

بسم الله الرحمن الرحيم

## برنامه ریزی حمل و نقل ریلی

فصل ۸: مدلسازی مساله بلاکینگ واگنها بر  
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# Modeling RBP as BDP



# Modeling RBP as BDP

- In this chapter, we show that the Railroad Blocking Problem (RBP) can be modeled as a Budget Design Problem (BDP)
- We present an MIP formulation (BLOCK) obtained by adding them to PATH for BDP.

# Definition of RBP

- We may define the RBP as:
  - Minimize the costs of delivering all commodities
  - by deciding which inter-terminal blocks to build
  - and specifying the assignment of commodities to these blocks,
  - while observing limits on the number and aggregate volume of the blocks assembled at each terminal
  - and limits on the number of blocks used to deliver a commodity.

# Modeling RBP as BDP

- The RBP can be modeled as a network design problem where
  - the nodes represent the railroad terminals and
  - the arcs represent potential blocks.
- As with the general BDP, the RBP seeks to minimize the flow costs of delivering all commodities.

# Maximum number of blocks

- The RBP constraint on the maximum number of blocks originating at a terminal has the same form as the node-budget constraints which we included in PATH (1.3) formulations for the BDP.
- In RBP, the fixed cost  $e_a = 1$  for all arcs.

# Balance equations

- The balance equations (1.2a) on the paths in PATH can be used to model the identical balance requirements for BLOCK.

## Different service constraints

- However, unlike BDP, the maximum number of arcs (blocks) which may be used in a commodity's path is restricted by a priority constraint.
- The possibility of having different service constraints for some of the traffic from an OD pair requires adding a priority class to the description of each commodity.
- Now commodities are identified by origin, destination, and a maximum number of handlings.



# Arc capacity

- Since blocks are assumed to be uncapacitated, the coefficients  $U_a$  in PATH are replaced by the maximum possible flow for arc  $a$ .

# Node Capacity

- However, where the node flows are assumed uncapacitated in BDP, in RBP the nodes (terminals) have a flow volume constraint to model the limit on the total number of cars which can be classified.

# Overall budget

- Finally, there is no overall budget for the blocking problem, so constraint (1.3d) from PATH may be omitted.

# Parameters

$G = (N, A)$  is the graph with node set  $N$  and candidate arc (block) set  $A$ .

$K$  is the set of all commodities  $k$  designated by an origin-destination pair of nodes and the number of intermediate handlings allowed.

$v^k$  is the volume of commodity  $k$  (in consistent units).

$orig(k)$  is the origin node for commodity  $k$ .  $orig(a)$  is the origin of arc  $a$ .

$dest(k)$  is the destination node for commodity  $k$ .  $dest(a)$  is the destination of arc  $a$ .

$Q(k)$  is the set of legal paths for commodity  $k$ .

# Parameters

$c_a \geq 0$  is the per unit cost of flow on arc  $a$  (assumed equal for all commodities).

$u_a$  is the capacity of arc  $a$ .

$B(i)$  is the number of blocks which may originate at node  $i$ .

$V(i)$  is the volume which may be classified at node  $i$ .

$PC_q^k$  path cost for flowing one unit of commodity  $k$  on path  $q$ .

# Decision variables

$f_q^k$  proportion of commodity  $k$  on path  $q$ ,  $\forall q \in Q(k), k \in K$ .

$y_a$  is the binary design variable for including arc (block)  $a$ .

$$y_a = \begin{cases} 1 & \text{if arc } a \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

# BLOCK Formulation

$$\min \sum_{k \in K} \sum_{q \in Q(k)} PC_q^k v^k f_q^k \quad (4.1a)$$

s.t.

$$\sum_{k \in K} \sum_{q \in Q(k)} v^k f_q^k \delta_a^q - u_a y_a \leq 0 \quad \forall a \in A \quad (4.1b)$$

$$\sum_{q \in Q(k)} f_q^k = 1 \quad \forall k \in K \quad (4.1c)$$

$$\sum_{\substack{a \in A \\ \text{orig}(a)=i}} y_a \leq B(i) \quad \forall i \in N \quad (4.1d)$$

$$\sum_{k \in K} \sum_{q \in Q(k)} \sum_{\substack{a \in A \\ \text{orig}(a)=i}} v^k f_q^k \delta_a^q \leq V(i) \quad \forall i \in N \quad (4.1e)$$

$$f_q^k \geq 0 \quad \forall q \in Q(k), k \in K$$

$$y_a \in \{0, 1\} \quad \forall a \in A$$

# Constraints

- Constraints (4.1b): ensure that flow is only allowed on arcs (blocks) included in the network
- Constraints (4.1c): ensure all of each commodity is delivered
- Constraints (4.1d): the limit on the number of blocks which may be built at each node is modeled by the node-budget constraints
- Constraints (4.1e): the volume of cars which may be classified at each terminal is modeled
- The balance equations and constraints on the maximum number of handlings are included on legal blocking paths for commodity  $k$ .



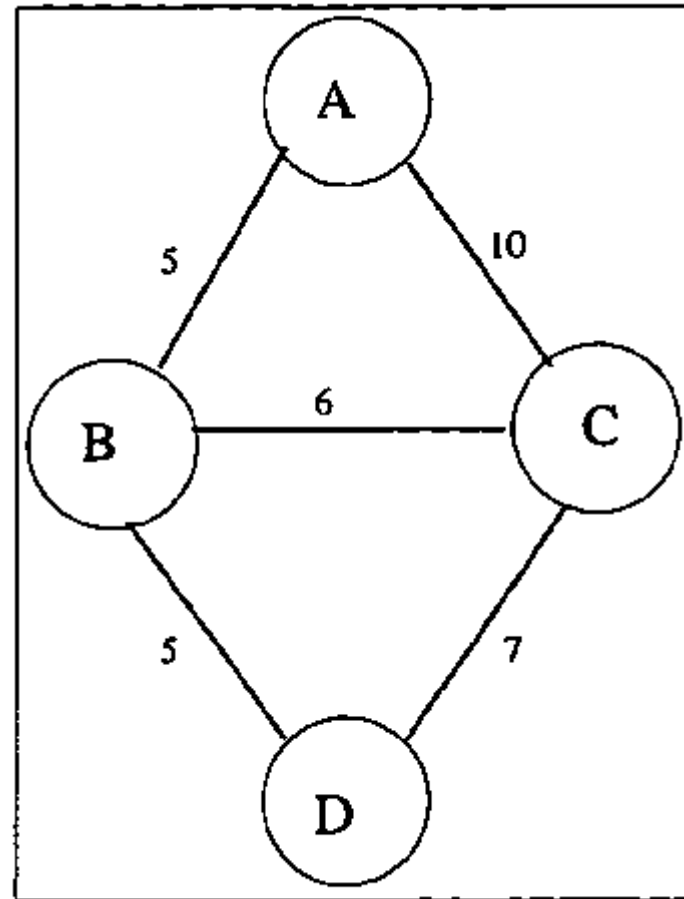
# Path cost

- Path costs could be included are:
  - Costs for car-miles (miles that a car following this blocking path will travel)
  - Costs for car-hours (hours of delay for classifications incurred along this blocking path)
  - Labor and equipment costs per car might be different depending on the links and blocks for a commodity

# Routing vs. Blocking Path

- We define  *routings*  to be paths through the physical network.
- It is convenient to describe a  *routing*  by the sequence of terminals visited.
- The terms  *blocking path*  or  *commodity blocking assignment*  on the other hand describe the path through the blocking network.
- If the blocking path is also identified by the sequence of terminals, a blocking path for a railcar will be a subsequence of the routing that it followed since blocking will be done at a subset of the terminals visited.

# An Example

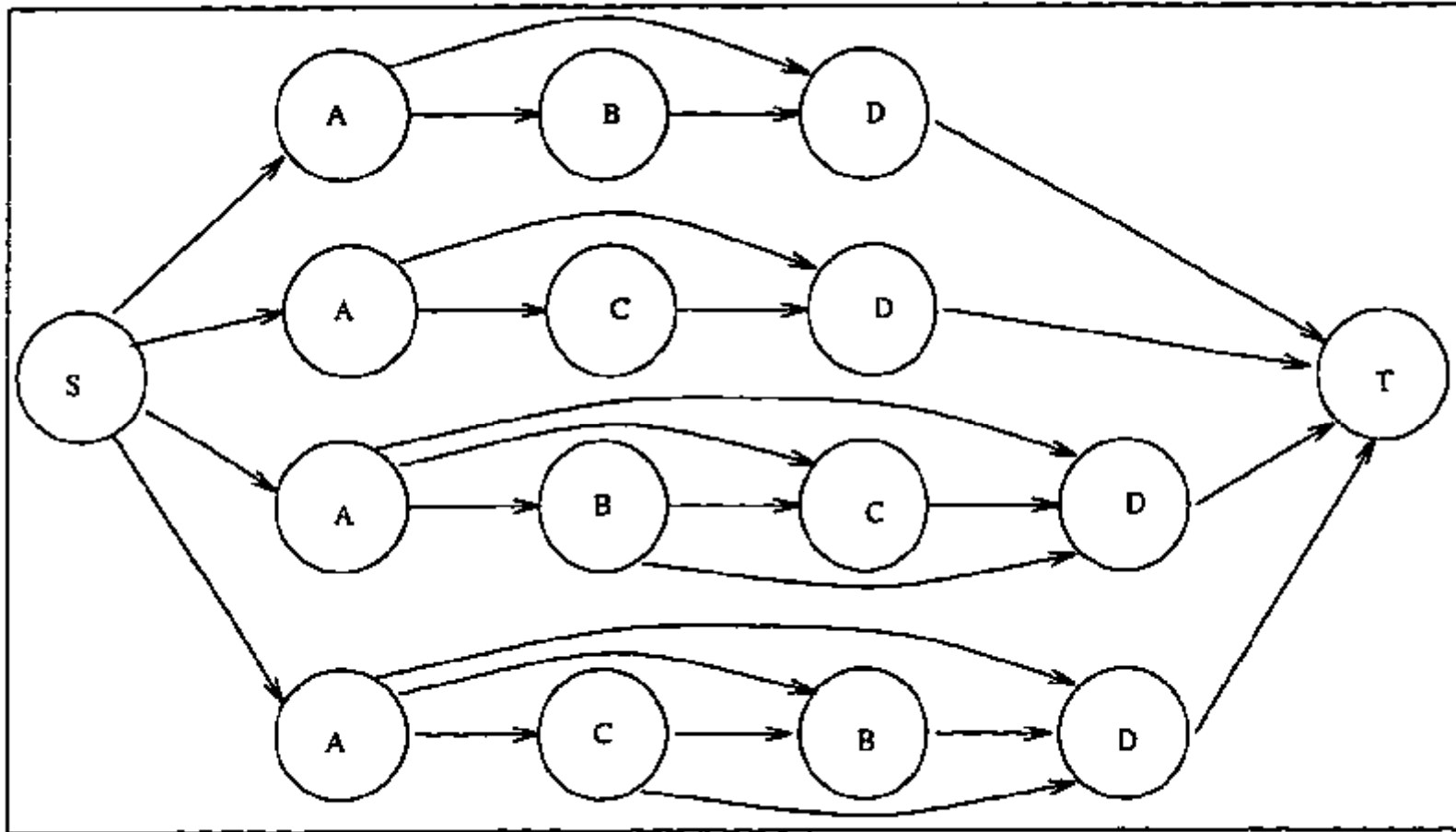


Physical network

# Routing

- Consider commodity A-D in the physical network
- The shortest four routings are
  - A-B-D (length 10)
  - A-C-D (length 17)
  - A-B-C-D (length 18)
  - A-C-B-D (length 21)

# Blocking paths



Commodity network for shortest 4 paths for commodity A→D



# References

# References

- H. N. Newton. **Network Design under Budget Constraints with Application to the Railroad Blocking Problem**. Ph.D. dissertation, Auburn University, USA, 1996.
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- Barnhart, C., H. Jin, P. H. Vance. **Railroad blocking: A network design application**, Operations Research, 48 603–614, 2000.

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