Data Mining

Part 5. Prediction

5.7 Regression Analysis

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Outline

- Introduction
- Linear Regression
- Other Regression Models
- References

Introduction

Introduction

- **Numerical prediction** is similar to **classification**
  - construct a model
  - use model to predict continuous or ordered value for a given input
- **Numeric prediction** vs. **classification**
  - Classification refers to predict categorical class label
  - Numeric prediction models continuous-valued functions

Introduction

- Regression analysis is the major method for numeric prediction
- **Regression analysis** model the relationship between
  - one or more **independent** or **predictor variables** and
  - a **dependent** or **response** variable
- **Regression analysis** is a good choice when all of the **predictor variables** are **continuous** valued as well.

Introduction

- In the context of data mining
  - The **predictor variables** are the **attributes** of interest describing the instance that are known.
  - The response variable is what we want to predict
- Some classification techniques can be adapted for prediction, e.g.
  - Backpropagation
  - k-nearest-neighbor classifiers
  - Support vector machines

Introduction

- **Regression analysis methods**:
  - Linear regression
    - Straight-line linear regression
    - Multiple linear regression
  - Non-linear regression
  - Generalized linear model
    - Poisson regression
    - Logistic regression
  - Log-linear models
  - Regression trees and Model trees

Linear Regression

Linear Regression

- **Straight-line linear regression**:
  - involves a response variable y and a single predictor variable x

    $$y = w_0 + w_1 x$$

  - w_0 : y-intercept
  - w_1 : slope
  - w_0 & w_1 are **regression coefficients**

Linear regression

- **Method of least squares**: estimates the best-fitting straight line as the one that minimizes the error between the actual data and the estimate of the line.

$$w_1 = \frac{\sum_{i=1}^{|D|}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{|D|}(x_i - \bar{x})^2} \qquad w_0 = \bar{y} - w_1\bar{x}$$

- *D:* a training set
- *x*: values of predictor variable
- *y*: values of response variable
- *|D|* : data points of the form($x1$, $y1$), ($x2$, $y2$),…, ($x|D|$, $y|D|$).
- $\bar{x}$ *:* the mean value of $x1$, $x2$, … , $x|D|$
- $\bar{y}$ *:* the mean value of $y1$, $y2$, … , $y|D|$

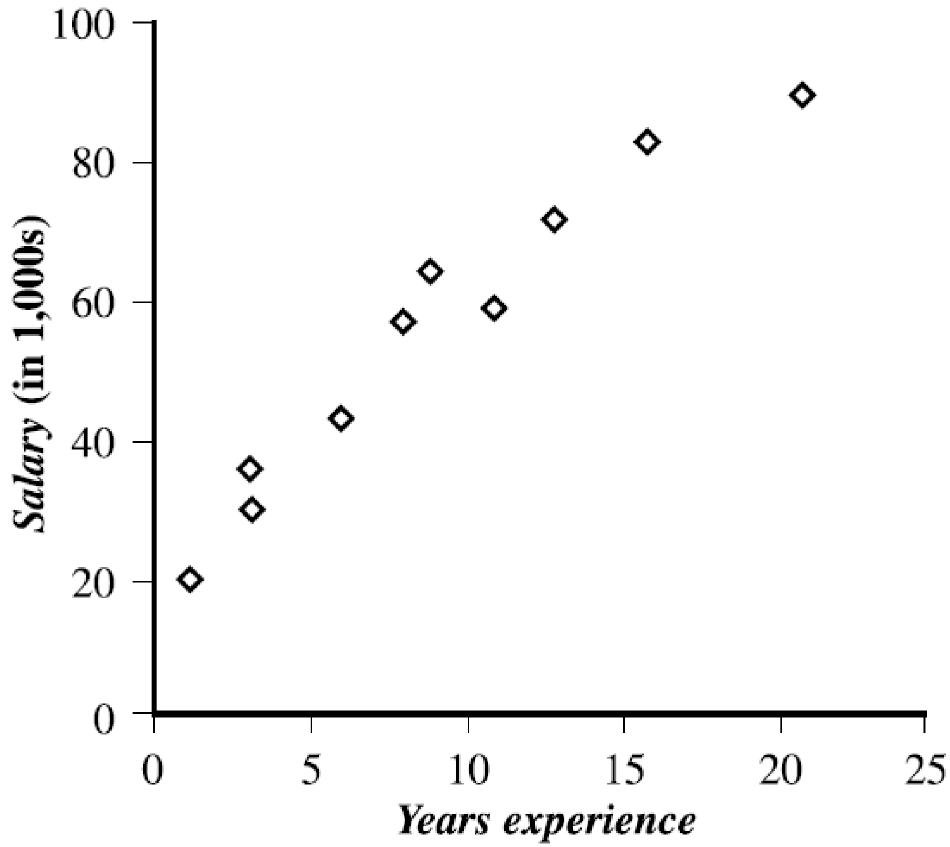
Example: Salary problem

- The table shows a set of paired data where *x* is the number of years of work experience of a college graduate and *y* is the corresponding salary of the graduate.

x years experience	y salary (in $1000s)
3	30
8	57
9	64
13	72
3	36
6	43
11	59
21	90
1	20
16	83

Linear Regression

- The 2-D data can be graphed on a **scatter plot**.
- The plot suggests a linear relationship between the two variables, *x and y.*



Example: Salary data

- Given the above data, we compute

$$\bar{x} = 9.1 \text{ and } \bar{y} = 55.4$$

- we get

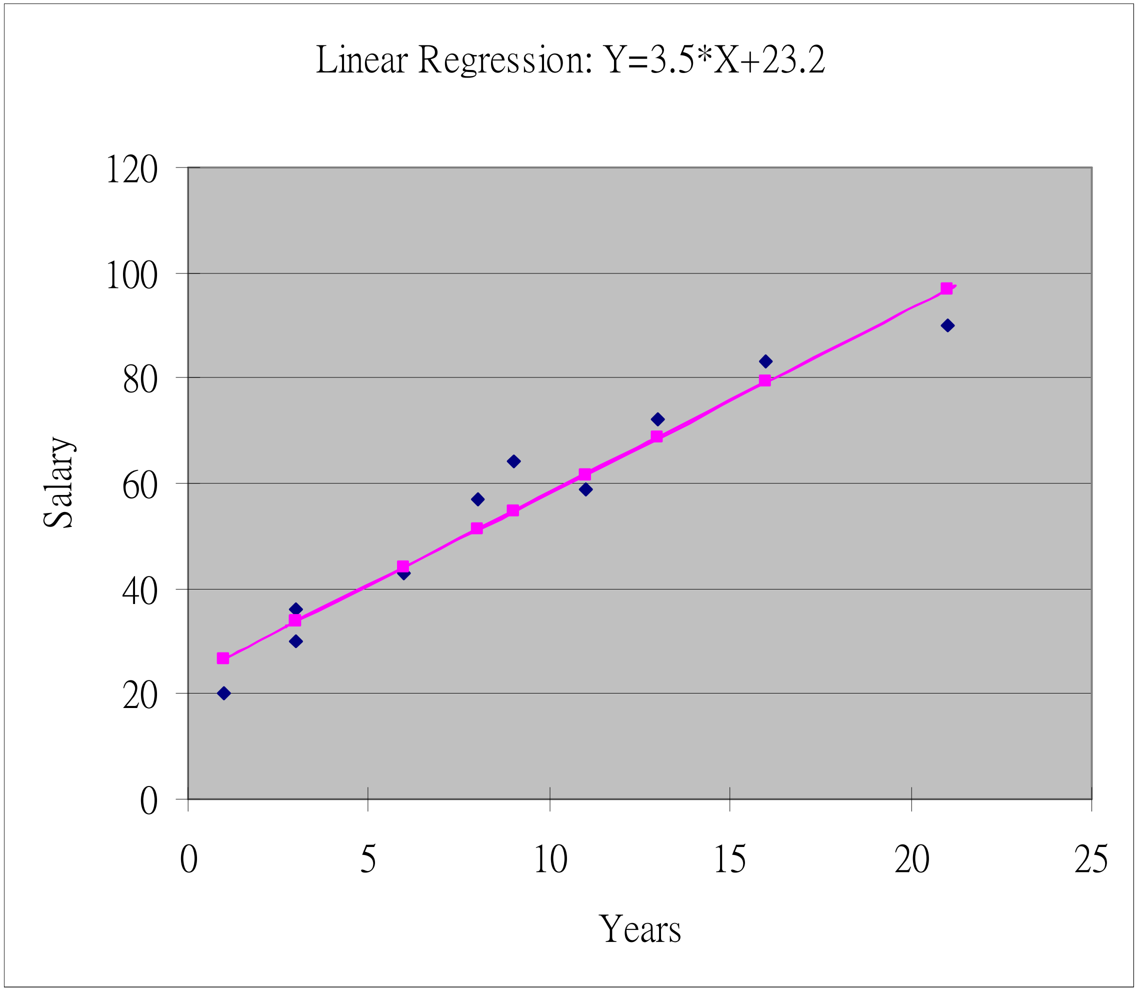
$$w_1 = \frac{(3-9.1)(30-55.4)+(8-9.1)(57-55.4)+\cdots+(16-9.1)(83-55.4)}{(3-9.1)^2+(8-9.1)^2+\cdots+(16-9.1)^2} = 3.5$$

$$w_0 = 55.4-(3.5)(9.1) = 23.6$$

- The equation of the least squares line is estimated by

$$y = 23.6 + 3.5x$$

Example: Salary data



Multiple linear regression

- **Multiple linear regression** involves more than one predictor variable
- Training data is of the form ($\mathbf{X}_1$, y_1), ($\mathbf{X}_2$, y_2),…, ($\mathbf{X}_{|D|}$, $y_{|D|}$)
- where the $\boldsymbol{X_i}$ are the *n*-dimensional training data with associated class labels, y_i
- An example of a multiple linear regression model based on two predictor attributes:

$$y = w_0 + w_1 x_1 + w_2 x_2$$

Example: CPU performance data

	Cycle time (ns) MYCT	Main memory (KB) Min. MMIN	Main memory (KB) Max. MMAX	Cache (KB) CACH	Channels Min. CHMIN	Channels Max. CHMAX	Performance PRP
1	125	256	6000	256	16	128	198
2	29	8000	32000	32	8	32	269
3	29	8000	32000	32	8	32	220
4	29	8000	32000	32	8	32	172
5	29	8000	16000	32	8	16	132
...							
207	125	2000	8000	0	2	14	52
208	480	512	8000	32	0	0	67
209	480	1000	4000	0	0	0	45

$$\begin{aligned} \text{PRP} = &-55.9 + 0.0489\ \text{MYCT} + 0.0153\ \text{MMIN} + 0.0056\ \text{MMAX} \\ &+ 0.6410\ \text{CACH} - 0.2700\ \text{CHMIN} + 1.480\ \text{CHMAX}. \end{aligned}$$

Multiple Linear Regression

- Various statistical measures exist for determining how well the proposed model can predict *y* (described later).
- Obviously, the greater the number of predictor attributes is, the slower the performance is.
- Before applying regression analysis, it is common to perform attribute subset selection to eliminate attributes that are unlikely to be good predictors for *y.*
- In general, regression analysis is accurate for numeric prediction, except when the data contain outliers.

Other Regression Models

Nonlinear Regression

- If data that does not show a linear dependence we can get a more accurate model using a nonlinear regression model
- For example,

$$y = w_0 + w_1 x + w_2 x^2 + w_3 x^3$$

Generalized linear models

- **Generalized linear model** is foundation on which linear regression can be applied to modeling **categorical response variables**
- Common types of generalized linear models include
  - **Logistic regression**: models the probability of some event occurring as a linear function of a set of predictor variables.
  - **Poisson regression**: models the data that exhibit a Poisson distribution

Log-linear models

- In the log-linear method, all attributes must be categorical
- Continuous-valued attributes must first be discretized.

Regression Trees and Model Trees

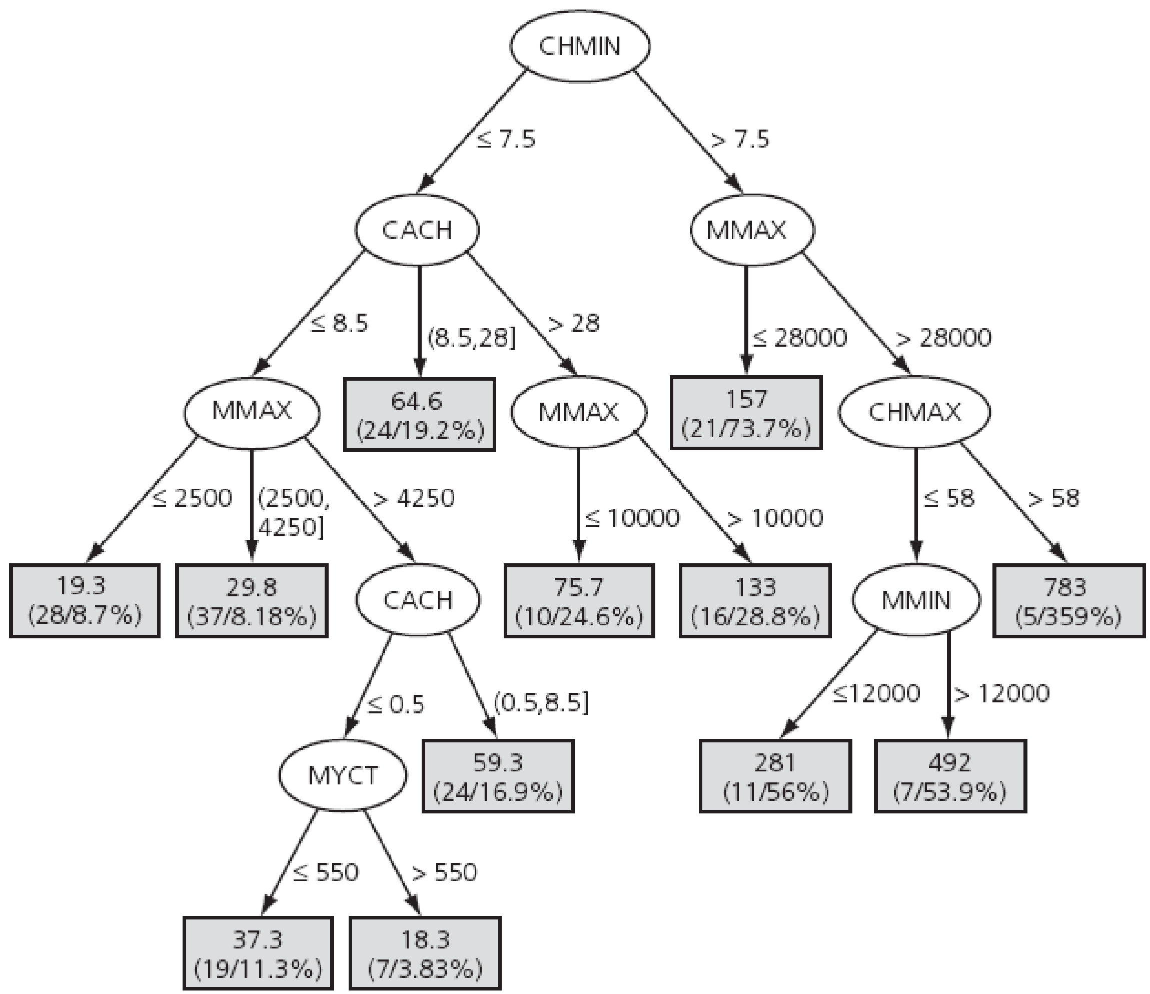
- Trees to predict continuous values rather than class labels
- **Regression and model trees** tend to be more accurate than linear regression when the data are not represented well by a simple linear model

Regression trees

- **Regression tree**: a decision tree where each leaf predicts a numeric quantity
- Proposed in CART system (Breiman et al. 1984)
  - CART: Classification And Regression Trees
- Predicted value is average value of training instances that reach the leaf

Example: CPU performance problem

- **Regression tree for the CPU data**



Example: CPU performance problem

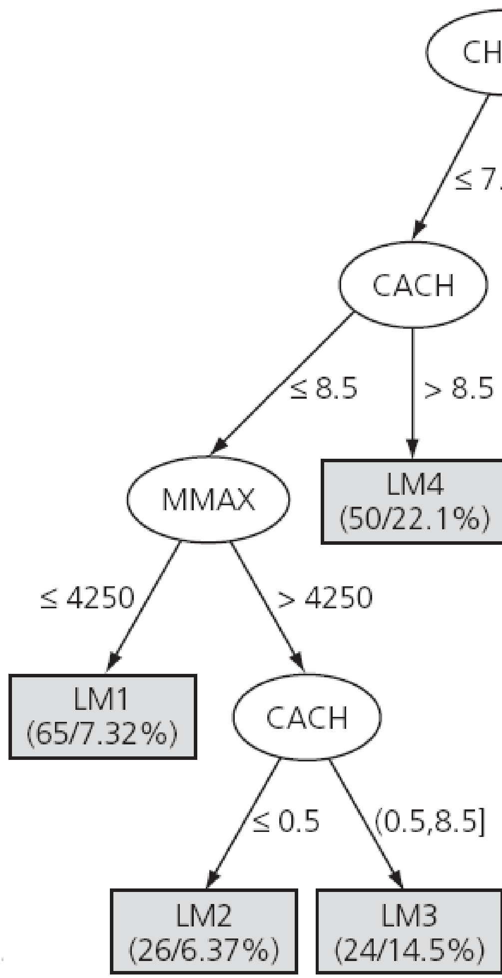
- We calculate the average of the absolute values of the errors between the predicted and the actual CPU performance measures
- It turns out to be significantly less for the tree than for the regression equation.

Model tree

- **Model tree**: Each leaf holds a regression model
- A multivariate linear equation for the predicted attribute
- Proposed by Quinlan (1992)
- A more general case than regression tree

Example: CPU performance problem

- Model tree for the CPU data

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LM1 PRP=8.29+0.004 MMAX+2.77 CHMIN
LM2 PRP=20.3+0.004 MMIN-3.99 CHMIN
        +0.946 CHMAX
LM3 PRP=38.1+0.012 MMIN
LM4 PRP=19.5+0.002 MMAX+0.698 CACH
        +0.969 CHMAX
LM5 PRP=285-1.46 MYCT+1.02 CACH
        -9.39 CHMIN
LM6 PRP=-65.8+0.03 MMIN-2.94 CHMIN
        +4.98 CHMAX
```

References

References

- J. Han, M. Kamber, **Data Mining: Concepts and Techniques**, Elsevier Inc. (2006). (Chapter 6)

- I. H. Witten and E. Frank, **Data Mining: Practical Machine Learning Tools and Techniques**, 2nd Edition, Elsevier Inc., 2005. (Chapter 6)

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