بسم الله الرحمن الرحیم

برنامه ریزی حرکت قطارها

فصل ۷: مساله طراحی شبکه بودجه ای

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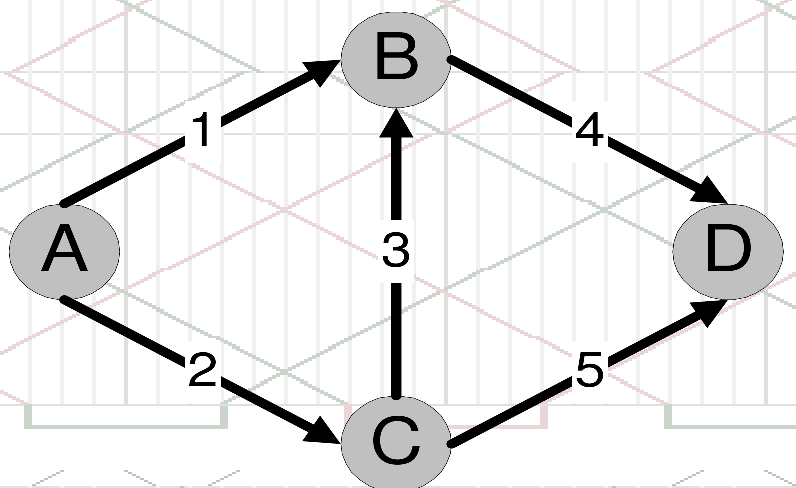
Network Budget Design Problem (BDP)



Outline

- Introduction
- Node-Arc Formulation
- Path-Based Formulation
- Comparing NODE and PATH

Introduction

Introduction

- In the network budget design problem (BDP) the cost of flowing a set of commodities through a network is minimized while observing budget constraints on the fixed costs of the links used.
- Applications of this problem are diverse and include problems such as deciding what transportation or communication infrastructure to build between cities or deciding on the topology of a computer network.

Introduction

- BDP can be used to construct a new network or modify an existing one.

- The links may be either directed or undirected.

Constraints

- **Balance equations**
  - In its simplest form, BDP includes **balance equations** for the flow of each commodity
- **Budget constraint**
  - A **budget constraint** on the sum of fixed costs of the links selected
- **Bundle constraints**
  - **Bundle constraints** to ensure that flow is allowed only on arcs which are built and that the maximum capacity of each arc is observed

Constraints

- Depending on the application, there may be other constraints, such as
  - **service level constraints** or
  - **requirements on the reliability** of the network.

Node-Arc Formulation

Node-Arc Formulation

- We first present a node-arc Mixed Integer Program (MIP) for BDP and show that the constraint matrix for this formulation is too large for the problems we wish to solve.

Parameters

$G = (N, A)$ is the graph with node set N and candidate arc set A.

K is the set of all commodities k designated by an origin-destination pair of nodes.

v^k is the volume of commodity k (in consistent units).

$orig(k)$ is the origin node for commodity k. $orig(a)$ is the origin of arc a.

$dest(k)$ is the destination node for commodity k. $dest(a)$ is the destination of arc a.

u_a is the capacity of arc a.

Parameters

$c_a \geq 0$ is the per unit cost of flow on arc a (assumed equal for all commodities).

e_a is the fixed cost for including arc a in the network.

B is the budget for fixed costs for the entire network.

$B(i)$ is the budget for fixed costs of arcs leaving i.

Variables

x_a^k is the proportion of commodity k's volume flowing on arc a.

$$y_a = \begin{cases} 1 & \text{if arc } a \text{ is included in the network,} \\ 0 & \text{otherwise.} \end{cases}$$

NODE Formulation

$$\min \sum_{k \in K} \sum_{a \in A} c_a v_k x_a^k \qquad (1.1a)$$

s.t.

Bundle
$$\sum_{k \in K} v_k x_a^k \leq u_a y_a \qquad \forall a \in A \qquad (1.1b)$$

Balance
$$\sum_{\substack{a \in A \\ orig(a)=i}} x_a^k - \sum_{\substack{a \in A \\ dest(a)=i}} x_a^k = \begin{cases} 1 & orig(k) = i \\ -1 & dest(k) = i \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N, k \in K \qquad (1.1c)$$

Budget
$$\sum_{a \in A} e_a y_a \leq B \qquad (1.1d)$$

Node Budget
$$\sum_{\substack{a \in A \\ orig(a)=i}} e_a y_a \leq B(i) \qquad \forall i \in N \qquad (1.1e)$$

$$y_a \in \{0,1\} \qquad \forall a \in A$$

$$0 \leq x_a^k \leq 1 \qquad \forall a \in A, k \in K$$

Objective

- The objective is to minimize the sum of the costs of delivering each commodity using the network formed by arcs for which $y_a = 1$.

Constraints

- For each arc, constraints (1.1b) prevent flow on arcs which are not built and enforce the upper bound U_a on flow for arcs which are built.
- For each node, constraints (1.1c) are balance equations for the flow of each commodity.
- The single budget constraint (1.1d) limits the sum of the fixed costs e_a for all arcs selected for the network to the total budget *B.*
- Constraints (1.1e) enforce the node-budget limit *B(i)* for the sum of the fixed costs of the arcs which leave the node.

BDP & Other Problems

- BDP is related to several other problems, such as, the fixed-charge network design problem and the multi-commodity flow problem.
- The fixed charge network design problem is different in that the fixed costs for the arcs appear in the objective function (minimize the total fixed and variable costs) and not in the constraints.
- The multi-commodity flow problem is a BDP with the binary arc selection variables fixed.

Node-Arc Formulation

- As is typical with node-arc formulations, NODE has a large number of constraints and variables.
- Let
  - |N| : denote the number of nodes
  - |K| : the number of commodities
  - |A| : the number of possible arcs

Node-Arc Formulation

- There will be |A| bundle constraints (1.1 b), one for each arc
- Since there is one mass balance constraint for each node-commodity pair, there will be:
  - |N| |K| equality constraints of form (1.1c)
  - with a non-zero coefficient for each potential arc which is adjacent to the node.
- The single (1.1 d) budget constraint
- |N| node-budget constraints (1.1 e), one for each node.
- Total of constraints

  $|A| + |N|\,|K| + 1 + |N|$

Variables

- |A||K| continuous flow variables x_a^k, one for the flow of each commodity on each arc
- |A| binary selection variables y_a

An Example

- For a network with
  - 100 nodes,
  - 1000 commodities, and
  - 5,000 arcs (half of the possible 10,000 node pairs)

An Example

- There would be:
  - 105,101 constraints,
    - bundle constraints: 5000
    - balance constraints: 100,000
    - total budget constraints: 1
    - node-budget constraints: 100
  - 5,000,000 continuous variables, and
  - 5,000 binary variables.

Result

- Given the huge size of the constraint set, storing and manipulating this formulation becomes difficult on a workstation.

Path-Based Formulation

Path-Based Formulation

Parameters

Q(k) is the set of all legal paths for commodity *k.*

PC_q^k is the path cost for flowing one unit of commodity *k* on path *q*.

δ_a^q is the incidence indicator that equals 1 if arc a is on path q and 0 otherwise.

Decision Variables

f_q^k proportion of commodity k on path q, $\forall q \in Q(k), k \in K$

PATH Formulation

$$\min \quad \sum_{k\in K}\sum_{q\in Q(k)} v^k \, PC_q^k \, f_q^k \qquad (1.3a)$$

s.t.

$$\sum_{k\in K}\sum_{q\in Q(k)} v^k \, f_q^k \, \delta_a^q - u_a y_a \le 0 \qquad \forall a \in A \qquad (1.3b)$$

$$\sum_{q\in Q(k)} f_q^k = 1 \qquad \forall k \in K \qquad (1.3c)$$

$$\sum_{a\in A} e_a y_a \le B \qquad (1.3d)$$

$$\sum_{\substack{a \in A \\ orig(a)=i}} e_a y_a \le B(i) \qquad \forall i \in N \qquad (1.3e)$$

$$f_q^k \ge 0 \qquad \forall q \in Q(k), k \in K$$

$$y_a \in \{0,1\} \qquad \forall a \in A.$$

Path-Based Formulation

- Substituting for $\sum_{q \in Q(k)} f_q^k \delta_a^q$ for x_a^k
- Constraints (1.3b) ensure that flow is only allowed on arcs included in the network
- Constraints (1.3c) ensure all of each commodity is delivered
- (1.3d) is the overall budget constraint
- (1.3e) are the budget constraints for each node

Path-Based Formulation

- For the same size example network as before (|N|=100, |K|=1000, |A|=5000), there would be:
  - 6,101 constraints
    - bundle constraints: 5000
    - balance constraints: 1000
    - total budget constraints: 1
    - node-budget constraints: 100
  - 5,000 binary y_a selection variables
  - an exponential number of continuous path variables

Comparing NODE and PATH

Comparing NODE and PATH

- Again let
  - |N| denote the number of nodes,
  - |K|, the number of commodities, and
  - |A|, the number of possible arcs.

Comparing NODE and PATH

- **The number of bundle constraints**
  – The number of bundle constraints (1.3b), $|A|$, is unchanged from the node-arc formulation.
- **Balance constraints**
  – However, the $|N|\ |K|$ mass-balance equations (1.1c) are replaced by $|K|$ constraints (1.3c).
- **Total budget constraints & node-budget constraints**
  – are unchanged from the node-arc formulation

Comparing NODE and PATH

- **Binary selection variables**
  - are unchanged from the node-arc formulation
- **Continuous variables**
  - the |A||K| continuous flow variables in node-arc formulation are replaced by the number of path variables is exponential in path-based formulation

Comparing NODE and PATH

- PATH can accommodate some constraints which could not be easily incorporated into NODE.
- For instance, limiting the number of arcs used in delivering a commodity to max_arcs may be incorporated into PATH by adding the constraint max_arcs to Q(k).
- However, this constraint may not be written as linear inequalities in the flow variables of NODE.
- Constraints of this nature might arise in planning airline itineraries for passengers (commodities) unwilling to make more than two plane changes.

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