $1, \ldots, M_f$.
  - Replace each x_{if} by its corresponding rank:

$$r_{if} \in \{1, \ldots, M_f\}$$

Ordinal Variables

- Step 2:
  - Since each ordinal variable can have a different number of states, it is often necessary to map the range of each variable onto [0.0, 1.0] so that each variable has equal weight.
  - This can be achieved by replacing the rank r_{if} of the ith object in the f th variable by:

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

- Step 3:
  - Dissimilarity can then be computed using any of the distance measures described for interval-scaled variables.

Ordinal Variables

- **Example:** Suppose that we have the sample data:

object identifier	*test-2 (ordinal)*
1	excellent
2	fair
3	good
4	excellent

- There are three states for test-2, namely fair, good, and excellent, that is $M_f = 3$.

Example: Dissimilarity between ordinal variables

- Step 1: if we replace each value for test-2 by its rank, the four objects are assigned the ranks 3, 1, 2, and 3, respectively.
- Step 2: normalizes the ranking by mapping rank 1 to 0.0, rank 2 to 0.5, and rank 3 to 1.0.
- Step 3: we can use, say, the Euclidean distance, which results in the following dissimilarity matrix:

$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 0.5 & 0.5 & 0 & \\ 0 & 1.0 & 0.5 & 0 \end{bmatrix}$$

Variables of Mixed Types

Variables of Mixed Types

- A database may contain different types of variables
  - interval-scaled, symmetric binary, asymmetric binary, nominal, and ordinal
- We can combine the different variables into a single dissimilarity matrix, bringing all of the meaningful variables onto a common scale of the interval [0.0, 1.0].

Variables of Mixed Types

- Suppose that the data set contains p variables of mixed type. The dissimilarity d(i, j) between objects i and j is defined as

$$d(i, j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

- $\delta_{ij}^{(f)} = 0$
  - if either (1) x_{if} or x_{jf} is missing (i.e., there is no measurement of variable f for object i or object j),
  - or (2) $x_{if} = x_{jf} = 0$ and variable f is asymmetric binary;
- otherwise $\delta_{ij}^{(f)} = 1$

Variables of Mixed Types

- The contribution of variable f to the dissimilarity between i and j, that is, $d_{ij}^{(f)}$
- If f is interval-based:
  - use the normalized distance so that the values map to the interval [0.0,1.0].
- If f is binary or categorical:
  - $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$, or $d_{ij}^{(f)} = 1$ otherwise
- If f is ordinal:
  - compute ranks r_{if} and

Example: Dissimilarity between variables of mixed type

- The sample data:

object identifier	*test-1 (categorical)*	*test-2 (ordinal)*
1	code-A	excellent
2	code-B	fair
3	code-C	good
4	code-A	excellent

Example: Dissimilarity between variables of mixed type

- For test-1 (which is categorical) is the same as outlined above
- For test-2 (which is ordinal) is the same as outlined above
- We can now calculate the dissimilarity matrices for the two variables.

$$\begin{pmatrix} 0.00 & & & \\ 1.00 & 0.00 & & \\ 0.75 & 0.75 & 0.00 & \\ 0.00 & 1.00 & 0.75 & 0.00 \end{pmatrix}$$

Example: Dissimilarity between variables of mixed type

- If we go back and look at the data, we can intuitively guess that objects 1 and 4 are the most similar, based on their values for test-1 and test-2.
- This is confirmed by the dissimilarity matrix, where d(4, 1) is the lowest value for any pair of different objects.
- Similarly, the matrix indicates that objects 1 and 2 and object 2 and 4 are the least similar.

References

References

- J. Han, M. Kamber, **Data Mining: Concepts and Techniques**, Elsevier Inc. (2006). (Chapter 7)

The end