In the name of God

Network Flows

8. Multicommodity Network Design Problems

8.3 Path Relinking, Cycle-based Neighbourhoods

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1. Introduction

Introduction

- **Reference:**
  - Ghamlouche, I., Crainic, T.G., Gendreau, M.: Path relinking, cycle-based neighbourhoods and capacitated multicommodity network design. Ann. Oper. Res. 131, 109–133 (2004)

Introduction

- **Path relinking**
  - has been proposed as a method to better explore the solution space of complex problems when a set of promising, or elite solutions is known
  - **The fundamental idea** is to build a solution path between two elite solutions, using their attributes to guide the generation of intermediary solutions.
  - This process is supposed to yield solutions that combine the "good" characteristics of the starting solutions and that improve the set of elite solutions and, eventually, the overall best.

Introduction

- The contributions of this work are:
  - 1) a new meta-heuristic for the CMND that outperforms the current best approximate methods
  - 2) a successful development of a path relinking method to a difficult combinatorial optimization problem
  - 3) an enhancement of our understanding of the behaviour of path relinking as a search method

2. Path Relinking

Path Relinking

- **Path Relinking**
  - Introduced by Glover (1997),
  - Path relinking is a meta-heuristic that operates on a set of elite solutions, called the **reference set**, and generates paths between solutions in this set to create improved new ones.
- **Initial solution & guiding solution**
  - Starting from an **initial solution**, the primary goal of the search is to find a path towards another, **guiding solution**, by performing moves that progressively introduce into the current solution attributes of the guiding solution.

Path Relinking

- **Move in Path Relinking**
  - The method does not progress by choosing a “best” move from the neighbourhood, but rather by selecting the “best” move from a restricted set of moves that incorporate some or all of the “good” attributes of the guiding solution.
  - This exploration allows the search to perform moves that may be considered unattractive according to the objective function value but which appear essential in reaching solutions with given characteristics.

Path Relinking

- **Path Relinking & Tabu Search**
  - Path relinking has been proposed in connection with tabu search.
  - Laguna, Marti and Campos (1999) propose an algorithm based on tabu search with strategies for search intensication and diversication for the linear ordering problem.
  - The method uses path relinking for search intensication.
  - Elite solutions are selected as the best solutions found during the entire search, while **initial solutions** are **local optima**.

Path Relinking

- **General Path Relinking Procedure**
  - This paper investigates the implementation of path relinking to the CMND.
  - The method starts with a tabu search phase using the algorithm of Ghamlouche, Crainic, and Gendreau (2001) to build the **reference set**.
  - When a predefined number of consecutive moves without improvement is observed, the method switches to a path relinking phase that stops when the reference set becomes empty (cardinality ≤ 1).
  - Then, either stopping conditions are verified, or the procedure is repeated to build a new reference set.

Path Relinking

- **General Path Relinking Procedure**
- 1: Generate a starting set of solutions using the cycle-based tabu search
- 2: Designate a subset of solutions to be included in the reference set
  - While the reference set cardinality > 1
    - Select two solutions from the reference set
    - Identify initial and guiding solutions
    - Remove initial solution from the reference set
    - Move from initial toward guiding solution, generating intermediate solutions
    - Update the reference set
- 3: Verify stopping criterion: Stop or go to 1:

Path Relinking

- Three main components of the path relinking method
  - Identification of the reference set.
  - Choice of the initial and guiding solutions.
  - Identification of the neighbourhood structure and the guiding attributes.

2.1 Identification of the Reference Set

Identification of the Reference Set

- What solutions are included in the reference set *R*, how good and how diversified they are, have a major impact on the quality of the new generated solutions.
- In our implementation, *R* is built during the tabu search phase and subsequently enriched during the path relinking phase.
- We study **six strategies** corresponding to different ways to build *R*.

Identification of the Reference Set

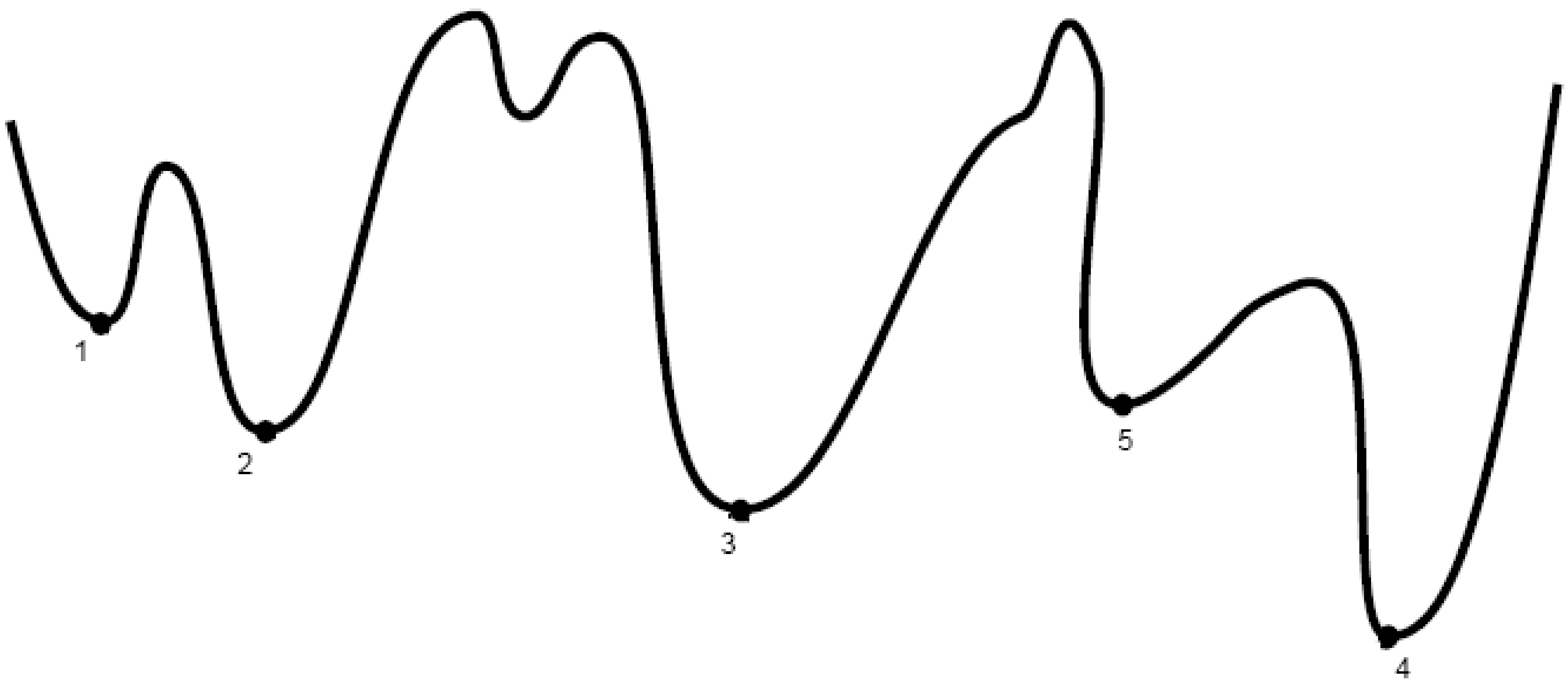
- **Strategy *S1***
  - R is built using each solution that, at some stage of the tabu search phase, improves the best overall solution and become the best one.
  - The goal of this strategy is to link overall improving solutions.

Identification of the Reference Set

- **Strategy *S2***
  - retains the “best” local minima found during the tabu search phase.
  - This strategy is motivated by the idea that local minimum solutions share characteristics with optimum solutions.
  - One then hopes that linking such solutions yields improved new ones.

Identification of the Reference Set

- **Strategy *S2***
  - In this Figure, solutions 2, 3, 4, and 5 are members of *R* when its dimension is limited to 4.

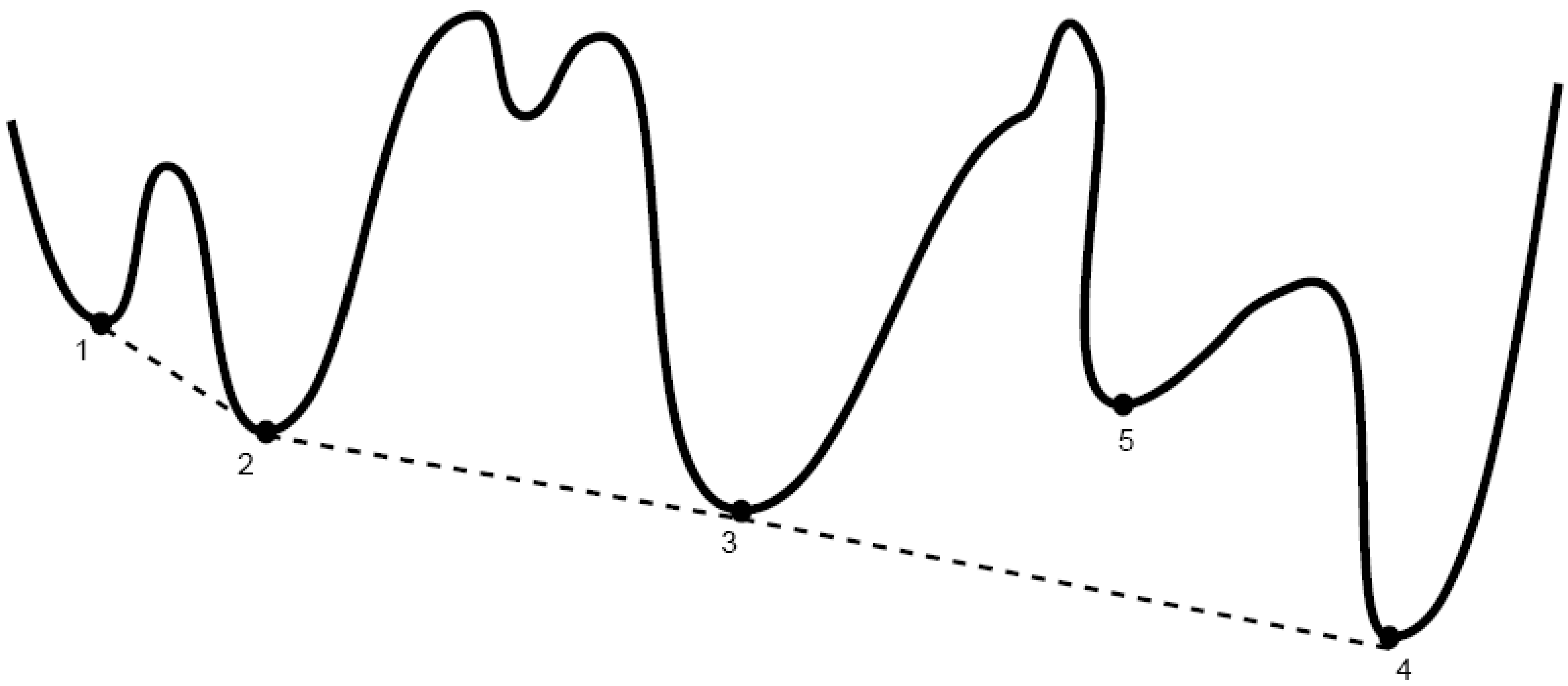


Identification of the Reference Set

- **Strategy *S3***
  - selects *R*-improving local minima, that is local minimum solutions that offer a better evaluation of the objective function than those already in *R*.
  - The idea is to introduce a time dimension in the selection process - often better solutions are encountered once the search has proceeded for some time - without loosing potentially good solutions found early on during the search.

Identification of the Reference Set

- **Strategy *S3***
  - if solutions 1 and 2 are already in R, only solutions 3 and 4 are admitted to this set.



Identification of the Reference Set

- **Strategy *S4***
  - represents a first attempt to allow solutions to be retained in *R* not only according to an attractive solution value but also according to a diversity, or dissimilarity criterion.
  - Define D^b_s, the level of dissimilarity between solution *s* and the best solution *b*, as the number of arcs with different status between the two solutions:

$$D_s^b = \sum_{(i,j)\in A} d_{ij},$$

  - where,

$$d_{ij} = \begin{cases} 0 & if \quad y_{ij}^s = y_{ij}^b \\ 1 & \quad \text{otherwise.} \end{cases}$$

Identification of the Reference Set

- Define also the median position of all solutions $s_i \in R$, relatively to the best solution b as:

$$Median = \frac{\sum_{s_i \in \mathcal{R}}^{s_i \neq b} D_{s_i}^b}{|\mathcal{R}| - 1},$$

  – Where $|R|$ denotes the dimension of the reference set.

- A solution s is then included in R if it improves the best overall solution, or if it improves the worst solution and its level of dissimilarity exceeds the median:

$$D_s^b > Median$$

Identification of the Reference Set

- **Building *R* According to Strategy *S4*:**
- For a given candidate solution s
  - If *R* is not full, Add *s* to R
  - Else, If *s* is the best overall solution,
  - Replace the worst solution in *R* by *s*
  - Else, If *s* is better than the worst solution in *R*
    - Compute Median
    - Compute D_s^b
    - If D_s^b > Median;
  - Replace the worst solution in *R* by *s*

Identification of the Reference Set

- **Strategy *S5***
  - aims to ensure both the quality and the diversity of solutions in *R*.
  - Starting with a large set of (“good”) solutions *S*, *R* is partially filled with the best solutions found, to satisfy the purpose of quality.
  - It is then extended with solutions that change significantly the structure of the solutions already in *R* to ensure diversity.

Identification of the Reference Set

- **Building *R* According to Strategy *S5*:**
  - 1. Initialize the reference set *R* with solutions satisfying strategy *S1*.
  - 2. For each solution $s \in \{S \setminus R\}$, compute the level of diversity Δ_s^R between solution *s* and all solutions $s_i \in R$

$$\Delta_s^R = \sum_{s_i \in \mathcal{R}} D_{s_i}^s / \mathcal{R}$$

  - 3. Extend *R* with solutions $s \in \{S \setminus R\}$ that maximize Δ_s^R

Identification of the Reference Set

- **Strategy *S6***
  - proceeds similarly to *S5* with the difference that *R* is extended with solutions close to those already in *R*.
  - This strategy aims to intensify the search by grouping good solutions with similar designs
  - To build *R*, strategy *S6* implements the first two steps used for strategy *S5*, then extends it with solutions $s \in \{S \setminus R\}$, that minimize Δ_s^R

2.2 Choice of the Initial and Guiding Solutions

Choice of the Initial and Guiding Solutions

- The choice of the initial and guiding solutions is also critical to the quality of the new solutions and, thus, the performance of the procedure.
- We investigate the effect of the six strategies

Choice of the Initial and Guiding Solutions

- **Strategy *C1***
  - Guiding and initial solutions are defined as the best and worst solutions, respectively.
- **Strategy *C2***
  - Guiding solution is defined as the best solution in the reference set, while the initial solution is the second best one.
- **Strategy *C3***
  - Guiding solution is defined as the best solution in the reference set, while the initial solution is defined as the solution with maximum distance from the guiding solution.

Choice of the Initial and Guiding Solutions

- **Strategy *C4***
  - Guiding and initial solutions are chosen randomly from the reference set.
- **Strategy *C5***
  - Guiding and initial solutions are chosen as the most distant solutions in the reference set.
- **Strategy *C6***
  - Guiding and initial solutions are defined respectively as the worst and the best solutions in the reference set.

2.3 Neighbourhood Structure and Guiding Attributes

Neighbourhood Structure and Guiding Attributes

- The two versions of the cycle-based neighbourhood used in the tabu search procedure:
  - **The basic version** that considers all flows on arcs,
    - This brings about significant modifications
  - **The intensification version** deviates only a commodity at the time.
    - This yields more limited changes.

Neighbourhood Structure and Guiding Attributes

- The aim of the path relinking procedure is to introduce progressively attributes of the guiding solution into new solutions obtained by moving away from an initial point.
- This requires small but purposeful movements and, thus, we use the second variant of the neighbourhood structure considering flow deviations of only one commodity at a time.

Neighbourhood Structure and Guiding Attributes

- Consequently, for the path relinking phase, the set Γ becomes Γ^p, defined as the set of the positive flow of commodity *p* on each of the open arcs of the corresponding network:

$$\Gamma^p(\bar{y}) = \left\{ x_{ij}^p > 0 \;:\; (i,j) \in \mathcal{A}(\bar{y}),\; p \in \mathcal{P} \right\}$$

Neighbourhood Structure and Guiding Attributes

- Moves from the current solution to a neighbouring one during path relinking must direct the search towards the guiding solution.
- Thus, cycles of interest are those that introduce **attributes** (i.e, **arcs**) present in the guiding solution.
- To apply this the set of the candidate links *C* becomes C_r, the set of all arcs with a different status in the initial solution s_i and the guiding solution s_g:

$$\mathcal{C}_r = \{(i,j) \in \mathcal{A} : y_{ij}^{s_i} \neq y_{ij}^{s_g}\}$$

Neighbourhood Structure and Guiding Attributes

- Similarly, sets $C_r(\gamma) \subseteq C_r$ include now all arcs $(i, j) \in C_r$ that can support a movement of units of flow.
- To select among the various possible paths between the two reference solutions, we move from the initial solution toward the guiding solution by selecting at each step the "best" move with respect to the improvement in the objective function.
- That is, cycles that include arcs in C_r are evaluated and a move is performed toward the "best" one.
- This neighbour becomes the new current solution, **even** if it is **worse** than the previous one.

2.4 The Overall Procedure

The Overall Procedure

- To sum up, the procedure combines cycle-based tabu search and path relinking and alternates between the two.
- After an initialization phase, the search proceeds with the tabu search to initialize the reference set *R* according to one of the mentioned strategies.
- The method then switches to a path relinking phase.
- Initial and guiding solutions are selected in *R* according to one of the mentioned criteria.
- The search proceeds by moving towards the guiding solution.

The Overall Procedure

- **Path Relinking Procedure**
- **Initialization**
  - Generate an initial feasible solution to initiate *BestSolution* and *CurrentSolution*
- **Main search loop**
  - Repeat the following until a stopping condition is met
  - Perform one tabu search iteration to get a new current solution
  - Add current solution to *R* if it satisfies the requirements of strategy S_i
  - If path relinking is to be launched,
    - Extend *R* if strategy *S5* or *S6* is used
    - Perform **Main Path Relinking Loop**

The Overall Procedure

- Notice that, due to the nature of the neighbourhoods used, there is no guarantee that the guiding solution will be reached.
- One cannot, therefore, stop the process only if the current and the guiding solutions are the same.
- We then define Δ_{IG} as the number of arcs with different status between the initial and the guiding solutions and we allow the search to explore a number of solutions not larger than Δ_{IG} .

3 Experimental Results

Experimental Results

- The objectives of the experiments is to calibrate and analyze the proposed path relinking procedure.
- The calibration phase aims:
  - Identifying appropriate values for the key parameters, and
  - Explore the impact on the solution quality of each combination of
    - Strategies to build the reference set, and
    - Strategies to select initial and guiding solutions.

Experimental Results

- The computer code is written in C++.
- The exact evaluation of the capacitated multicommodity network flow problems is done using the linear programming solver of CPLEX 6.5 (1999).
- All tests were conducted on
  - one 400MHz processor of a 64-processors Sun Enterprise 10000
  - with 64 Gigabyte of RAM,
  - operating under Solaris 2.7.
- Computing times are reported in CPU seconds.

3.1 Parameter Settings

Parameter Settings

- Two parameters have to be calibrated:
  - The cardinality of the reference set |R|
    - Three values, 6, 8, and 15, were tested for |R|
  - The criteria to launch a path relinking phase (*PRstart*)
    - The path relinking phase is started after a number of iterations of the tabu search procedure without improvement of the best value of the objective function.
    - We retained 10, 20, and 50 iterations for this parameter,
- Thus, in total, 9 sets of parameters were tested.

Parameter Settings

- Strategies *S3* and *C1*, for building the reference set and selecting initial and guiding solutions, respectively, were used for these tests.
- Experiments were conducted on 10 problems covering networks from 100 to 700 design arcs and from 10 to 400 commodities.
- This is similar to Ghamlouche, Crainic, and Gendreau (2001).

Parameter Settings

- For each parameter set, we run the path relinking algorithm 3 times on each problem.
- The average gap for each problem was calculated as the average of the gaps between the path relinking solutions and the best known solution for the same problem
  - that of the branch-and-bound procedure of CPLEX 6.5, when available, or
  - that obtained by Ghamlouche, Crainic, and Gendreau 2001, otherwise.

Parameter Settings

- The average gap assigned for each parameter set is the average of the average gaps of all problems.
- Moreover, we assign to each problem a score from 9 to 1 for each parameter set.
- This score depends on the value of the average gap and is determined as follows:
  - For each problem, a score of 9 is assigned to the parameter set giving the lowest average gap over 3 runs,
  - a score of 8 is assigned to the parameter set giving the second lowest average gap, and so on.
  - The total score assigned to a parameter set is the sum of scores assigned to this set for each problem.

Parameter Settings

- Average gaps and scores for each parameter set are:

$\lvert\mathcal{R}\rvert$, $PRstart$	Av. Gap	score
6, 10	2.07%	60
8, 10	2.90%	39
15, 10	3.30%	28
6, 20	1.42%	75
8, 20	2.04%	53
15, 20	2.62%	42
6, 50	1.91%	55
8, 50	1.95%	64
15, 50	2.20%	49

Parameter Settings

- The results indicate that increasing the dimension of the reference set is not necessarily beneficial.
- In fact, the results point to the parameter set $|R| = 6$ and $PRstart = 20$ as the most effective:
  - it offers the lowest average gap and the highest score.
- This setting was used in all the experiments reported in the rest of this section.

3.2 Path Relinking Strategies

Path Relinking Strategies

- We tested all combinations of strategies in order to identify the best way to build
  - the reference set (strategy S_i) and
  - to select initial and guiding solutions (strategy C_j).
- The best (S_i, C_j) combination was then used for the extensive experimental analysis of the path relinking procedure.

Path Relinking Strategies

- The same 10 problems used to calibrate the parameter settings were also used here.
- Moreover, each run is repeated 3 times.
- Thus, since there are 36 possible combinations of strategies, a total of 10 * 3 * 36 =1080 runs were performed.
- All strategies have been run for the same number of iterations.
- The performance of each combination of strategies is measured as the average improvement in solution quality, compared to the corresponding result of the cycle-based tabu search.

Path Relinking Strategies

- Average improvement over cycle-based tabu search

	$C1$	$C2$	$C3$	$C4$	$C(5)$	$C(6)$
$S1$	-0.51%	-0.91%	-0.84%	-0.47%	-0.55%	-0.42%
$S2$	-0.23%	-0.18%	-0.73%	-0.39%	-0.50%	-0.19%
$S3$	-0.54%	-0.51%	-0.93%	-0.69%	-0.99%	-0.80%
$S4$	-0.39%	-0.80%	-0.75%	-0.39%	-0.59%	-0.29%
$S5$	-0.36%	-0.69%	-0.61%	-0.41%	-0.45%	-0.43%
$S6$	-0.46%	-0.46%	-0.01%	-0.47%	0.08%	-0.16%

Path Relinking Strategies

- The conclusion
  - is that path relinking offers a consistently better performance in terms
- The table identifies strategies *S3* and *C5* as offering the best results.
- This set was therefore retained for our experimental analyses.

Path Relinking Strategies

- The combination (*S3*, *C5*) offers a combination of
  - Strategy *S3*: very good solutions obtained over time - local optima better than the current best in R - and
  - Strategy *C5*: guiding and initial solutions are chosen as the most distant solutions in the reference set.
- This combination proceeds towards the best solution in the reference set.

Path Relinking Strategies

- It is noteworthy that the second best couple of strategies, (S3, C3) is of a very similar structure.
- The success of these combinations confirms the importance of,
  - First, the presence of elite solutions in the reference set and,
  - Second, relinking strategies that diversify the solutions considered.
- This emphasizes the diversification effect of path relinking and supports some of the conclusions of Laguna and Armentano (2001).

3.3 Influence of Sampling

Influence of Sampling

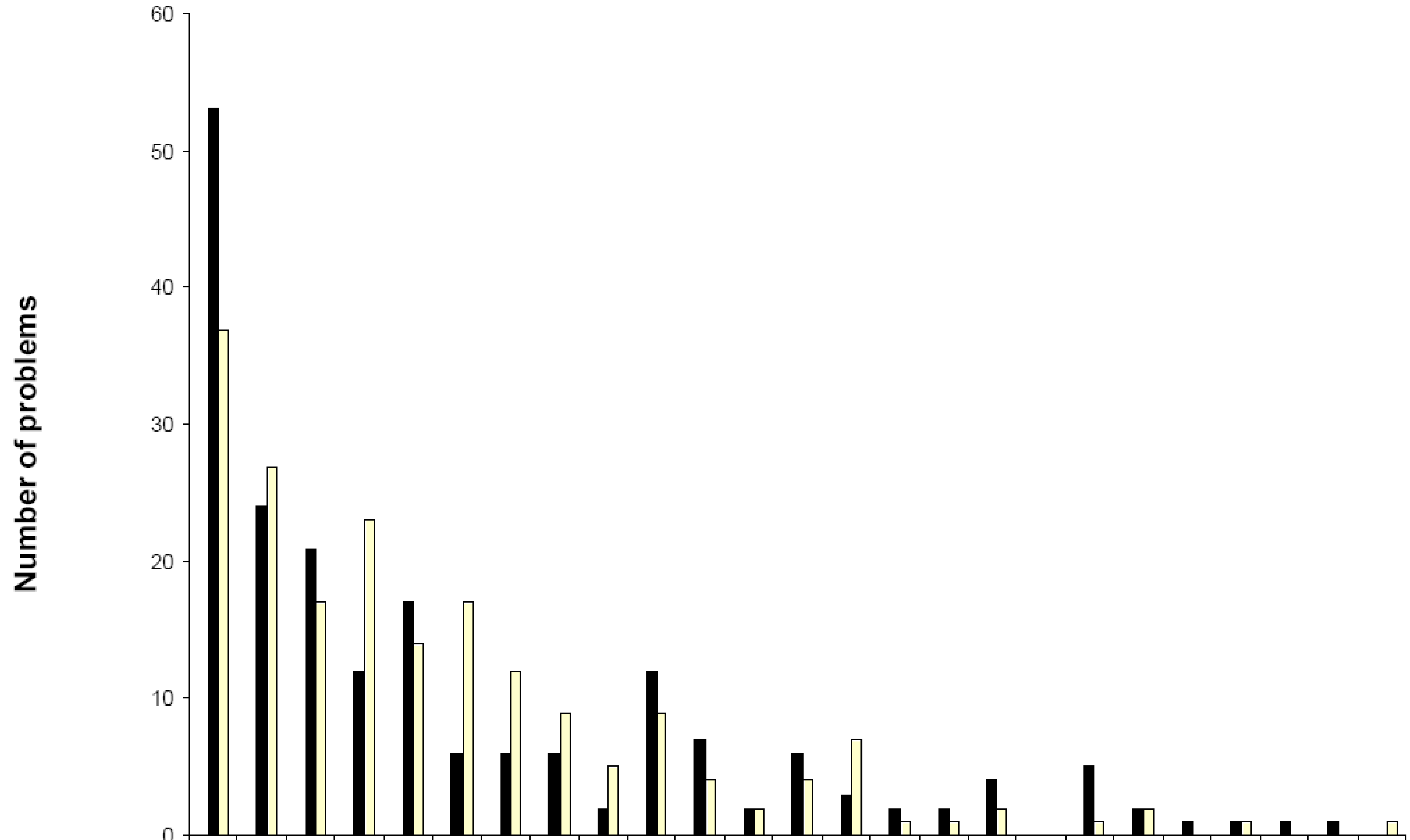
- We investigate the impact of randomness on the robustness of the cycle-based tabu search and path relinking procedures.
- The solutions produced in different runs are generally different.
- Relatively large differences usually signal a significant impact of the random factors and a somewhat less robust method.

Influence of Sampling

- We run both the path relinking and the cycle-based tabu search meta-heuristics 3 times on each of the problems used for experimentation.
- For each problem, the relative gap between solutions produced over the 3 runs is computed as the percentage gap between the worst and the best solution relative to the worst solution.

Influence of Sampling

- This figure displays the relative gap distributions for the two procedures.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Range of variation	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	5.75	5	5.25	5.5	5.75	6	6.25
■ Path relinking	53	24	21	12	17	6	6	6	2	12	7	2	6	3	2	2	4	0	5	2	1	1	1	1	0
□ Tabu cycle	37	27	17	23	14	17	12	9	5	9	4	2	4	7	1	1	2	0	1	2	0	1	0	0	1

Influence of Sampling

- Relative gaps are grouped within intervals of 0.25 points of percentage.
- In each interval, the black and white columns are associated to the path relinking and tabu search, respectively, and represent the number of problems with results in the specic range.

Influence of Sampling

- Two conclusions emerge from the figure
  - First, the relative gaps for both meta-heuristics are tight and the methods appear robust.
    - In fact 74% of the problems solved with both meta-heuristics present a relative gap less than or equal to 2%.
  - Second, path relinking is even less influenced by sampling and is more robust.
- This follows from the fact that sampling is replaced during the path relinking phase by a deterministic way to select the candidate links in C.

3.4 Result Analysis

Result Analysis

- To evaluate the performance of the path relinking algorithm proposed in this paper, we compare its output to
  - the results of the cycle-based tabu search (Ghamlouche, Crainic, and Gendreau 2001) and
  - to the optimal solutions obtained using the branch-and-bound algorithm of CPLEX 6.5
- The same two data sets of networks used by Ghamlouche, Crainic, and Gendreau (2001) were also used to test the path relinking method.

Result Analysis

PROB	OPT	TC	PR	AV. TC	AV. PR	IMPROV
25,100,10,V,L	14712	14712	14712	14769.33	14712	-0.39%
	(0.36)	(19.48)	(12.54)	(19.48)	(12.33)	
25,100,10,F,L	14941	14941	14941	14941	15081.33	0.94%
	(53.64)	(22.33)	(14.13)	(22.32)	(14.14)	
25,100,10,F,T	49899	50529	49899	50619.67	50154	-0.92%
	(40.58)	(33.84)	(24.06)	(33.12)	(23.52)	
25,100,30,V,T	365272	365385	365385	365385	365385	0.00%
	(16.62)	(141.51)	(101.44)	(142.37)	(103.40)	
25,100,30,F,L	37055	37515	37654	38672.33	37704	-2.50%
	(1727.96)	(112.48)	(75.15)	(112.48)	(73.91)	
25,100,30,F,T	85530	87325	86428	88095.33	86492.33	-1.82%
	(534.18)	(132.92)	(96.98)	(130.95)	(97.45)	
20,230,40,V,L	423848	430628	424385	430768.33	424789.67	-1.39%
	(10.43)	(214.12)	(148.82)	(217.16)	(145.90)	
20,230,40,V,T	371475	372522	371811	372888	372162.33	-0.19%
	(52.37)	(241.44)	(156.92)	(239.58)	(160.36)	
20,230,40,F,T	643036	652775	645548	657803.67	646378.67	-1.74%
	(671.76)	(259.53)	(172.16)	(259.53)	(163.47)	
20,300,40,V,L	429398	432007	429398	432168.33	429523.67	-0.61%
	(3.76)	(304.60)	(224.91)	(307.50)	(218.45)	
20,300,40,F,L	586077	602180	590427	604072.67	594590.67	-1.57%
	(145.55)	(335.59)	(228.33)	(341.06)	(234.53)	

Result Analysis

20,300,40,V,T	464509	466115	464509	466546.33	464822.67	-0.37%
	(83.88)	(378.70)	(247.89)	(378.42)	(244.04)	
20,300,40,F,T	604198	615426	609990	616859.67	609995.33	-1.11%
	(59.88)	(349.48)	(214.43)	(354.30)	(219.52)	
20,230,200,V,L	94386	100001	100404	101481.33	101469.33	-0.01%
	(t)	(2585.94)	(2494.92)	(2596.49)	(2502.82)	
20,230,200,F,L	141737.40	148066	147988	148975	151352	1.60%
	(t)	(3141.91)	(2878.27)	(3141.91)	(2760.73)	
20,230,200,V,T	97914	106868	104689	107589.33	105598.67	-1.85%
	(t)	(2729.57)	(2210.86)	(2729.57)	(2304.53)	
20,230,200,F,T	137271	147212	147554	147868	148044.33	0.12%
	(t)	(3634.10)	(3385.75)	(3656.85)	(3505.63)	
20,300,200,V,L	74972.40	81367	78184	82186.67	79095.33	-3.76%
	(t)	(4085.89)	(3565.98)	(4007.80)	(3649.88)	
20,300,200,F,L	117306	122262	123484	123247.67	124924.67	1.36%
	(t)	(4210.14)	(4012.64)	(4336.93)	(4173.73)	
20,300,200,V,T	74991	80344	78866.80	80724	79212.27	-1.87%
	(t)	(4203.84)	(3924.21)	(4140.07)	(3714.29)	
20,300,200,F,T	108252	113947	113584	115342	114632.33	-0.62%
	(t)	(4854.61)	(3857.14)	(4643.75)	(3486.90)	

Result Analysis

PROB	OPT	TC	PR	AV.TC	AV.PR	IMPROV
100,400,10,V,L	28423	28786	28485	28836.67	28529	-1.07%
	(84.81)	(252.15)	(89.21)	(254.99)	(83.97)	
100,400,10,F,L	24436	24022	24022	24022	24022	0.00%
	(t)	(196.50)	(82.86)	(201.38)	(79.20)	
100,400,10,F,T	66364	67184	65278	67376.33	65539.33	-2.73%
	(t)	(451.28)	(209.93)	(474.44)	(177.84)	
100,400,30,V,T	385544	385508	384926	385512.67	385181.67	-0.09%
	(t)	(1199.88)	(492.76)	(1220.42)	(496.29)	
100,400,30,F,L	50496	51831	51325	52176.33	51875.67	-0.58%
	(t)	(717.42)	(314.97)	(735.50)	(328.19)	
100,400,30,F,T	141278	147193	141359	147478	143403.67	-2.76%
	(t)	(1300.79)	(480.86)	(1300.79)	(522.44)	
30,520,100,V,L	53966	56603	54904	56722	55043.67	-2.96%
	(t)	(2260.69)	(1194.12)	(2260.69)	(1199.35)	
30,520,100,F,L	95294	103657	102054	105276.33	104222.33	-1.00%
	(t)	(2683.74)	(1459.99)	(2623.55)	(1508.39)	
30,520,100,V,T	52085	54454	53017	54537	53212	-2.43%
	(t)	(2715.76)	(1513.66)	(2711.55)	(1402.92)	
30,520,100,F,T	98357	105130	106130	107885.33	107575	-0.29%
	(t)	(2891.93)	(1522.68)	(2877.52)	(1478.54)	
30,700,100,V,L	47603	50041	48723	50098.33	48894.33	-2.40%
	(1736.05)	(2959.51)	(1860.61)	(2959.51)	(1815.84)	

Result Analysis

30,700,100,F,L	60525	64581	63091	64802.67	63436.33	-2.11%
	(t)	(3181.72)	(1837.50)	(3181.72)	(1754.95)	
30,700,100,V,T	45944.50	48176	47209	48249.67	47288.33	-1.99%
	(t)	(3745.82)	(1894.08)	(3688.35)	(1832.10)	
30,700,100,F,T	55709	57628	56575.50	58111.33	56808.83	-2.24%
	(t)	(3547.09)	(1706.06)	(3547.09)	(1713.94)	
30,520,400,V,L	112997.50	122673	119416	123277.33	119624	-2.96%
	(t)	(55771.20)	(27477.40)	(55720.50)	(33716.57)	
30,520,400,F,L	X	164140	163112	165458	163377	-1.26%
	(t)	(40070.40)	(36669.30)	(41567.30)	(36600.43)	
30,520,400,V,T	X	122655	120170	123210.33	120764.33	-1.99%
	(t)	(4678.80)	(23089.10)	(21437.40)	(25705.77)	
30,520,400,F,T	X	169508	163675	170301.33	164921.33	-3.16%
	(t)	(49886.80)	(52173.20)	(51400.35)	(44862.23)	
30,700,400,V,L	X	107727	105116	108459.33	105403.33	-2.82%
	(t)	(38856.90)	(22314.80)	(36282.25)	(19742.33)	
30,700,400,F,L	X	150256	145026	150909	147887.33	-2.00%
	(t)	(68214.20)	(75664.90)	(73899.05)	(66489.40)	
30,700,400,V,T	X	101749	101212	103112.33	102119.33	-0.96%
	(t)	(51764.10)	(24288.90)	(49239.65)	(28664.70)	
30,700,400,F,T	X	144852	141013	146705	141446.67	-3.58%
	(t)	(79053.40)	(44936.40)	(79053.40)	(49824.83)	

Result Analysis

- **PROB column**
  - Problems are identified in the first column by the number of nodes, arcs, and commodities, as well as two letters summarizing the fixed cost and capacity information.
- **The OPT column**
  - corresponds to the solution of the branch-and-bound algorithm solved using CPLEX 6.5 (1999) on the same computer.
  - A limit of 10 hours was imposed.
  - An X indicates that the procedure has failed to produce a feasible solution within this time limit
  - A t indicates that the procedure stopped due to a time limit condition.

Result Analysis

- To facilitate comparisons in terms of solution quality and computation time, the cycle-based tabu search and the path relinking algorithm were run 3 times for each problem.
- **TC and AV.TC columns**
  - hold respectively the best and the average solution of the cycle-based tabu search,
- **PR and AV.PR columns**
  - display the best and the average solution found by the path relinking approach.

Result Analysis

- The figures in parentheses in the first three columns represent total computation times in CPU seconds.
- The corresponding figures in columns AV.TC and AV.PR represent the average total computation times in CPU seconds.
- **IMPROV column**
  - displays the percentage of improvement of the average path relinking solution relative to that of the cycle-based tabu search (i.e AV.TC).

Result Analysis

- **First Observation**
  - The path relinking procedure displays a satisfactory behavior.
  - Comparing columns TC and PR, one observes that path relinking obtains the optimal solutions for 5 problems compared to 2 for TC and improves the best solutions found by cycle-based tabu search for 37 problems out of 41.
  - The same behavior is also observed when average solution values are compared:
    - The overall performance is clearly superior for the path relinking approach.
    - The performance of path relinking also appears consistent over the range of problems, including relatively large ones in terms of number of arcs (e.g., 700) and commodities (e.g., 400).

Result Analysis

- When branch-and-bound failed to identify a feasible solution within 10 hours CPU time, the proposed algorithm has identified best-known solutions for these problems with a reasonable computational effort.

Result Analysis

- **Second Observation**
  - The second observation concerns the computing effort.
  - Indeed, despite of the fact that the path relinking procedure is stopped on the same criterion as the cycle-based tabu search, 400 iterations,
  - The tables indicate that the computational time for the path relinking procedure is often significantly shorter.
  - This follows principally from the reduced number of resolutions of the multicommodity capacitated network flow problem and the fact that the dimension of the neighbourhood used during the path relinking phase is smaller.

Result Analysis

- Distribution of relative gaps

	PSet	X	OPT	IMP	(0-2%]	(2-4%]	(4-6%]	(6-8%]	(8-10%]	>10%
TC	C	7	2	2	11	4	9	4	4	-
PR	C	7	5	3	14	2	7	5	-	-
AV.TC	C	7	1	2	9	5	9	5	4	1
AV.PR	C	7	1	3	16	4	5	5	2	-

Result Analysis

- The table displays the distribution of the optimality gaps relative to CPLEX branch-and-bound for both the cycle-based tabu search and the path relinking procedures.
- Values are calculated for both the best and average solutions.

Result Analysis

- **Columns**
  - Columns PSet identifies the problems set,
  - the X column indicates the number of problems where branch-and-bound did not find a feasible solution (in 10 hours),
  - the opt column indicates the number of optimal solutions found by the meta-heuristics, and
  - the **imp column** displays the number of problems where the meta-heuristic solution is better than the one found by branch-and-bound after 10 hours of computation.
  - The next six columns correspond to the indicated gap intervals.

Result Analysis

- The results displayed in the table show that path relinking obtains a larger number of optimal solutions than the cycle-based tabu search.
- Moreover, the distribution of the gaps indicates that path relinking solutions are closer to the optimum than those of the cycle-based tabu search, in terms of both best and average solution values.
- On average, for problems in sets C, path relinking obtains gaps of 2.32%, respectively,

The End