بسم الله الرحمن الرحیم

برنامه ریزی حرکت قطارها

فصل ۱۶: یک مدل بهینه سازی خطوط مسافری

A Line Optimization Model

مدرس: دکتر مسعود یقینی

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Outline

- Introduction
- Modeling Railway Networks and Lines
- The Mixed Integer Linear Programming Formulation
- An Example

Introduction

Introduction

- **Line**
  - In a railway system with periodic timetable a **line** connecting two stations runs several times in a fixed time interval e.g. one hour, across the network.
  - This number is called the **frequency** of the line.
- **Line Optimization Problem**
  - consists of choosing some lines with their frequencies to serve passenger demand and to optimize a given objective
- **Objective functions**
  - to minimize operational costs for a fixed service
  - to maximize service quality for fixed operational costs

Introduction

- **Service Quality**
  - One way to improve the service is to minimize the total travel time of all passengers
  - At this stage of planning there is no timetable hence you cannot determine the exact waiting period while changing lines
  - Changing of lines itself is a major inconvenience, hence one possible way to optimize service is to minimize the total number of changes or even simpler to maximize the total number of travelers on direct connections or simply direct travelers.

Modeling Railway Networks and Lines

Modeling Railway Networks and Lines

- $G = (V, E)$:
  - An undirected graph where
  - V denotes the set of vertices which describe the stations
  - E is the set of edges which define direct connections or links between two stations
    - T : the travel time on a single link
    - D : the travel distance on a single link
  - We assume that passengers from a to b come back to a

Modeling Railway Networks and Lines

- ***Possible lines***
  - L_0 : denote the set of all possible lines
- ***Classification Yards***
  - $\mathcal{CY} \subseteq V$ classification yards, that describe the stations in which a line may start / end, and they have a special equipment (e.g. sidings to compose trains)
  - Only paths in G with start and endpoint in $\mathcal{CY}$ are possible lines
- ***Frequency of lines***

Modeling Railway Networks and Lines

- $tr(a, b)$ / $tr(t)$
  - $a, b \in V, a \# b$
  - Volume of traffic between the stations
- $\mathcal{T} := \{\{a, b\} \mid a, b \in V, a \# b, tr(\{a, b\}) \# 0$
  - denote the set of origin-destination pairs with nonzero volume of traffic

Modeling Railway Networks and Lines

- **Assumption**
  - Travelers between a and b $(a, b) \in V$ use a shortest path between a and b in G with respect to some edge evaluation i.e. travel time T or travel distance D
- P_t : denote the shortest path in G with respect to some edge evaluation between a and b ($t = \{a, b\} \in \mathcal{T}$)
- $tl(e)$: the traffic load is given by:

$$tl(e) := \sum_{\substack{\{a,b\}=t\in\mathcal{T} \\ e\in P_t}} tr(t)$$

Modeling Railway Networks and Lines

- $lfr(e)$: line frequency requirement, the minimum number of trains / lines, which have to run along link e to serve the demand for transportation
- A reasonable calculation of the line frequency requirement would be:

$$lfr(e) := \left\lceil \frac{tl(e)}{C} \right\rceil$$

- C : train capacity, a vague estimation of the real situation

Modeling Railway Networks and Lines

- **Decision Variables:**
- $d_{t,l}$: denote the number of direct travelers between $t \in T$ ($t = \{a, b\}$ using line l.
- f_l : denotes the frequency of line $l \in L_0$
  - the frequencies of the possible lines are in a fixed time interval (e.g. in one hour)

The Mixed Integer Linear Programming Formulation

The MIP Formulation

- The MIP formulation of the line optimization problem

$$
\begin{aligned}
D^* \quad &= \max \sum_{l \in \mathcal{L}} \sum_{\substack{t \in \mathcal{T} \\ P_t \subseteq l}} d_{t,l} \\
s.t. \quad & \sum_{\substack{l \in \mathcal{L} \\ P_t \subseteq l}} d_{t,l} \quad \leq \quad tr(t) \qquad (\text{for all } t \in \mathcal{T}) \\
& \sum_{\substack{t \in \mathcal{T} \\ e \in P_t \subseteq l}} d_{t,l} \quad \leq \quad C \cdot f_l \qquad (\text{for all } e \in E, l \in \mathcal{L}) \\
& \sum_{\substack{l \in \mathcal{L} \\ e \in l}} f_l \quad = \quad lfr(e) \qquad (\text{for all } e \in E) \\
& d_{t,l}, f_l \in \mathbf{Z}_+ \qquad (\text{for all } t \in \mathcal{T}, l \in \mathcal{L})
\end{aligned}
$$

The MIP Formulation

- We will allow fractional travelers $d_{t,\,l}$ we relax $d_{t,\,l} \in \mathbf{Z}_+$ to $d_{t,\,l} \geq 0$,
- The number of direct travelers is huge, therefore it seems not to be very important to find the exact integral optimum
- It may be sufficient to base our evaluation on its linear programming relaxation

The MIP Formulation

- Inequality,

$$\sum_{\substack{l \in \mathcal{L} \\ P_t \subseteq l}} d_{t,l} \leq tr(t) \quad (\text{for all } t \in \mathcal{T})$$

  - restricts the number of direct travelers between $t \in T$ by the total number of travelers

- By inequality

$$\sum_{\substack{t \in \mathcal{T} \\ e \in P_t \subseteq l}} d_{t,l} \leq C \cdot f_l \quad (\text{for all } e \in E, l \in \mathcal{L})$$

  - no line can be overloaded

The MIP Formulation

- Equation,

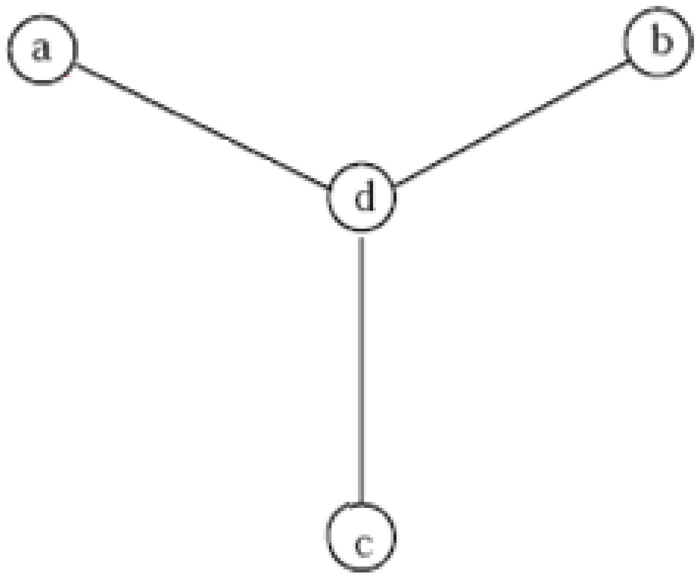
$$\sum_{\substack{l \in \mathcal{L} \\ e \in l}} f_l \quad = \quad lfr(e) \quad (\text{for all } e \in E)$$

- ensures that the edges are covered with a sufficient number of lines / frequencies.

An Example

An Example

- Considering the following railway network:



An Example

- Demands (*tr*):

tr	V1	V2	Volume
1	a	B	50
2	a	c	50
3	a	d	50
4	b	c	50
5	b	d	50
6	c	d	50

An Example

- Lines / services (f), and C = 100 persons

l	V1	V2
1	a	B
2	a	C
3	a	d
4	b	c
5	b	d
6	c	d

An Example

- Decision variables $d_{t,l}$ that denote the number of direct travelers between $t \in T$ $(t = \{a, b\}$ using line l.

t	l
1 (a, b)	1 (a, b)
2 (a, c)	2 (a, c)
3 (a, d)	1 (a, b)
3 (a, d)	2 (a, c)
3 (a, d)	3 (a, d)
4 (b, c)	4 (b, c)
5 (b, d)	1 (a, b)
5 (b, d)	4 (b, c)
5 (b, d)	5 (b, d)
6 (c, d)	2 (a, c)
6 (c, d)	4 (b, c)
6 (c, d)	6 (c, d)

An Example

$$\max \sum_{l \in \mathcal{L}} \sum_{\substack{t \in \mathcal{T} \\ P_t \subseteq l}} d_{t,l}$$

$$\max\ d_{1,1} + d_{2,2} + d_{3,1} + d_{3,2} + d_{3,3} + d_{4,4} + d_{5,1} + d_{5,4} + d_{5,5} + d_{6,2} + d_{6,4} + d_{6,6}$$

An Example

- Inequality restricts the number of direct travelers between $t \in T$ by the total number of travelers

$$\sum_{\substack{l \in \mathcal{L} \\ P_t \subseteq l}} d_{t,l} \leq tr(t) \quad (\text{for all } t \in \mathcal{T})$$

$$d_{1,1} \leq 50$$

$$d_{2,2} \leq 50$$

$$d_{3,1} + d_{3,2} + d_{3,3} \leq 50$$

$$d_{4,4} \leq 50$$

$$d_{5,1} + d_{5,4} + d_{5,5} \leq 50$$

$$d_{6,2} + d_{6,4} + d_{6,6} \leq 50$$

An Example

- By inequality no line can be overloaded

$$\sum_{\substack{t \in T \\ e \in P_t \subseteq l}} d_{t,l} \leq C \cdot f_l \quad (\text{for all } e \in E, l \in \mathcal{L})$$

l	V1	V2	e1	e2
1	a	b	(a-d)	(d-b)
2	a	c	(a-d)	(d-c)
3	a	d	(a-d)	
4	b	c	(b-d)	(d-c)
5	b	d	(b-d)	
6	c	d	(d-c)	

e	*L*	*t*
(a-d)	1 (a, b)	1 (a, b)
(a-d)	1 (a, b)	3 (a, d)
(a-d)	2 (a, c)	2 (a, c)
(a-d)	2 (a, c)	3 (a, d)
(a-d)	3 (a, d)	3 (a, d)
(b-d)	1 (a, b)	1 (a, b)
(b-d)	1 (a, b)	5 (b, d)
(b-d)	4 (b, c)	4 (b, c)
(b-d)	4 (b, c)	5 (b, d)
(b-d)	5 (b, d)	5 (b, d)
(d-c)	2 (a, c)	2 (a, c)
(d-c)	2 (a, c)	6 (c, d)
(d-c)	4 (b, c)	4 (b, c)
(d-c)	4 (b, c)	6 (c, d)
(d-c)	6 (c, d)	6 (c, d)

An Example

$$d_{1,1} + d_{3,1} \le 100 \times f_1$$

$$d_{2,2} + d_{3,2} \le 100 \times f_2$$

$$d_{3,3} \le 100 \times f_3$$

$$d_{1,1} + d_{5,1} \le 100 \times f_1$$

$$d_{4,4} + d_{5,4} \le 100 \times f_4$$

$$d_{5,5} \le 100 \times f_5$$

$$d_{2,2} + d_{6,2} \le 100 \times f_2$$

$$d_{4,4} + d_{6,4} \le 100 \times f_4$$

$$d_{6,6} \le 100 \times f_6$$

An Example

- This equation ensures that the edges are covered with a sufficient number of lines / frequencies.

$$\sum_{\substack{l \in \mathcal{L} \\ e \in l}} f_l \quad = \quad lfr(e) \quad (\text{for all } e \in E)$$

- *lfr*(*e*) : line frequency requirement, the minimum number of trains / lines, which have to run along link *e* to serve the demand for transportation

$$lfr(e) := \left\lceil \frac{tl(e)}{C} \right\rceil$$

An Example

Arcs	*tr*(a-b)	*tr*(a-c)	*tr*(a-d)	*tr*(b-c)	*tr*(b-d)	*tr*(c-d)	total	*lfr*(*e*)
(a-d)	50	50	50	0	0	0	150	2
(b-d)	50	0	0	50	50	0	150	2
(d-c)	0	50	0	50	0	50	150	2

$$f_1 + f_2 + f_3 = 2$$

$$f_1 + f_4 + f_5 = 2$$

$$f_2 + f_4 + f_6 = 2$$

An Example

- Solution:

$$d_{1,1} = 50 \qquad f_1 = 1$$

$$d_{2,2} = 50 \qquad f_2 = 1$$

$$d_{3,2} = 50 \qquad f_4 = 1$$

$$d_{4,4} = 50$$

$$d_{5,4} = 50$$

$$d_{6,4} = 50$$

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