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Modeling RBP as BDP (Newton Model)



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Modeling RBP as BDP

- In this chapter, we show that the Railroad Blocking Problem (RBP) can be modeled as a Budget Design Problem (BDP)
- We present an MIP formulation (BLOCK) obtained by adding them to PATH for BDP.



Outline



Assumptions



Definition of RBP

- We may define the RBP as:
 - Minimize the costs of delivering all commodities
 - by deciding which inter-terminal blocks to build
 - and specifying the assignment of commodities to these blocks,
 - while observing limits on the number and aggregate volume of the blocks assembled at each terminal
 - and limits on the number of blocks used to deliver a commodity.



Modeling RBP as BDP

- The RBP can be modeled as a network design problem where
 - the nodes represent the railroad terminals and
 - the arcs represent potential blocks.
- As with the general BDP, the RBP seeks to minimize the flow costs of delivering all commodities.



Maximum number of blocks

- The RBP constraint on the maximum number of blocks originating at a terminal has the same form as the node-budget constraints which we included in PATH (1.3) formulations for the BDP.
- In RBP, the fixed cost $e_a = 1$ for all arcs.



Balance equations

- The balance equations (1.2a) on the paths in PATH can be used to model the identical balance
 - requirements for BLOCK.



Different service constraints

- However, unlike BDP, the maximum number of arcs (blocks) which may be used in a commodity's path is restricted by a priority constraint.
 - The possibility of having different service constraints for some of the traffic from an OD pair requires adding a priority class to the description of each commodity.
- Now commodities are identified by origin, destination, and a maximum number of handlings.

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Arc capacity

• Since blocks are assumed to be uncapacitated, the coefficients U_a in PATH are replaced by the maximum possible flow for arc a.



Node Capacity

• However, where the node flows are assumed uncapacitated in BDP, in RBP the nodes (terminals) have a flow volume constraint to model the limit on the total number of cars which can be classified.



Overall budget

- Finally, there is no overall budget for the blocking problem, so constraint (1.3d) from PATH may be
 - omitted.



Formulation



Parameters

G = (N, A) is the graph with node set N and candidate arc (block) set A.

K is the set of all commodities k designated by an origin-destination pair of nodes and the number of intermediate handlings allowed.

 v^k is the volume of commodity k (in consistent units).

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orig(k) is the origin node for commodity k. orig(a) is the origin of arc a.

dest(k) is the destination node for commodity k. dest(a) is the destination of arc a. Q(k) is the set of legal paths for commodity k.

Parameters

 $c_a \geq 0$ is the per unit cost of flow on arc a (assumed equal for all commodities).

 u_a is the capacity of arc a.

B(i) is the number of blocks which may originated at node *i*.

V(i) is the volume which may be classified at node *i*.

 PC_q^k path cost for flowing one unit of commodity k on path q.



Decision variables



BLOCK Formulation

			<u>Strain</u>	∇
	$min \qquad \sum_{k \in K} \sum_{q \in Q(k)} PC_q^k v^k f_q^k$		(4.1a)	
\mathbb{P}	s.t.			\geq
X	$\sum_{k \in K} \sum_{q \in Q(k)} v^k f_q^k \delta_a^q - u_a y_a \le 0$	$\forall a \in A$	(4.1b)	
	$\sum_{q \in Q(k)} f_q^k = 1$	$\forall k \in K$	(4.1c)	<
\geq	$\sum_{\substack{a \in A \\ orig(a)=i}} y_a \leq B(i)$	$\forall i \in N$	(4.1d)	
A, K	$\sum_{k \in K} \sum_{q \in Q(k)} \sum_{\substack{a \in A \\ orig(a) = i}} v^k f_q^k \delta_a^q \leq V(i)$	$\forall i \in N$	(4.1e)	
	$f_q^k \ge 0$	$\forall q \in Q(k), k \in K$		\leq
	$y_a \in \{0,1\}$	$\forall a \in A$		
1	Q	\sim		

Constraints

- Constraints (4.1b): ensure that flow is only allowed on arcs (blocks) included in the network
- Constraints (4.1c): ensure all of each commodity is delivered
- Constraints (4.1d): the limit on the number of blocks which may be built at each node is modeled by the node-budget constraints
- Constraints (4.1e): the volume of cars which may be classified at each terminal is modeled
- The balance equations and constraints on the maximum number of handlings are included on legal blocking paths for commodity *k*.

Path cost

- Path costs could be included are:
 - Costs for car-miles (miles that a car following this blocking path will travel)
 - Costs for car-hours (hours of delay for classifications incurred along this blocking path)
 - Labor and equipment costs per car might be different depending on the links and blocks for a commodity



Routing vs. Blocking Path

- We define *routings* to be paths through the physical network.
- It is convenient to describe a *routing* by the sequence of terminals visited.
- The terms *blocking path* or *commodity blocking assignment* on the other hand describe the path through the blocking network.

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• If the blocking path is also identified by the sequence of terminals, a blocking path for a railcar will be a subsequence of the routing that it followed since blocking will be done at a subset of the terminals visited.

An Example



Routing

- Consider commodity A-D in the physical network
 - The shortest four routings are
 - A-B-D (length 10)
 - A-C-D (length 17)
 - A-B-C-D (length 18)
 - A-C-B-D (length 21)



Blocking paths



References



References

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- H. N. Newton, C. Barnhart and P. H. Vance, <u>Constructing Railroad Blocking Plans to Minimize</u> <u>Handling Costs</u>, Transportation Science, 32:330-345, 1998.
- Barnhart, C., H. Jin, P. H. Vance. <u>Railroad blocking:</u>
 <u>A network design application</u>, Operations Research, 48 603–614, 2000.

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