

بسم الله الرحمن الرحيم

## برنامه ریزی حرکت قطارها

فصل ۱۳: مدل ترکیبی برنامه ریزی بلاکینگ  
واگنها و تشکیل قطارها

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*In the name of God*

# **Combined Car Blocking and Train Makeup Model**

## **(Keaton Model)**

# Outline

- **Introduction**
- **Car Time**
- **Blocking Policy**
- **The Service Network**
- **Mathematical Formulation**



# Introduction

# Combined Model

- Combined car blocking and makeup model is to determine:
  - (1) which pairs of terminals are to be provided ,with direct train connections,
  - (2) the frequency of service,
  - (3) how the individual cars are routed through the available configuration of trains and intermediate terminals, and
  - (4) how cars are physically grouped or "blocked" within trains.
- The model is intended for an intermediate planning horizon (6-24 months), not for controlling day-to-day operations.

# Objective

- The objective is to minimize
  - the sum of train fixed costs,
  - car time costs, and
  - classification yard costs.

# Train fixed cost

- There is a fixed cost associated with operating a train, independent of the number of cars assigned to the train.
- This fixed cost consists:
  - the wages of the train crew, plus
  - the cost of a units of motive power

# Variable cost

- There is a variable cost, consisting of fuel, which increases with train size.



# Classification yard costs

- Because of the level of fixed train costs, railroad managers have a powerful incentive to operate long trains.
- It is not generally economical to provide all pairs of terminals with direct train connections, and thus many cars must change trains in intermediate terminals.
- and will usually encounter a considerable amount of delay in the process (often in excess of 18 hours).
- There are additional costs associated with the labor, fuel, motive power, and classification yard facilities required to switch cars from one train to another.

# Constraints

- There are physical limits to the number of cars which can be moved in a single train.
- yard volumes



# Car Time

# Car Time

- The time required to move from origin terminals to destination terminals includes time spent:
  - (1) Car time on moving in trains,
  - (2) Car time in origin terminals (waiting for departure from origin yards),
  - (3) Car time in intermediate terminals (making connections in intermediate terminals, if necessary)
    - (i) In classification and assembly operations
    - (ii) Car time is spent waiting for the departure of the appropriate outbound train after classification has been completed.
    - (iii) There may be congestion-induced yard delays.
  - (4) Car time in destination terminals (waiting in destination terminals for delivery to final customers)

# Car Time

- Time spent moving in trains is a relatively small portion of total car time.
- The time spent in terminals is far more significant and important to the choice of an train makeup plan.

# Car time in origin terminals

- Cars are typically brought into the origin yard over the course of a day by local freight trains (which collect cars from individual shippers) or trucks
- The total number of cars in the yard will generally accumulate at an irregular rate over the course of the day.
- At various intervals, outbound trains will leave the yard, taking the cars which have accumulated up to that point.
- More frequent departures will generally lead to a reduction in the average delay per car.
- If cars accumulate at a constant rate over time, and outbound trains depart at regular intervals, the average delay per car will be equal to one-half of the interval of time between consecutive departures (i.e. 12 hours with one daily train, 6 hours with two, etc.).

# Car time in intermediate terminals

- Car time in intermediate terminals where cars are transferred from one train to another is spent:
  - (i) In *classification* and *assembly* operations
  - (ii) Car time is spent waiting for the departure of the appropriate outbound train after classification has been completed.
  - (iii) There may be congestion-induced yard delays.

# Car time in intermediate terminals

- (i) Car time in classification and assembly operations.
  - Classification is the process of sorting the cars of incoming trains onto a series of parallel yard tracks according to their next downstream destination. This activity can take several hours.
  - The assembly operation involves consolidating groups of cars to form outbound trains. This operation can consume several more hours



# Car time in intermediate terminals

- (ii) Car time is spent waiting for the departure of the appropriate outbound train after classification has been completed.
  - This is typically the main source of delay in intermediate terminals.
  - Each inbound train will connect with a number of outbound trains; some inbound-outbound pairs will be scheduled close together in time,
  - while other pairs may have almost 24 hours between arrival and departure times.
  - Thus, one might expect that train schedules (timetables) would significantly influence the delay to specific cars

# Car time in intermediate terminals

- Based on data from a number of yards that average yard times typically range from about 15 to 27 hours with one daily outbound connection.
- The minimum delay occurs with approximately a 10-hour interval between train arrival and departure.
- The minimum and maximum delay can be reduced with 2 or more departures.
- In busy terminals, with train arrivals and departures spread throughout the day, all pairs of inbound and outbound connections cannot be scheduled so that each pair can achieve the minimum delay.
- The average delay to a typical inbound-outbound pair will lie somewhere between the minimum and maximum delays, and we will assume that a reasonable average can be determined.
- This figure could be based on historical data, simulation, or the judgment of railroad operating personnel.

# Car time in intermediate terminals

- (iii) There may be congestion-induced yard delays.
  - The model proposed here does not explicitly relate yard delay to yard volumes
  - We have no formal procedure to quantify congestion-induced yard delays
  - To prevent overloading individual yards, we limit the number of blocks which can be formed at each yard

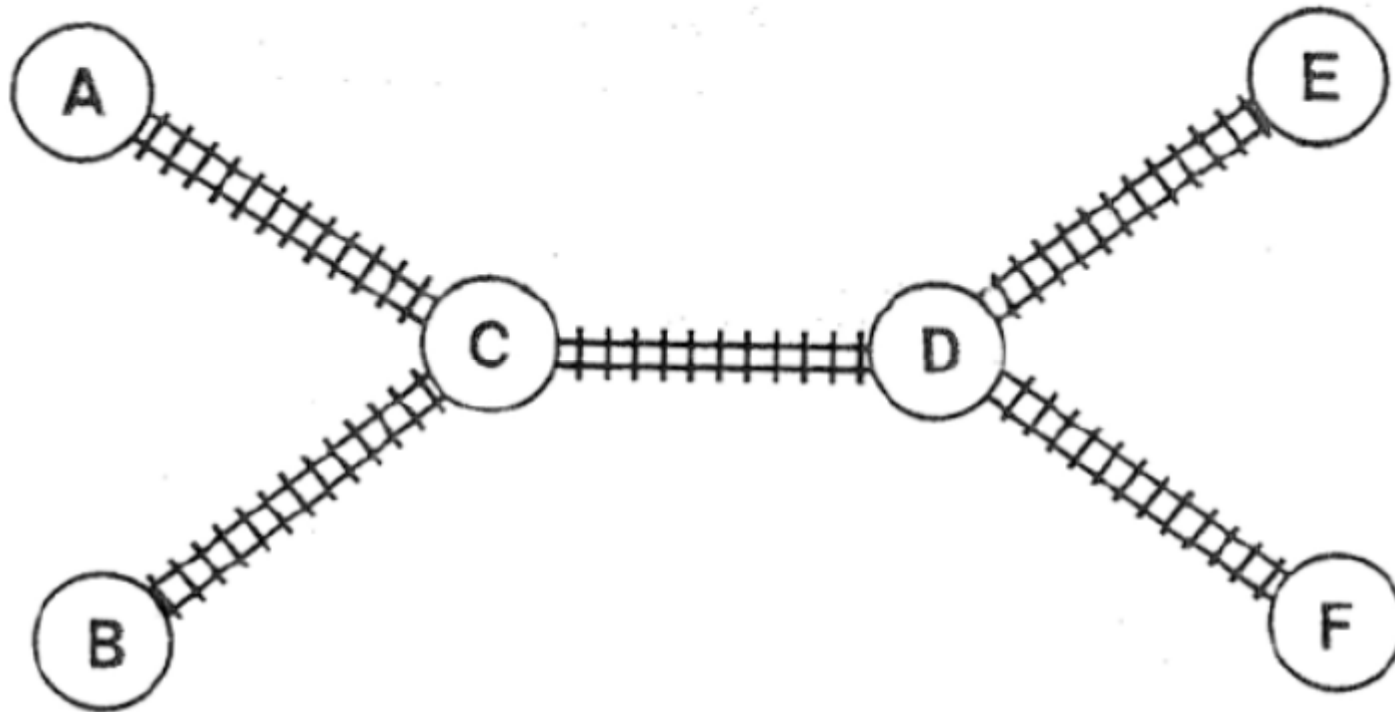
# Car time in destination terminals

- There are delays following train arrivals in destination terminals.
- Depending on the nature of the local freight operations, which deliver cars to the customers, more frequent inbound trains may reduce average car delay.



# Blocking Policy

# Train Makeup Model



# Blocking Policy

- Car time in intermediate yards is influenced by the way in which the cars in inbound trains are grouped together, or blocked.
- Consider the network of Fig. 1, and assume that cars leaving terminal A are sorted into two blocks, with all cars terminating at B and C in one physically contiguous block, and all cars terminating at D, E, and F in another block.
- With this practice, a train from A to D can set out the (BC) block at terminal C, and proceed on to D with the (DEF) block.
- Thus, the cars destined for D, E, and F can, in effect, bypass the delay at C.
- The train could pick up another block of cars at C, replacing the block set out there, and delivery them on to D.

# Blocking Policy

- This process could be carried one step further, with the cars bound for B sorted into a third block before leaving terminal A.
- At yard C, these cars could be transferred directly to a train from C to B, bypassing the classification operations at C, and thereby experiencing a reduction in delay.
- This operation is called a "block-swap."
- Although it is difficult to estimate the average delay in block swap operations, this practice is common in the industry.



# Blocking Policy

- Because we are also limiting train sizes, it is sufficient to simply limit the number of departing train connections to ensure that yard capacities are not exceeded;
- the total number of blocks is essentially irrelevant.
- If some tracks are not long enough to accommodate all the cars on a train,  $N$  must be adjusted accordingly.
- Similarly, we must recognize that the number of arriving cars varies from day to day, and further adjust  $N$  to ensure that track capacity is not exceeded more than a given proportion of time.)

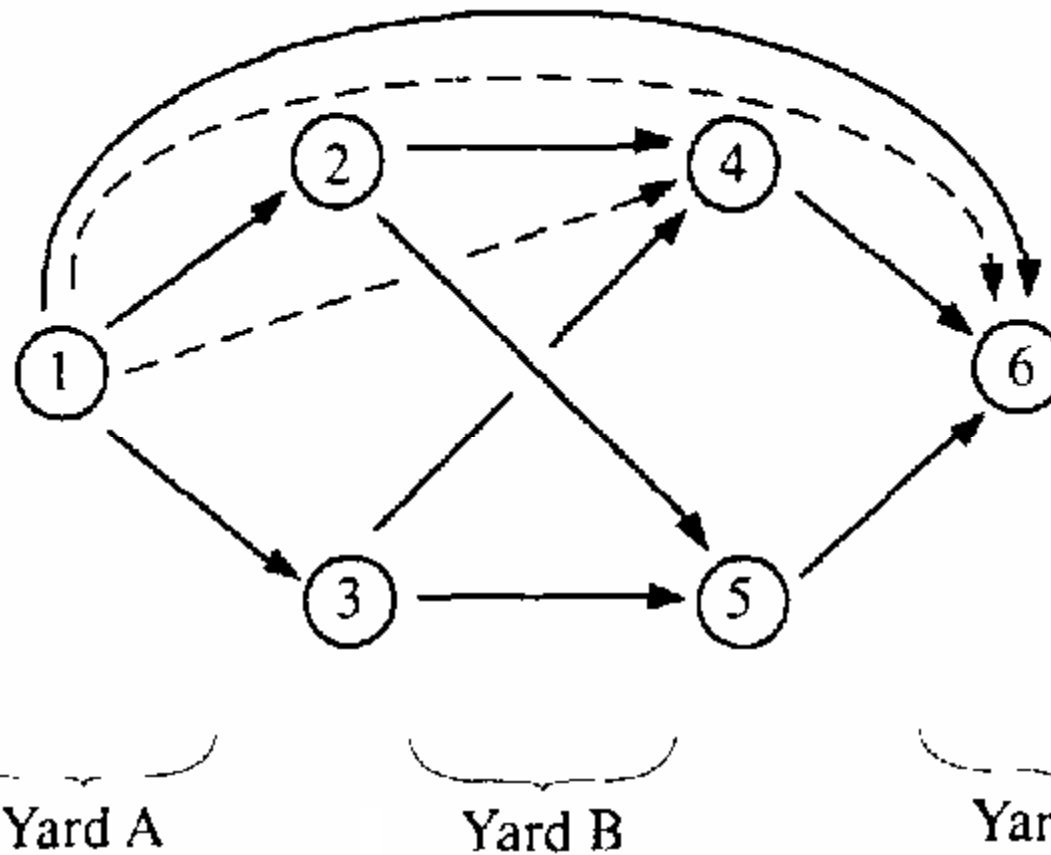


# The Service Network

# The Service Network

- Car time and classification cost in the model are represented by a *service network*  $G(N, A)$  where  $N$  denotes the node set, and  $A$  the arc set.
- There is a sub-network in  $G$  for each O-D pair  $p$ ,  $p = 1, N$ , denoted  $G_p = (N_p, A_p)$ .
- The sub-networks are disjoint; that is,  $A_p \cap A_q = \emptyset$  for O-D pair  $p \neq q$ .
- Hence,  $G = G_1 \cup G_2 \cup \dots \cup G_N$ .
- The arcs in each  $A_p$  correspond to the movement of cars in trains between yards, and also on the interchange of cars between connecting trains within yards.
- The cost on each arc is the monetary value of the car time associated with a train or an interchange, plus the expense of physically switching a car.

# Figure 1: Simplified service network



# Simplified service network

- This scheme is illustrated in Figure 1. Consider cars going from origin yard A to destination yard C, with the opportunity to change trains in intermediate yard B.

## Solid arc (1,6)

- Solid arc (1,6) corresponds to one direct train per day from A to C.
- The cost on this arc represents the monetary value of car time in (a) the origin yard, (b) moving in the train, and (c) in the destination yard, including frequency delay waiting for delivery to customers.
- Frequency delay at yard A will be 12 hours with one daily train to C.
- Assuming one daily local freight out of yard C, delivering cars to customers, frequency delay will also be 12 hours in the destination yard.

## Dashed arc (1,6)

- The dashed arc (1,6) represents one train per day from A which sets out a block of cars at B and proceeds on to C.
- This train can also pick up a block at B and move it to C.
- The cost on this arc includes all the items above, plus car time cost for the added delay of setting out at B.

## Arc (1,2)

- Arc (1,2) corresponds to one train per day from yard A to intermediate yard B, where cars will connect with another train for C.
- The cost on this arc includes (a) car time in yard A, and (b) train time.



## Arc (1,3)

- Arc (1,3) corresponds to twice-a-day service from A to B; the cost includes a reduced frequency delay of 6 hours.
- The final train makeup plan can contain at most one of these alternatives, since train frequency choices are mutually exclusive.

## Arcs (4,6) and (5,6)

- Arcs (4,6) and (5,6) corresponds to one and two connections per day between yards B and C.
- The costs include (a) train time, plus (b) car time in yard C.

## Arcs (2,4) and (3,4)

- Arcs (2,4) and (3,4) represent the interchange of cars in yard B between inbound trains and a single daily outbound.
- The costs on both arcs include (a) car time costs, plus (b) the cost of physically switching a car.

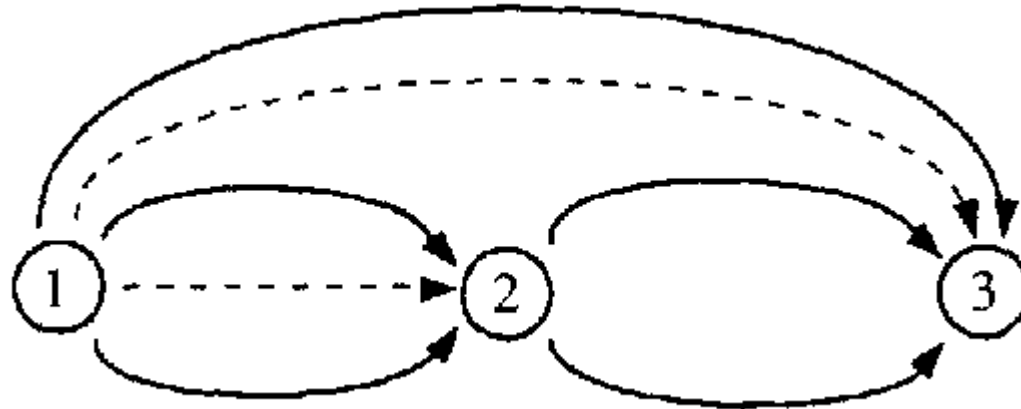
## Arcs (2, 5) and (3, 5)

- Arcs (2, 5) and (3,5) represent inter-change with two daily outbound trains
- The costs include a reduced frequency delay of 6 hours.

## Dashed Arc (1,4)

- If all cars bound for yard C are blocked together at yard A, a block switch can be made in yard B, avoiding the classification, assembly, and some of the frequency delay.
- Arc (1,4) corresponds to such a operation
- The cost on this arc will include the 12 hour frequency delay at A, plus an additional frequency delay associated with the block switch at B.

## Figure 2: Compact form of the network



- A more compact form of the network, shown in this Figure, is actually used in the analysis.
- The interchange arcs are combined with the train arcs.
- The 12 hour frequency delay at B associated with a once-a-day departure is added to the train arc (4,6).
- Similarly, a 6 hour frequency delay is added to twice-a-day train arc (5,6).

# The objective

- The specific trains which appear in the final plan are determined by:
  - minimizing train car time, and
  - classification cost for all O-D moves,

# Decision variables

- A 0-1 decision variable is associated with each potential train service
- There is a unique variable for each frequency choice.
- The term train used hereafter will refer not only to the yards connected, but to frequency as well.



# Constraints

- Constraints:
  - upper limits on O-D trip times,
  - limits on train size, and
  - limits on the number of blocks which can be formed in each yard.
- Indirectly, by restricting the number of blocks, also limit the volume of cars which can pass through a yard.

# Potential blocks

- The potential blocks which can be built at each yard are determined by the potential train connections out of the yard.
- In the example of Figure 1, a block for C will be built at yard A only if one or more of the following trains are operated:
  - (a) a direct A-C train,
  - (b) an A-B-C train, setting out at B, or
  - (c) an A-B train, exchanging a block for C with a B-C train.

# Potential blocks

- The blocks to be carried by each train are specified in advance.
- The model does not determine the blocks to be built at each yard, and then assign them to trains.
- Instead, the choice of trains determines the blocks which must be built.
- In order to obtain the best possible solution, a large set of potential train connections must be available.

# Pure Strategy

- One feature of the model is that all cars between each O-D pair must follow the same routing.
- This is common practice in the industry is referred to this as a *pure strategy* constraint.



# Mathematical Formulation

# Mathematical Formulation

## • Indices:

$p$	index for O-D pairs
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$k$	index for trains
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$m$	index for yards
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## • Sets:

$N_p$	node set for O-D pair $p$
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$A_p$	arc set for O-D pair $p$
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$\Omega$	set of restrictions on train variables
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$P_k$	set of O-D pairs which can take train $k$
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$Y_m$	the set of trains which can be blocked at yard $m$
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# Mathematical Formulation

## • Parameters:

$r^p$	the number of cars to be moved between O-D pair $p$
$c_{ij}^p$	the cost of sending $r^p$ cars over $arc(i, j) \in A_p$ , including the monetary value of car time cost and classification cost
$s_{ij}^p$	the transit time in hours to pass over $arc(i, j) \in A_p$
$d_k$	the cost of providing train $k$
$\tau_k$	the maximum number of cars which can move in train $k$
$\alpha_k$	the number of blocks in train $k$
$\beta_m$	the maximum number of blocks which can be built at yard $m$
$\sigma_p$	the maximum transit time allowable for O-D pair $p$
$K_{ij}^p$	is defined to be the number of the train corresponding to $arc(i, j) \in A_p$ . Each arc in $A_p$ corresponds to a unique train, and each train appears at most once in $A_p$ .

# Mathematical Formulation

- **Decision Variables:**

$x_{ij}^p$	a binary variable; 1 if cars for O-D pair $p$ use $arc(i, j) \in A_p$ , 0 otherwise
$t_k$	a binary variable; 1 if train $k$ is provided, 0 otherwise



# Mathematical Formulation

$\text{minimize } \sum_p \sum_{(i,j) \in A_p} c_{ij}^p x_{ij}^p + \sum_k d_k t_k$	(1)
<p>subject to:</p> $\sum_{j \in N_p} x_{1j}^p = 1$ $\sum_{j \in N_p} x_{ij}^p - \sum_{i \in N_p} x_{ji}^p = 0 \quad \text{for } i \neq 1 \text{ and } j \neq  N_p $ $\sum_{i \in N_p} x_{i N_p }^p = 1$	
<p>for all O-D pairs <math>p</math></p>	(2)
$x_{ij}^p - t_k \leq 0$	for $k = K_{ij}^p$ , for all $(i, j) \in A_p$ , for all $p$ (3)
$t_k \in \Omega$	for all trains $k$ (4)
$\sum_{p \in P_k} r^p x_{ij}^p \leq \tau_k$	for $k = K_{ij}^p$ , for all trains $k$ (5)
$\sum_{k \in y_m} \alpha_k t_k \leq \beta_m$	for all yards $m$ (6)
$\sum_{(i,j) \in A_p} s_{ij}^p x_{ij}^p \leq \sigma_p$	for all O-D pairs $p$ (7)
$x_{ij}^p = (0,1)$	for all $(i, j) \in A_p$ , for all $p$ (8)
$t_k = (0,1)$	for all $k$ (9)

# The objective (1)

- The objective (1) is to minimize car time, classification, and train costs.

## Constraints (2)

- Constraints (2) ensure that all cars reach their destinations.
- To enforce the pure strategy constraints, all (x) variables are restricted to 0-1 integer variables.

## Constraints (3)

- Constraints (3) prevent cars from moving on a train unless it is available.
- Note that there is a separate constraint for each O-D pair / train combination.

## Constraints (4)

- Constraints (4) impose several types of restrictions on the train variables  $t_k$
- Recall that there is a distinct  $t_k$  for each of the mutually exclusive train frequency choices between any pair of terminals.
- Constraints (4) ensure that, at most, only one of these frequency choices can be used.
- Also, we may wish to obtain a solution in which specific train connections are available. Constraints (4) will then specify that the appropriate  $t_k = 1$ .

## Constraints (5), (6) & (7)

- Constraints (5) limit the number of cars which can move on each train,
- Constraints (6) limit the number of blocks which can be formed in each yard.
- Finally, the operating plan must provide trip times which meet the desired service standards. This condition is enforced by constraints (7).



# References

# References

- M. H. Keaton. “**Designing Optimal Railroad Operating Plans: Lagrangian Relaxation and Heuristic Approaches.**” Transportation Research, 23B:415-431 (1989).
- M. H. Keaton. “**Designing Railroad Operating Plans: A Dual Adjustment Method for Implementing Lagrangian Relaxation.**” Transportation Science, 26:263-279 (1992).



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