Absolute Capacity Determination

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Reference:

Techniques for absolute capacity determination in railways,

By R.L. Burdett, E. Kozan, Transportation Research Part B 40 (2006) 616–632.

1. Terminology

- **Capacity**: is the maximum number of trains that can traverse the entire railway or certain critical (bottleneck) section(s) in a given duration of time.

- **Absolute capacity**: is a theoretical value (overestimation) of capacity that is realised when only critical section(s) are saturated (i.e., continuously occupied).

- **Actual Capacity:** is the amount that occurs when interference delays are incorporated on the critical section(s).

- Capacity may be different for each proportional mix of trains (train types).

- **A Railway Corridor:** is generally a single serial line (track) that is made up of one or more sections of specific length that are sequentially traversed.

- **A Section:** is the length of rail between two locations.

- **A Location**: is any fixed point of reference along the rail such as the end of a section of rail, the start or end of a crossing loop, crossover or junction, or the position of a signal device.

- Traffic movement on each line (track) can potentially be in one or both directions, that is, traffic is **uni-directional** or **bi-directional**.

- One train only however may generally occupy a section of rail at any given time (section occupation condition (SOC)) for safety reasons.

- Parallel tracks with the same topography have also been referred to as corridors.

- **A Railway System:** is a single corridor or a collection of separate and or interrelated corridors.

- **Non-serial Track**: is a railway line that branches outwards and hence inwards in the opposite direction.

- Hence a network of interrelated corridors is a non-serial track.

- Train types that use the railway are electric or diesel powered and may be used for moving passengers or freight.

- **Transit Time**: is the total journey time for a train.

- **Sectional Running Time (SRT)**: is the time that a train takes to traverse a given section

- **Section Occupation Time (SOT)**: is the total time spent by a train on a section, which may include pre planned dwell time and scheduled/unscheduled delays.

- **A Trains Path**: is a passage through the system from one input–output (IO) point to another.

- **A Fleet**: A group of trains that travel in the same direction (sequentially) on the railway without being disturbed by traffic in the other direction.

- **Proportional Distribution:** is the percentage of total traffic that consists of each train type

- **Directional Distribution**: is the percentages for travel in each direction

- For every distinct proportional and directional distribution (i.e., percentage train mix), the capacity of the corridor will be potentially different.

2. Train Mixes

I	Train type index. The set of trains is $I = \{1, 2, . . .\}$
T	Time period duration.
l, k	Location index.
$\rightarrow$	This symbol signifies uni-directional travel between two locations, for example, $l \rightarrow k$.
$-$	This is used to signify a non-directional or generic link as occurs when defining a corridor, for example $l - k$.
Φ	The set of input/output (IO) points which are locations where trains may enter and leave the network. Any location may be defined as an IO point.
N	The set of valid sections in the railway network.
C	The set of valid corridors in the railway network. Note: $C = \{(l,k) : l,k \in \Phi\}$
p_{l-k}	Set of locations traversed on corridor l – k.
S_{l-k}	Set of sections traversed on corridor l – k.

CS_{l-k}	The critical section of the corridor that lies between l and k (i.e., corridor *l*–*k*).
$C_{abs}^{(l,k)}$	The absolute capacity of section $l - k$.
C_{abs}^{l-k}	The absolute capacity of corridor l – k. $C_{abs}^{l-k} = \min_{(l,k)\in S_{l-k}}(C_{abs}^{(l,k)})$
η_i^{l-k}	Proportion of train type *i* on corridor *l* - *k* where $(l,k) \in C$
$\mu_i^{l\to k}$	The proportion of train type *i* in the direction of *k* on corridor *l* - *k*. $\mu_i^{l\to k} + \mu_i^{k\to l} = 1$, Note that $\mu_i^{l\to k} = 0$ if movement from *l* to *k* is restricted (for example, as occurs in uni-directional systems).
$SRT_i^{l\to k}$	The sectional running time of train i when travelling on section (*l*, *k*) in the direction of location *k*.
$x_i^{l\to k}$	Number of trains of type *i* that travel from location *l* to *k*.
X_i^{l-k}	Number of trains of type *i* that use corridor *l* - *k*. $X_i^{l-k} = x_i^{l\to k} + x_i^{k\to l}$.

For a line with boundaries (*l*, *k*) and particular proportional and directional distributions, the proportion of traffic in each direction, respectively, is determined from the balance equation:

$$\sum_i (\eta_i^{l-k} (\mu_i^{l \to k} + \mu_i^{k \to l})) = 1$$

$$\sum_i (\eta_i^{l-k} \mu_i^{l \to k}) + \sum_i (\eta_i^{l-k} \mu_i^{k \to l}) = 1$$

Coridor A-D:

Proportional and directional distributions:

	Train Type			
	1	2	3	4
$x_i^{l \to k}$	22	8	5	1
$x_i^{k \to l}$	22	9	2	2
$SRT_i^{l \to k} / SRT_i^{k \to l}$	8	10	12	14
η_i^{l-k}	0.62	0.24	0.10	0.04
$\mu_i^{l \to k}$	0.50	0.47	0.71	0.33
$\mu_i^{k \to l}$	0.50	0.53	0.29	0.67

$$\sum_i (\eta_i^{l-k} (\mu_i^{l \to k} + \mu_i^{k \to l})) = 1$$

(0.62 (0.50+0.50)) + (0.24 (0.47+0.53)) + (0.10 (0.71+0.37)) + (0.04 (0.33+0.67)) = 1

$$\sum_i (\eta_i^{l-k} \mu_i^{l \to k}) + \sum_i (\eta_i^{l-k} \mu_i^{k \to l}) = 1$$

((0.62 * 0.50) + (0.24 * 0.47) + (0.10 * 0.71) + (0.04 * 0.33)) + ((0.62 * 0.50) + (0.24 * 0.53) + (0.10 * 0.29) + (0.04 * 0.67)) = 1

- A section of rail that is fully utilised is referred to as saturated, and this occurs when it is fully occupied.
- Under these conditions and for feasibility, the time-period must be greater than or equal to the section occupancy time (i.e., number of trains in each direction multiplied by their respective SRT in that direction).

$$\sum_i (x_i^{l\to k} SRT_i^{l\to k} + x_i^{k\to l} SRT_i^{k\to l}) \le T$$

- If the number of type i trains in each direction, respectively, is:

$$x_i^{l\to k} = (\eta_i^{l-k} \mu_i^{l\to k}) C_{abs}^{(l-k)}$$

$$x_i^{k\to l} = (\eta_i^{l-k} \mu_i^{k\to l}) C_{abs}^{(l-k)}$$

- Then the substitution of these terms and the re-arrangement of the equation gives the following expression for absolute capacity:

$$C_{abs}^{(l-k)} = \frac{T}{\sum_{\forall i} \eta_i^{l-k} (\mu_i^{l\to k} SRT_i^{l\to k} + \mu_i^{k\to l} SRT_i^{k\to l})}$$

- Free capacity (i.e., $F_i^{l\to k}$) in terms of a particular train type and for a particular direction can then be determined by:

$$F_i^{l\to k} = \frac{T - \sum_{\forall i'} (x_{i'}^{l\to k} SRT_{i'}^{l\to k} + x_{i'}^{k\to l} SRT_{i'}^{k\to l})}{SRT_i^{l\to k}}$$

- Absolute utilisation levels (not incorporating congestion and interaction effects) can also be computed by:

$$U_{abs}^{(l,k)} = \frac{\sum_{\forall i} (x_i^{l \to k} SRT_i^{l \to k} + x_i^{k \to l} SRT_i^{k \to l})}{T}$$

- Consequently, the percentage capacity that is theoretically free on section (l, k) can be calculated by:

$$1 - U_{abs}^{(l,k)}$$

Coridor A-D:

Proportional and directional distributions:

	Train Type			
	1	2	3	4
$x_i^{l\to k}$	22	8	5	1
$x_i^{k\to l}$	22	9	2	2
$SRT_i^{l\to k} / SRT_i^{k\to l}$	8	10	12	14
η_i^{l-k}	0.62	0.24	0.10	0.04
$\mu_i^{l\to k}$	0.50	0.47	0.71	0.33
$\mu_i^{k\to l}$	0.50	0.53	0.29	0.67
$\eta_i^{l-k} (\mu_i^{l\to k} \; SRT_i^{l\to k} + \mu_i^{k\to l} \; SRT_i^{k\to l})$	4.96	2.4	1.2	0.56
$(x_{i'}^{l\to k} SRT_{i'}^{l\to k} + x_{i'}^{k\to l} SRT_{i'}^{k\to l})$	352	170	84	42

Coridor A-D:

Absolute capacity:

$$C_{abs}^{(l-k)} = \frac{1440}{4.96 + 2.4 + 1.2 + 0.56} = \frac{1440}{9.12} = 157.89$$

Free capacity for train type 1:

$$F_1^{l \to k} = \frac{1440 - 648}{8} = 99$$

Utilization rate:

$$U_{abs}^{(l,k)} = \frac{648}{1440} = 0.45$$

3. Signals and Reference Locations

- In previous equation is assumed that crossing loops occur at the section boundaries thus allowing traffic to passeach other.

- In practice one or both section boundaries may have signal devices.

- Signal devices allow throughput in one direction to be increased, because it allows additional trains to safely occupy a section.

- However, utilisation of capacity can be lost when consecutive trains travel in opposite directions as occurs when bi-directional flow is allowed.

- This is because trains cannot pass each other at the section boundary. They can only pass each other at the nearest crossing loop.

- Consequently **enforced headways** on that section must be incurred and these may be of considerable duration in some railways.

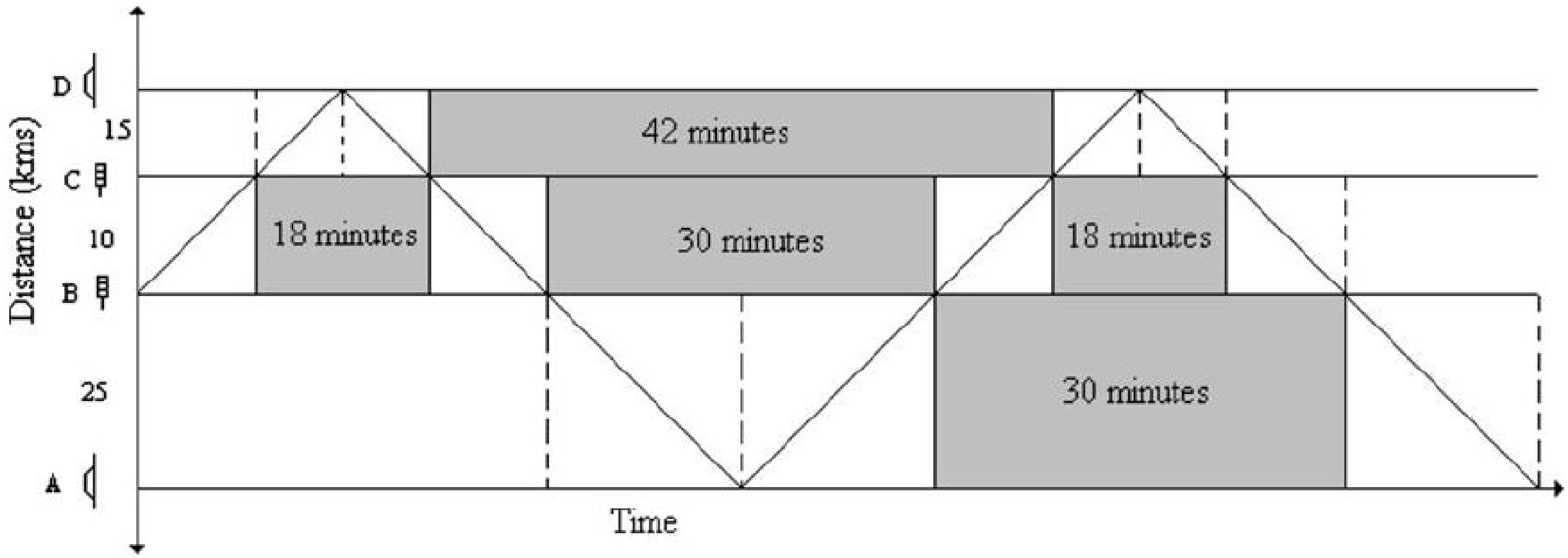

Fig. 1. Enforced headways on a corridor containing signals.

- In particular, the absolute capacity will be greatest (i.e., the upper bound) when there are two fleets, one in each direction.

- Absolute capacity will be smallest when there are many fleets of one train (i.e., trains commonly alternate in the sequence in each direction).

- The lower bound value is determined by noting that the worst sequence containing alternating trains is of length 2z where z = min(#up, #dn).
- The remaining trains form a fleet. For example, the two possible resulting sequences have the following format: (d u d u d . . . d d . . . d) or (u d u d u . . . u u . . . u) where u and d refer to an up or down train, respectively.
- Consequently there will be at most 2z enforced headways that must be included.

- The upper bound is determined by choosing the two fleets (i.e., all up and then all down, or all down and then all up) that has the smallest enforced headway between them.

- $h^{(l,k)}$: is the enforced headway time on section (l, k) between traffic moving in opposite directions when there is no crossing loop at location k.

- $h^{(k,l)}$: is the enforced headway time on section (l, k) between traffic moving in opposite directions when there is no crossing loop at location l.

- The enforced headway between an $l \rightarrow k$ and a k→l train is $h^{(l,k)}$, and the headway between a $k \rightarrow l$ and an $l \rightarrow k$ train is $h^{(k,l)}$.

- $h^{(l,k)}$ and $h^{(k,l)}$ are zero if location l and k, respectively, are crossing loops.

- $\overline{SRT}^{l \to k}$ & $\overline{SRT}^{l - k}$: is defined as weighted average travelling times in a direction and in a section.

- The enforced headways are calculated as the sum of two weighted average travelling times to and from the nearest crossing loop location *l'*:

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$$h^{(k,l)} = \overline{SRT}^{l \to l'} + \overline{SRT}^{l' \to l}$$

- The weighted average travelling times to and from the nearest crossing loop location *l'* may be different depending on the particular proportional and directional distributions.

- The weighted average sectional running time in a particular direction is calculated as follows:

$$\overline{SRT}^{l\to k} = \left(\frac{1}{\sum_{\forall i} (\eta_i^{l-k} \mu_i^{l\to k})} \right) \sum_{\forall i} (\eta_i^{l-k} \mu_i^{l\to k} SRT_i^{l\to k})$$

- The weighted average sectional running time in a section is calculated as follows:

$$\overline{SRT}^{l-k} = \sum_{\forall i} (\eta_i^{l-k} \mu_i^{l\to k}) \overline{SRT}^{l\to k} + \sum_{\forall i} (\eta_i^{l-k} \mu_i^{k\to l}) \overline{SRT}^{k\to l}$$

- To derive equations for the lower and upper bounds on the absolute section capacity we note that for feasibility, the following inequalities (which represent section saturation conditions) must be satisfied for the lower and upper bound cases, respectively:

Lower Bound:

$$\sum_i (x_i^{l\to k} SRT_i^{l\to k} + x_i^{k\to l} SRT_i^{k\to l}) + (h^{(l,k)} + h^{k,l)})z \le T$$

Upper Bound:

$$\sum_i (x_i^{l\to k} SRT_i^{l\to k} + x_i^{k\to l} SRT_i^{k\to l}) + \min(h^{(l,k)} + h^{k,l)}) \le T$$

- where z is the number of trains travelling in the direction with the least number trains and is computed as

if $\sum_{\forall i} (\eta_i \mu_i^{l\to k}) \le \sum_{\forall i} (\eta_i \mu_i^{k\to l})$ then $z = \sum_{\forall i} (x_i^{l\to k})$

Otherwise $z = \sum_{\forall i} (x_i^{k\to l})$

- We know that:

$$x_i^{l\to k} = \eta_i^{l-k}\mu_i^{l\to k}C_{abs}^{(l-k)}$$

$$x_i^{k\to l} = \eta_i^{l-k}\mu_i^{k\to l}C_{abs}^{(l-k)}$$

- After substitution of these values the resulting intermediate equations are as follows:

Lower Bound:

$$C_{abs}^{(l,k)}\sum_i(\eta_i^{l-k}\mu_i^{l\to k}SRT_i^{l\to k} + \eta_i^{l-k}\mu_i^{k\to l}SRT_i^{k\to l}) + (h^{(l,k)} + h^{k,l)})z \le T$$

Upper Bound:

$$C_{abs}^{(l,k)}\sum_i(\eta_i^{l-k}\mu_i^{l\to k}SRT_i^{l\to k} + \eta_i^{l-k}\mu_i^{k\to l}SRT_i^{k\to l}) + \min(h^{(l,k)}, h^{k,l)}) \le T$$

- The summation term is actually the weighted average sectional running time $\overline{SRT}^{l-k}$ which can be substituted.
- Simple rearrangement in terms of $C_{abs}^{(l,k)}$ and the replacement of the inequality with an equality sign gives the following equations (LB and UB replace $C_{abs}^{(l,k)}$ to avoid confusion):

Lower Bound:

$$LB = \frac{T - z(h^{(l,k)} + h^{(k,l)})}{\overline{SRT}^{l-k}}$$

Upper Bound:

$$UB = \frac{T - \min(h^{(l,k)}, h^{k,l)})}{\overline{SRT}^{l-k}}$$

Capacity of section B-C:

		Train Type				Sum
		1	2	3	4	
1	$x_i^{l \to k}$	22	8	5	1	36
2	$x_i^{k \to l}$	22	9	2	2	35
3	$SRT_i^{l \to k} / SRT_i^{k \to l}$ (A-B)	6	8	9	11	
4	$SRT_i^{l \to k} / SRT_i^{k \to l}$ (B-C)	8	10	12	14	
5	$SRT_i^{l \to k} / SRT_i^{k \to l}$ (C-D)	9	11	13	15	
6	η_i^{l-k}	0.62	0.24	0.10	0.04	
7	$\mu_i^{l \to k}$	0.50	0.47	0.71	0.33	
8	$\mu_i^{k \to l}$	0.50	0.53	0.29	0.67	
9	$\eta_i^{l-k} \ \mu_i^{B \to A}$	0.31	0.13	0.03	0.03	0.49
10	$\eta_i^{l-k} \ \mu_i^{A \to B}$	0.31	0.11	0.07	0.01	0.51

11	$\eta_i^{l-k}\ \mu_i^{B\to A}\ SRT_i^{B\to A}$	1.86	0.90	0.64	0.15	3.55
12	$\eta_i^{l-k}\ \mu_i^{A\to B}\ SRT_i^{A\to B}$	1.86	1.02	0.26	0.29	3.43
13	$\eta_i^{l-k}\ \mu_i^{C\to D}\ SRT_i^{C\to D}$	2.79	1.40	0.38	0.40	4.97
14	$\eta_i^{l-k}\ \mu_i^{D\to C}\ SRT_i^{D\to C}$	2.79	1.24	0.92	0.20	5.15
15	$\eta_i^{l-k}\ \mu_i^{B\to C}\ SRT_i^{B\to C}$	2.48	1.13	0.85	0.18	4.64
16	$\eta_i^{l-k}\ \mu_i^{C\to B}\ SRT_i^{C\to B}$	2.48	1.27	0.35	0.38	4.48

- Calculating enforced headways:

$$\overline{SRT}^{A\to B} = \frac{3.43}{0.51} = 6.72$$

$$\overline{SRT}^{A\to B} = \frac{3.55}{0.49} = 7.24$$

$$h^{(B,A)} = \overline{SRT}^{B\to A} + \overline{SRT}^{A\to B} = 6.72 + 7.24 = 13.96$$

$$\overline{SRT}^{C\to D} = \frac{4.49}{0.51} = 8.80$$

$$\overline{SRT}^{D\to A} = \frac{5.15}{0.49} = 10.51$$

$$h^{(C,D)} = \overline{SRT}^{C\to D} + \overline{SRT}^{D\to C} = 8.80 + 10.51 = 19.31$$

- Lower bound and upper bound capacity:

$$\overline{SRT}^{B \to C} = \frac{4.64}{0.51} = 9.10$$

$$\overline{SRT}^{C \to B} = \frac{4.48}{0.49} = 9.14$$

$$\overline{SRT}^{B-C} = 0.51 * 9.10 + 0.49 * 9.14 = 9.12$$

$$LB = \frac{T - z(h^{(l,k)} + h^{k,l)})}{\overline{SRT}^{l-k}} = \frac{1440 - 35(13.96 + 19.31)}{9.12} = 30.21$$

$$UB = \frac{T - \min(h^{(l,k)}, h^{k,l)})}{\overline{SRT}^{l-k}} = \frac{1440 - 13.96}{9.12} = 156.36$$

Absolute Capacity Determination