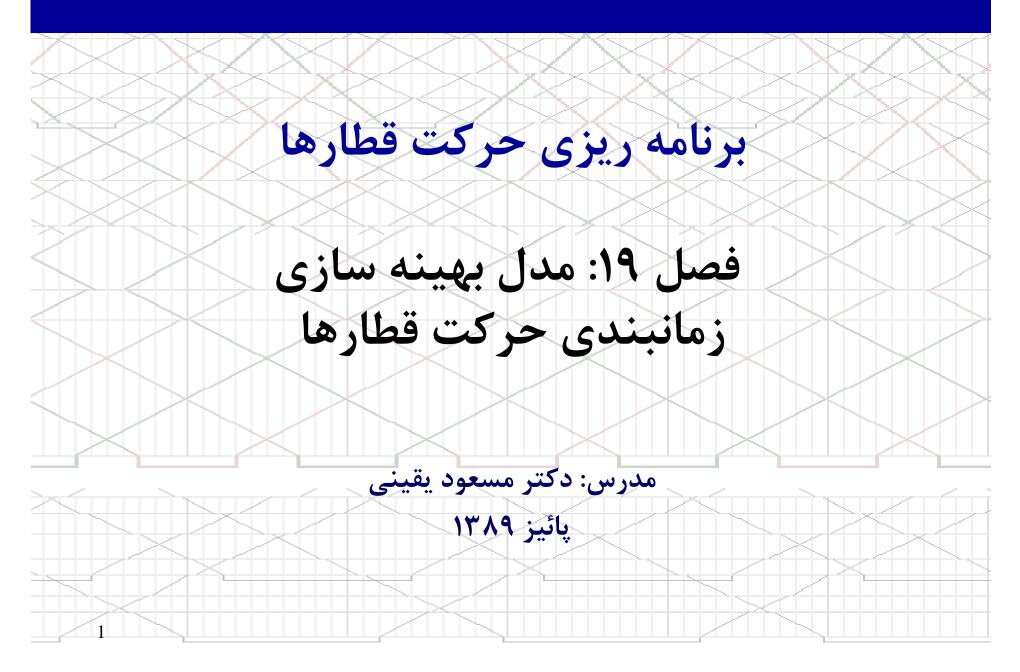
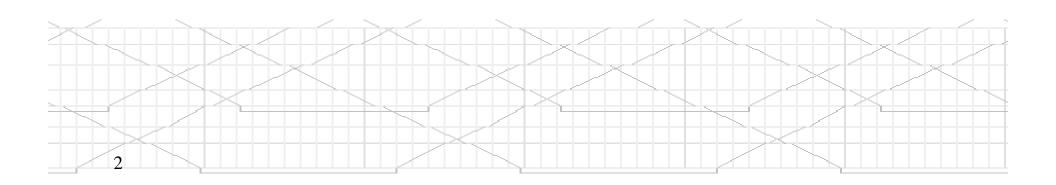
بسم الله الرحمن الرحيم



# **Train Scheduling Problem**

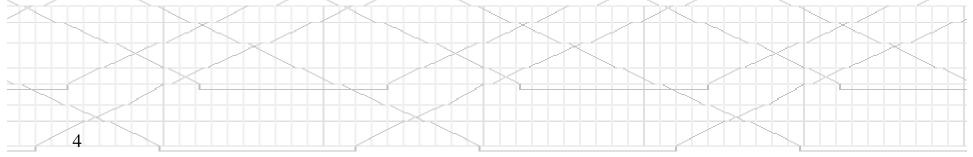
#### (Higgins Model)



# Main Objective

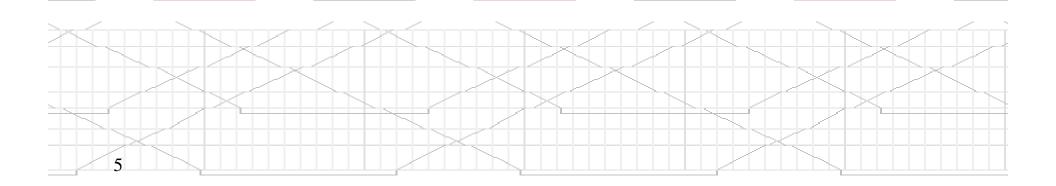
#### The main objective:

- is to assist train controllers or time table planners to resolve train conflicts at sidings so as to ensure safe operation and at the same time keep the overall delay due to train conflicts (or total travel time) as short as possible.
- The model is not designed to replace the train controller.
- The model is applied to a single or partially double line track corridor for which sidings are placed at various positions along this corridor.
- When two trains are to interfere with each other (cross or overtake), the main decision variable for the train controller is to decide which train is to be delayed at a siding.

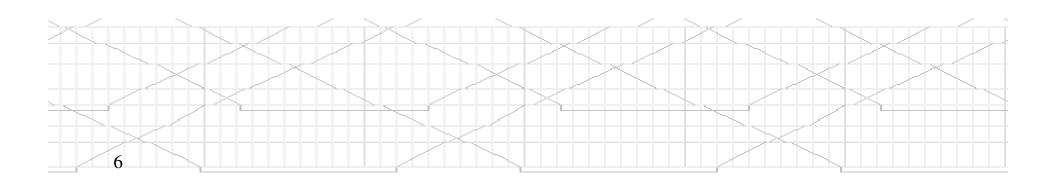


#### **Track section (segment):**

- The track is divided into segments which are separated by sidings.
- For double track sections, it is assumed one lane will be allocated for inbound trains and one lane will be allocated for outbound trains. Usually, signal points will be set up this
  - way.



- Siding (crossing loop or passing loop):
  - A section of track which can be used for the crossing and passing of trains under single track operations.
  - A train station on a single line track will usually contain a siding.
  - Scheduled stops are permitted at any intermediate siding for any train.



#### Conflict delay:

 Is the delay to a train if it must wait at a siding for another train to cross or overtake.

#### Minimum headway:

The minimum length of time separating two trains on a single line track.

#### **Planned schedule:**

 Is generated at a tactical level (medium term planning) by rail planners to be used on a daily or weekly basis.

#### • Actual schedule:

 The actual schedule will be the same as the planned schedule if no unforeseen events occur.

#### Train conflict:

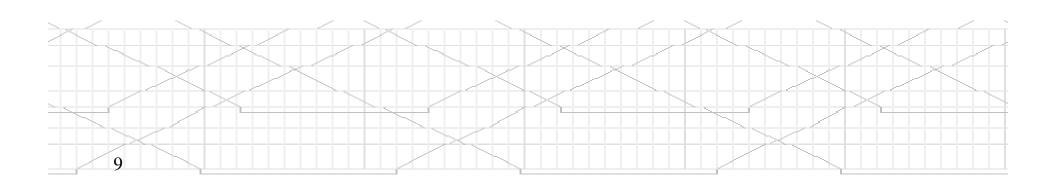
- There are two situations:
  - When two trains approach each other on a single line track and the continuation of either or both trains' journey would not be possible
  - When a faster train catches up a slower train travelling in the same direction

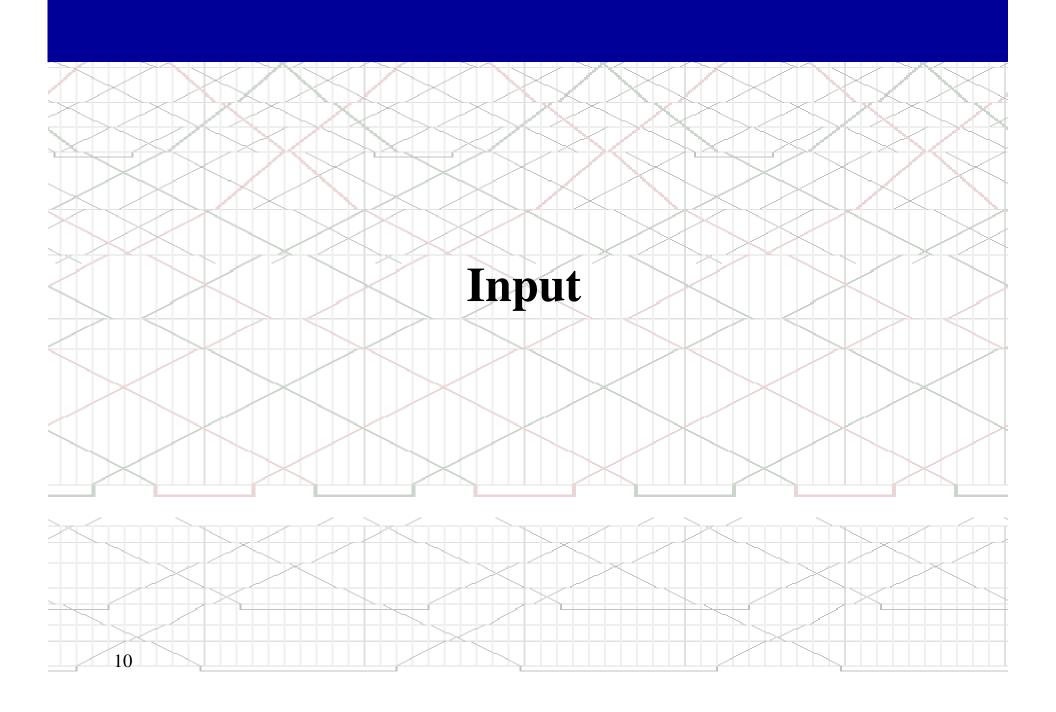
#### • Resolving a conflict:

 Is when one of the trains involved in the conflict must be forced to the siding for the other train to cross or pass.

- This ensures safe operation (feasible schedule).

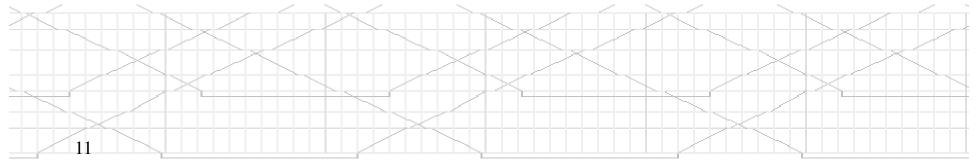
- Total weighted travel time:
  - The total travel time from origin to destination (including conflict delays) for all trains, weighted by train priority.
- Total weighted conflict delay:
  - Is total weighted travel time minus the free travel time.

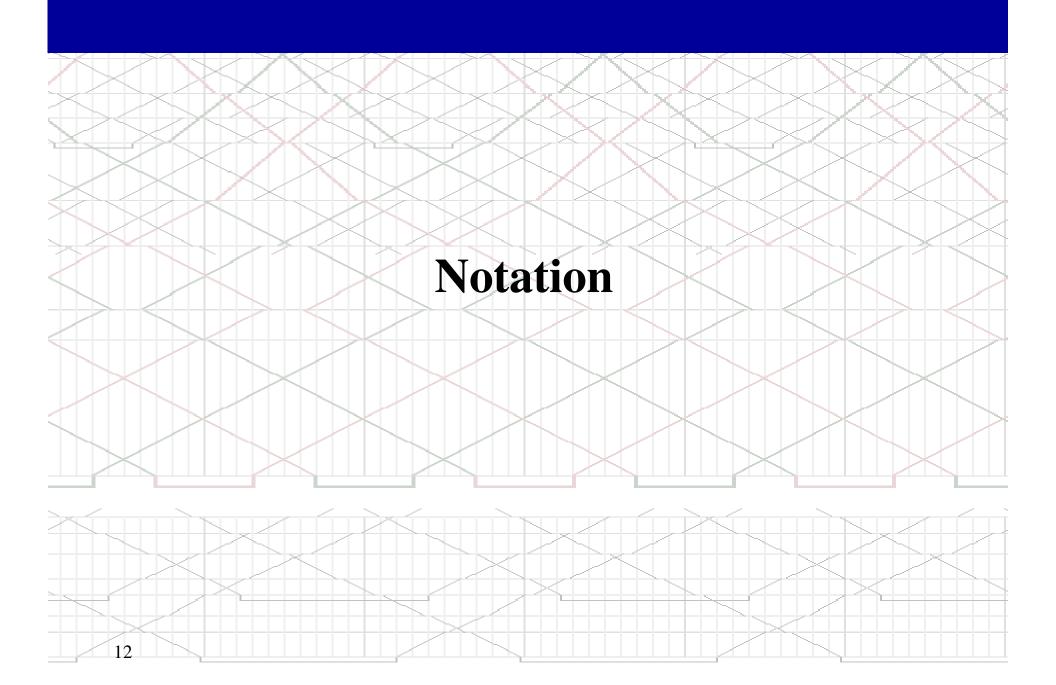




#### **Required Information**

- The times of any scheduled train stops. These stops may include loading/unloading, refuelling and crew changes.
  - The priorities of each train. These are determined by several factors such as the type of train, customer contract agreements and train load.
- The upper and lower velocity limits for each train (which are dependent on the physical characteristics of the track segment and the train).

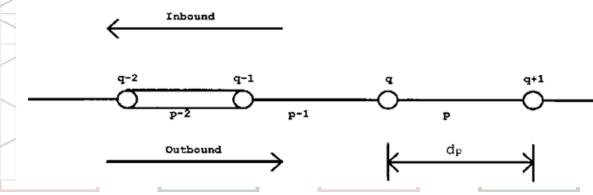


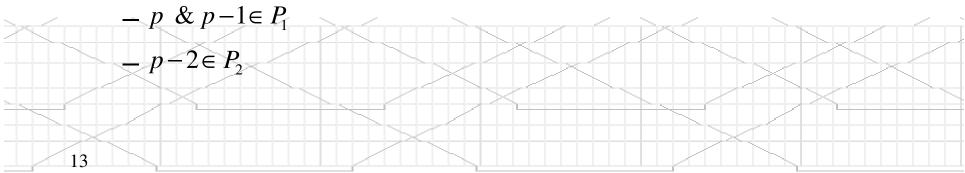


#### **Set of Line Tracks & Sidings**

• The set of sidings is represented by  $Q = \{1, 2, ..., NS\}$ 

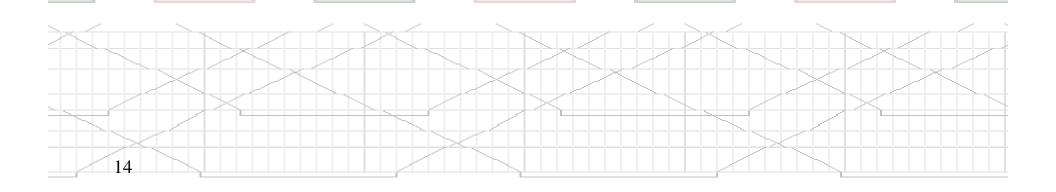
- Let  $P = \{P_1, P_2\}$  where
  - $-P_1$  = set of single line track sections (segments)
  - $-P_2$  = set of double line track sections (segments)



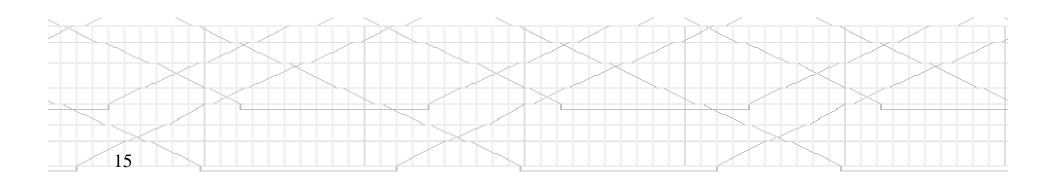


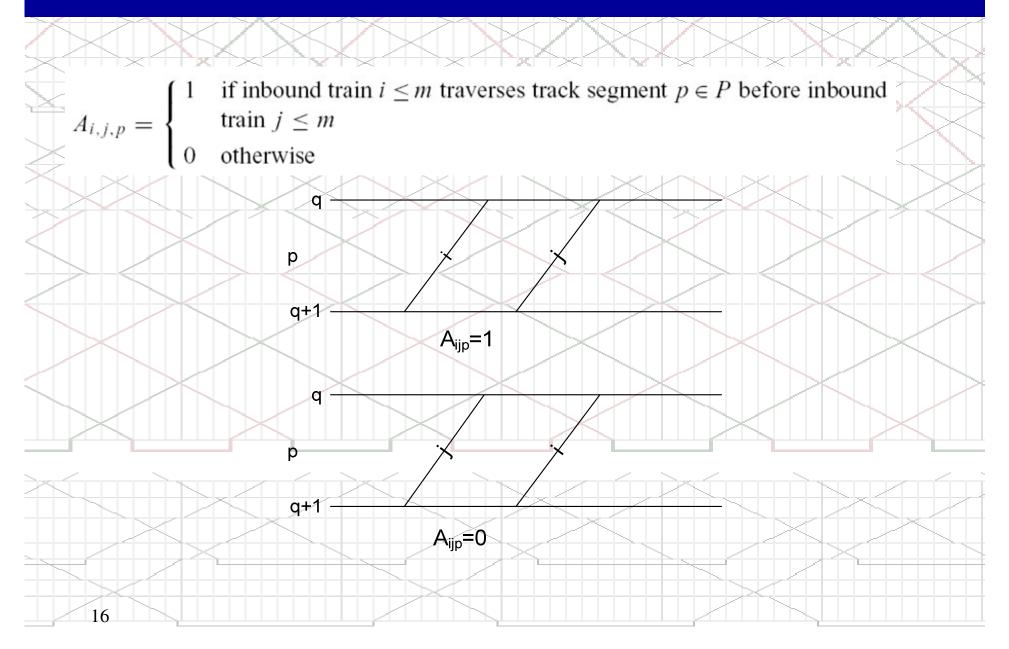
### **Set of Trains**

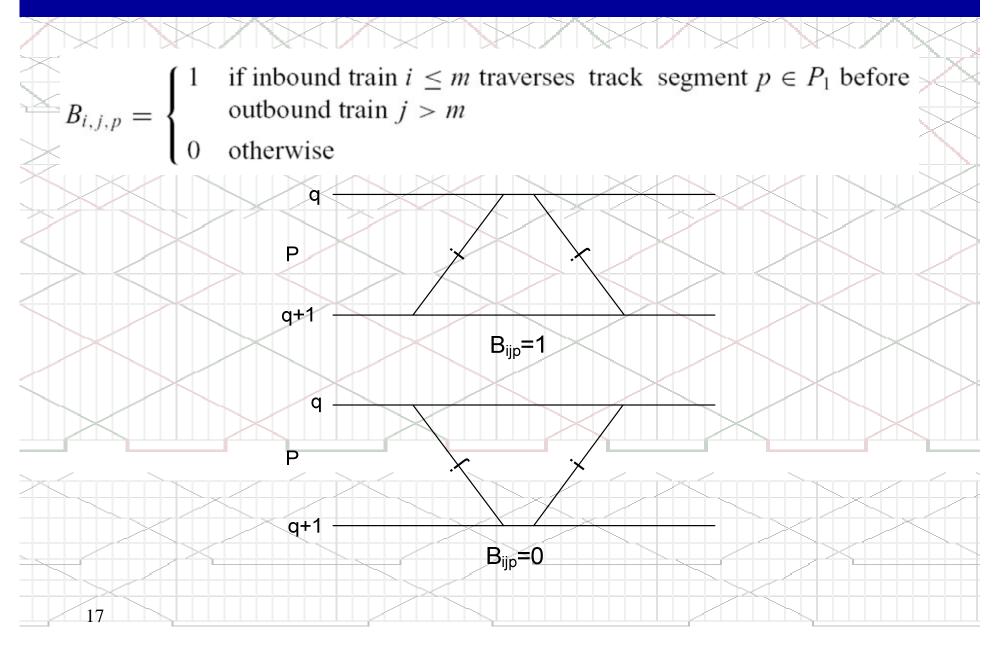
- I={1, 2,..., m, m+1, m+2, ..., N}
  - -The set of trains
  - -Inbound trains are numbered from 1 to *m* and
  - -Outbound trains are numbered from m + 1 to N.
- The ordering of the trains in this set is considered in
  - terms of earliest departure time from the origin station.

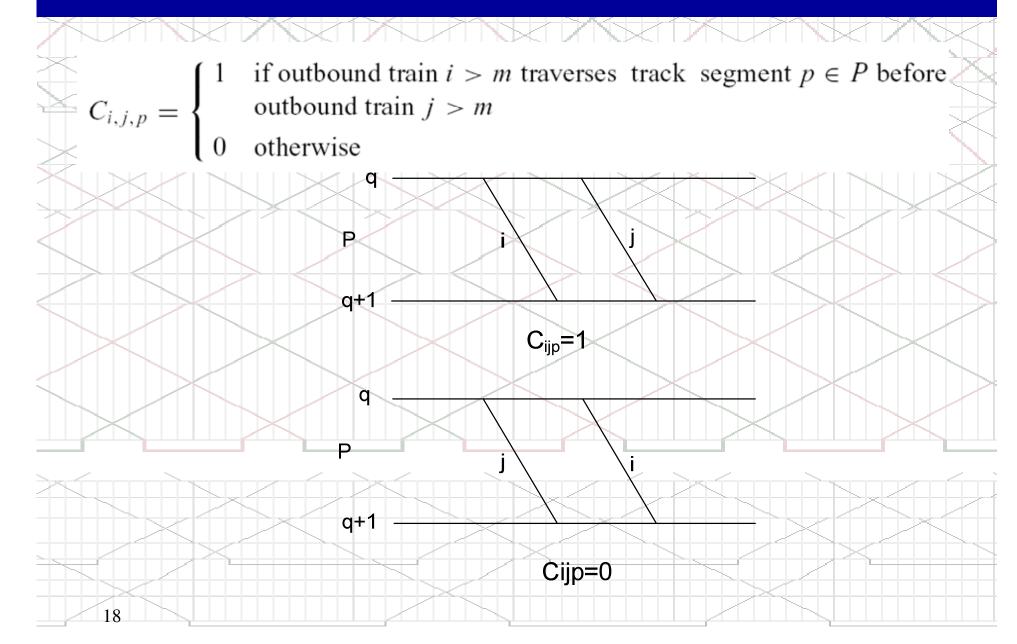


- The integer decision variables for determining which train traverses a track segment first (and the determination of where conflicts are resolved)
- The correct setting of these integer decision variables
  - will ensure the safe operation of conflicting trains.









• The arrival and departure time continuous decision variables are as follows:

 $XA_q^i$  = arrival time of train  $i \in I$  at siding  $q \in Q$  $XD_q^i$  = departure time of train  $i \in I$  from siding  $q \in Q$  $XD_{O_i}^i$  = departure time of train  $i \in I$  from its origin station  $XA_{D_i}^i$  = arrival time of train  $i \in I$  at its destination station

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### **Required Input Parameters**

• The required input parameters for the model are defined as follows:

= origin station of train  $i \in I$ 

 $O_i$ 

 $D_i$ 

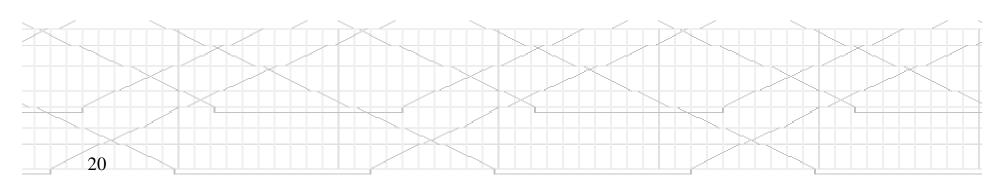
 $d_p$ 

 $ar{v}_p^i$ 

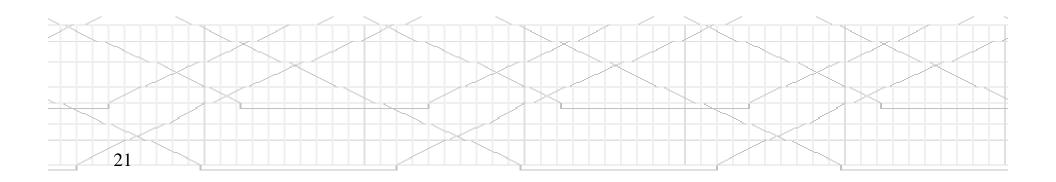
 $W^i$ 

 $S_q^i$ 

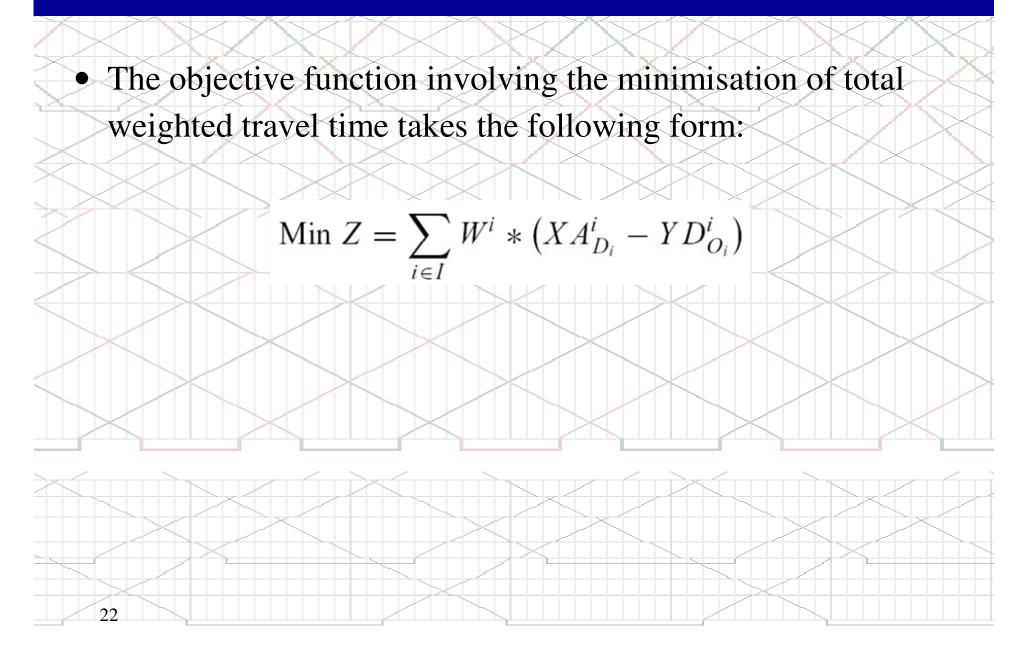
- = destination station of train  $i \in I$
- = minimum headway between trains  $i, j \in I$  on segment  $p \in P$
- = length of segment  $p \in P$
- $YD_{O_i}^i$  = earliest allowable departure time of train  $i \in I$  from its origin station
  - = minimum allowable average velocity of train  $i \in I$  on segment  $p \in P$
  - = maximum achievable average velocity of train  $i \in I$  on segment  $p \in P$
  - = priority of train  $i \in I$  (highest for passenger trains)
  - = scheduled stop time for train  $i \in I$  at siding  $q \in Q$



# **Objective Function and Constraints**



#### **Objective Function**

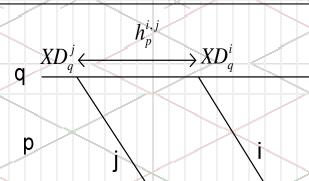


• The following and overtake constraints for outbound trains, if train j travels first:

q+1

 $\frac{M * C_{i,j,p} + XA_{q+1}^{i} \ge XA_{q+1}^{j} + h_{p}^{i,j}}{M * C_{i,j,p} + XD_{q}^{i} \ge XD_{q}^{j} + h_{p}^{i,j}} \bigg\} \forall p \in P \text{ and } i, j > m$ 

- The first inequality implies that if outbound train j is to travel along track segment p first, train i must arrive at siding q after train j plus the minimum headway.
- The second inequality implies that if outbound train j is to travel along track segment p first, then outbound train i must depart siding q after train j plus the minimum headway between the two trains.





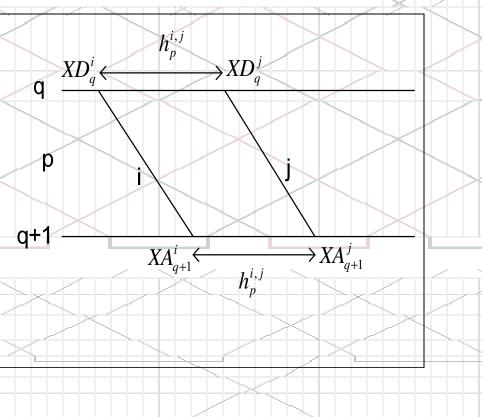
23

• The following and overtake constraints for outbound trains, if train i travels first:

 $\left\{ \begin{array}{l} M * (1 - C_{i,j,p}) + XA_{q+1}^{j} \ge XA_{q+1}^{i} + h_{p}^{i,j} \\ < M * (1 - C_{i,j,p}) + XD_{q}^{j} \ge XD_{q}^{i} + h_{p}^{i,j} \end{array} \right\} \forall p \in P \text{ and } i, j > m$ 

- The first inequality implies that if outbound train i is to travel along track segment p first, train j must arrive at siding q after train i plus the minimum headway.
- The second inequality implies that if outbound train i is to travel along track segment p first, then outbound train j must depart siding q after train i plus the minimum headway between the two trains.

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• The following and overtake constraints for inbound trains, if train j travels first:

$$\left\{ \begin{array}{l} M * A_{i,j,p} + XA_{q}^{i} \geq XA_{q}^{j} + h_{p}^{i,j} \\ M * A_{i,j,p} + XD_{q+1}^{i} \geq XD_{q+1}^{j} + h_{p}^{i,j} \end{array} \right\} \forall p \in P \text{ and } i, j \leq m$$

The following and overtake constraints for inbound trains, if train i travels first:

 $M * (1 - A_{i,j,p}) + XA_{q}^{j} \ge XA_{q}^{i} + h_{p}^{i,j}$  $M * (1 - A_{i,j,p}) + XD_{q+1}^{j} \ge XD_{q+1}^{i} + h_{p}^{i,j}$  $\forall p \in P \text{ and } i, j \le m$ 25

q

q+1

 $XA_{q+1}^{j} \longleftrightarrow XD_{q+1}^{i}$ 

• The constraints for when two trains approach each other if outbound train j travels along track segment p first, are:

$$\left. \begin{array}{l} h_{p}^{i,j} + XA_{q+1}^{j} \leq XD_{q+1}^{i} + M \ast B_{i,j,p} \\ h_{p}^{i,j} + XA_{q}^{i} \leq XD_{q}^{j} + M \ast (1 - B_{i,j,p}) \end{array} \right\} \forall p \in P_{1}, (i \leq m, j > m)$$

 The first inequality implies that if outbound train j travels along track segment p first, inbound train i must depart siding q + 1 after train j arrives plus a minimum headway.

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 $\forall p \in P_1, (i \leq m, j > m)$ 

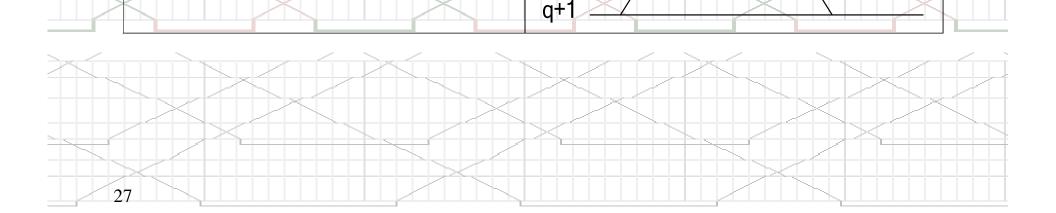
 $h_p^{i,j}$ 

 $XA_a^i \longleftrightarrow XD_a^j$ 

• The constraints for when two trains approach each other if inbound train i travels along track segment p first, are:

$$\left. \begin{array}{l} h_{p}^{i,j} + XA_{q+1}^{j} \leq XD_{q+1}^{i} + M * B_{i,j,p} \\ h_{p}^{i,j} + XA_{q}^{i} \leq XD_{q}^{j} + M * (1 - B_{i,j,p}) \end{array} \right\}$$

 The second inequality implies that if inbound train i travels along track segment p first, outbound train j must depart siding q after train i arrives plus a minimum headway.

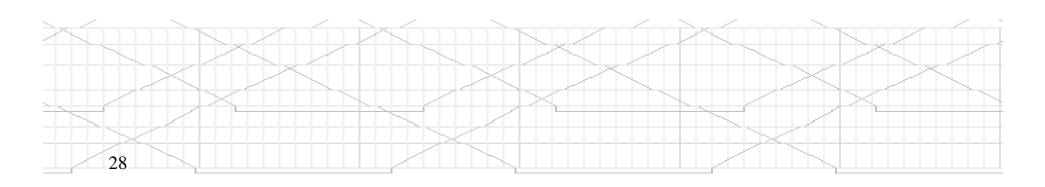


q

р

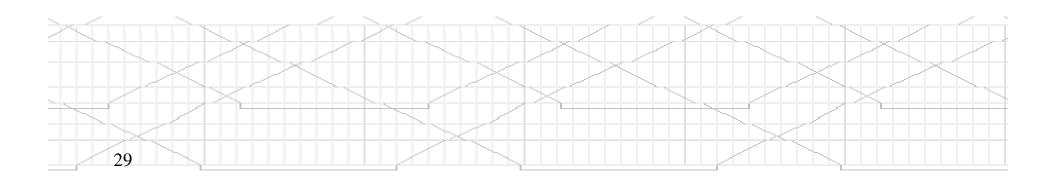
• Given the upper and lower average achievable velocities for each train on each segment, the upper and lower limits for travel time of train i on segment p are given by:

$$\begin{aligned} \frac{d_p}{\bar{v}_p^i} &\leq XA_{q+1}^i - XD_q^i \leq \frac{d_p}{\underline{v}_p^i} \quad i > m, \, p \in P \\ \frac{d_p}{\bar{v}_p^i} &\leq XA_q^i - XD_{q+1}^i \leq \frac{d_p}{\underline{v}_p^i} \quad i \leq m, \, p \in P \end{aligned}$$

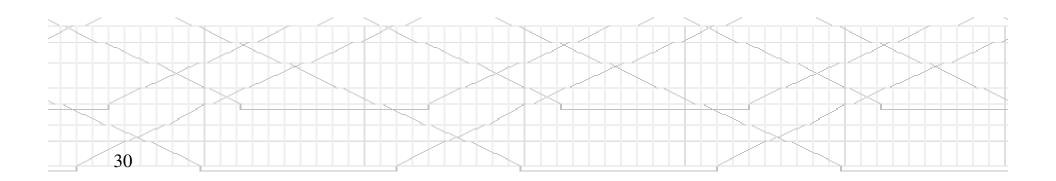


• To stop any train from departing before its earliest departure time and any train departing an intermediate siding before it arrives, the following constraints are included:

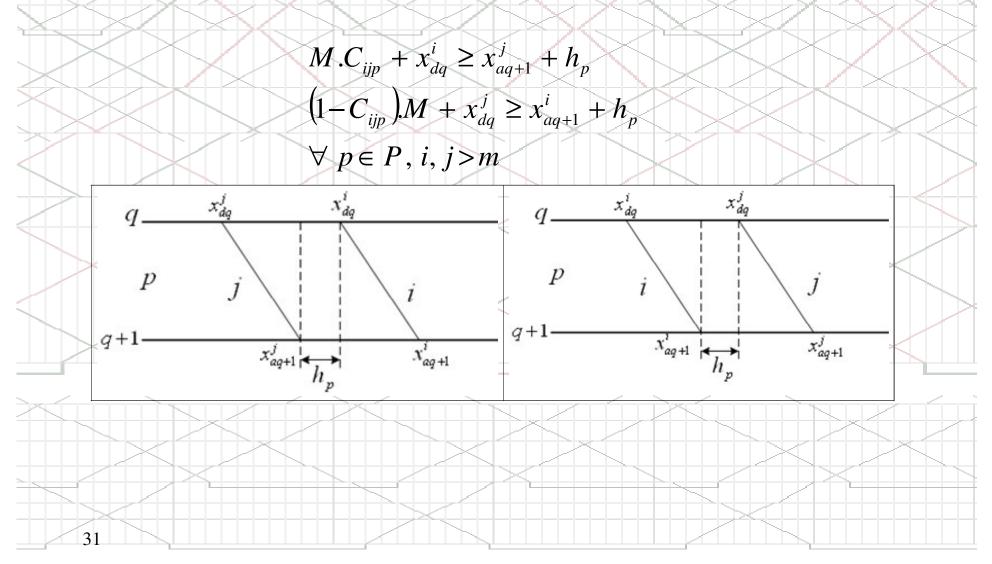
$$\begin{array}{c} XD_{O_i}^i \geq YD_{O_i}^i \\ XA_q^i + S_q^i \leq XD_q^i \end{array} \} \forall i \in I, q \in Q$$



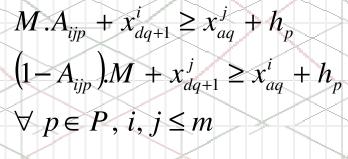
# **A Revised Model**

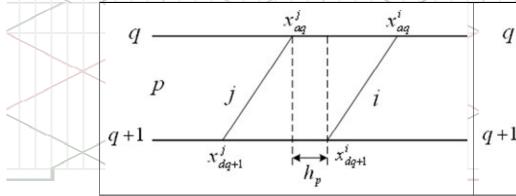


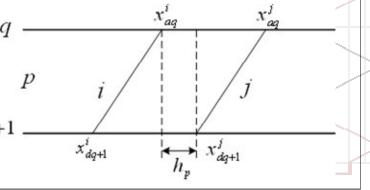
• The following and overtake constraints for outbound trains:

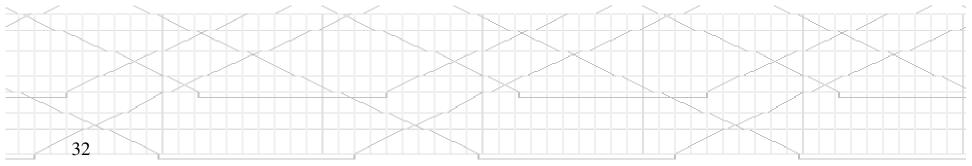


• The following and overtake constraints for inbound trains:







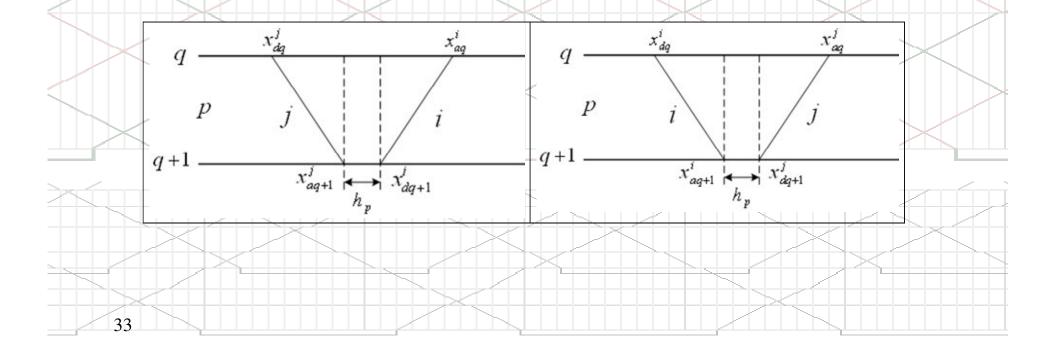


• The constraints for when two trains approach each other:

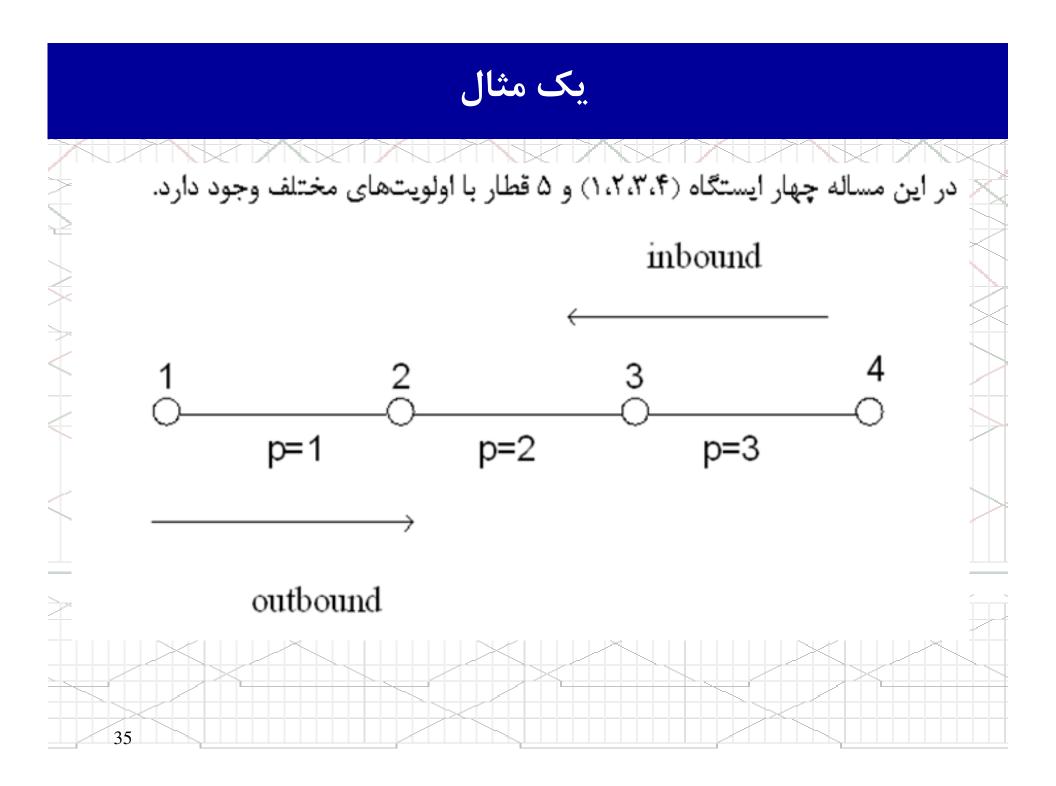
$$M.B_{ijp} + x^{i}_{dq+1} \ge x^{j}_{aq+1} + h_{p}$$

$$(1 - B_{ijp})M + x^{j}_{dq} \ge x^{i}_{aq} + h_{p}$$

$$\forall p \in P, i \le m, j > m$$







، مثال	یک
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جدول اعزام قطارها										
زمان توقف برنامه ای		زمان سیر در	زمان سیر در	زمان سیر در	زودترين زمان اعزام	مقصد	مبدا	اولويت قطار	شمارہ قطار	
ایستگاه ۳	ایستگاه ۲	بلاک ۳	بلاک ۲	بلاک ۱	از مبدا					
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•		۱۴	۱۳	١٣	۰ ۵۰	۴	١	۲	۴	
	١٠		۱۷	۲.	7:79	٣	١	١	۵	

فرضيات عساله:

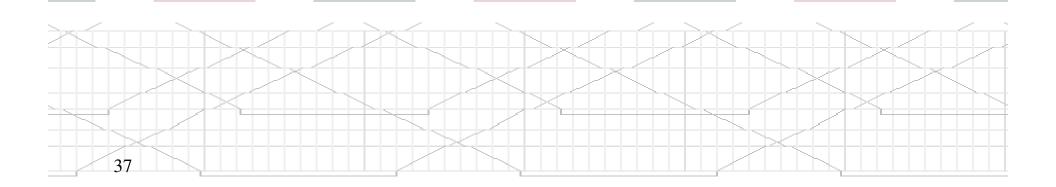
 کلیه بلاکها یک خطه هستند.
 در این مسئله سرعت قطارها در بلاکها ثابت فرض شده است.
 در این مسئله سرعت قطارها در بلاکها ثابت فرض شده است.
 زمان آزاد سازی بلاک برای قطارهای هم جهت ۵ دقیقه و برای قطارهای غیر هم جهت ۴ دقیقه فرض شده است.

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تابع هدف

 $\min = (XA_1^1 - YD_4^1) + (XA_2^2 - YD_4^2) + (XA_4^3 - YD_1^3) + 2(XA_4^4 - YD_1^4) + (XA_3^5 - YD_1^5)$ 

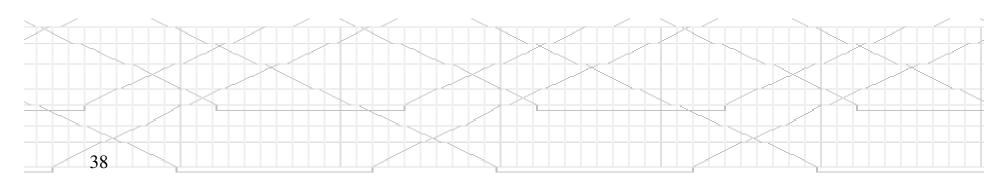
 $\min = (XA_1^1 - 60) + (XA_2^2 - 110) + (XA_4^3 - 80) + 2(XA_4^4 - 110) + (XA_3^5 - 146)$ 



## محدوديتهاي رعايت حداقل فواصل زماني قطارها در مسير رفت

برای بلاک اول (بین ایستگاه ۱ و ۲) –قطار ۳ و ۴:

$$\begin{split} &1000^*C_{341} + x_{d1}^3 \ge x_{d1}^4 + 18 \\ &1000^*C_{341} + x_{a2}^3 \ge x_{a2}^4 + 25 \\ &1000^*(1-C_{341}) + x_{d1}^4 \ge x_{d1}^3 + 25 \\ &1000^*(1-C_{341}) + x_{a2}^4 \ge x_{a2}^3 + 18 \\ &1000^*(1-C_{341}) + x_{a2}^4 \ge x_{a2}^4 + 18 \\ &1000^*(1-C_{341}) + x_{a3}^4 + 1$$



محدودیتهای رعایت حداقل فواصل زمانی قطارها در مسیر رفت

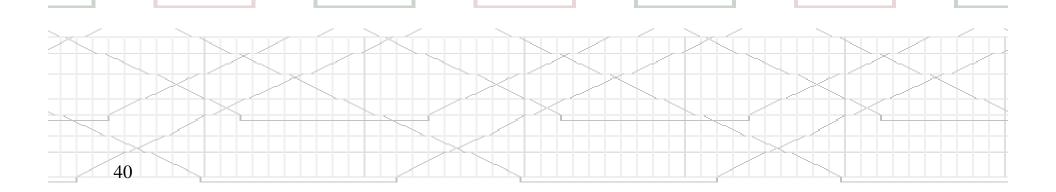
### برای بلاک اول (بین ایستگاه ۱ و ۲) –قطار ۳ و ۵:

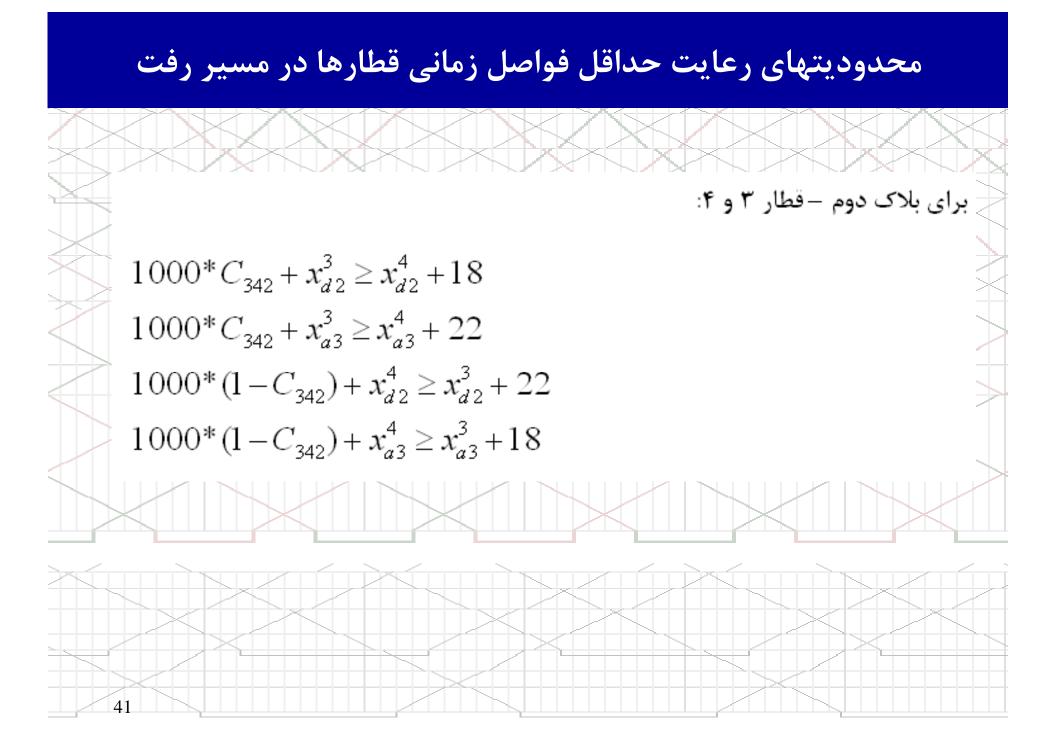
 $1000^*C_{351} + x_{d1}^3 \ge x_{d1}^5 + 25$   $1000^*C_{351} + x_{a2}^3 \ge x_{a2}^5 + 25$   $1000^*(1 - C_{351}) + x_{d1}^5 \ge x_{d1}^3 + 25$  $1000^*(1 - C_{351}) + x_{a2}^5 \ge x_{a2}^3 + 25$ 

محدودیتهای رعایت حداقل فواصل زمانی قطارها در مسیر رفت

### برای بلاک اول (بین ایستگاه ۱ و ۲) –قطار ۴ و ۵:

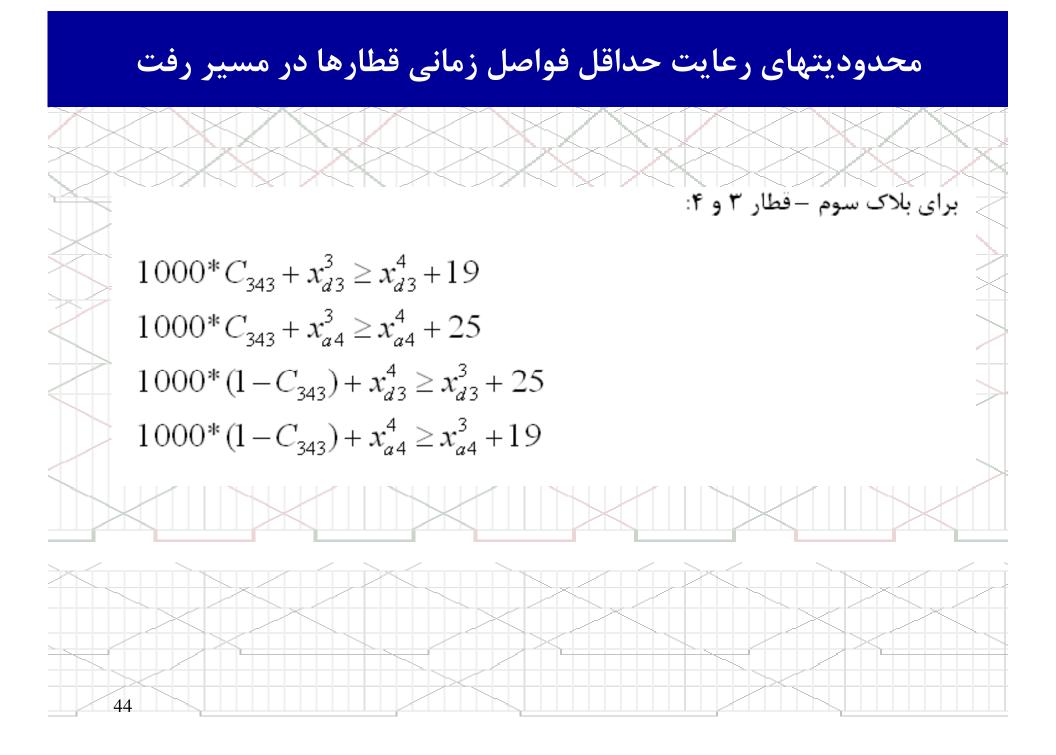
 $1000^{*}C_{451} + x_{d1}^{4} \ge x_{d1}^{5} + 25$   $1000^{*}C_{451} + x_{a2}^{4} \ge x_{a2}^{5} + 18$   $1000^{*}(1 - C_{451}) + x_{d1}^{5} \ge x_{d1}^{4} + 18$  $1000^{*}(1 - C_{451}) + x_{a2}^{5} \ge x_{a2}^{4} + 25$ 





محدودیتهای رعایت حداقل فواصل زمانی قطارها در مسیر رفت برای بلاک دوم – قطار ۳ و ۵:  $1000^*C_{352} + x_{d2}^3 \ge x_{d2}^5 + 22$  $1000^*C_{352} + x_{a3}^3 \ge x_{a3}^5 + 22$  $1000^*(1-C_{352}) + x_{d2}^5 \ge x_{d2}^3 + 22$  $1000^*(1-C_{352}) + x_{a3}^5 \ge x_{a3}^3 + 22$ 42

محدودیتهای رعایت حداقل فواصل زمانی قطارها در مسیر رفت برای بلاک دوم –قطار ۴ و ۵:  $1000^*C_{452} + x_{d2}^4 \ge x_{d2}^5 + 22$  $1000^*C_{452} + x_{a3}^4 \ge x_{a3}^5 + 18$  $1000^*(1-C_{452}) + x_{d2}^5 \ge x_{d2}^4 + 18$  $1000^*(1-C_{452}) + x_{a3}^5 \ge x_{a3}^4 + 22$ 43



محدودیتهای رعایت حداقل فواصل زمانی قطارها در مسیر برگشت

برای بلاک اول: هیچ در قطار برگشت نداریم که بخواهیم حداقل فاصله زمانی برای برگشت آنها را کنترل کنیم. برای بلاک دوم – قطار ۱ و ۲:  $1000 * A_{122} + x_{d3}^1 \ge x_{d3}^2 + 20$  $1000 * A_{122} + x_{a2}^1 \ge x_{a2}^2 + 20$  $1000*(1-A_{122})+x_{d2}^2 \ge x_{d2}^1+20$  $1000*(1-A_{122})+x_{a2}^2 \ge x_{a2}^1+20$ 

محدودیتهای رعایت حداقل فواصل زمانی قطارها در مسیر برگشت برای بلاک سوم –قطار ۱ و ۲:  $1000 * A_{123} + x_{d4}^1 \ge x_{d4}^2 + 25$  $1000 * A_{123} + x_{a3}^1 \ge x_{a3}^2 + 25$  $1000*(1-A_{123})+x_{d4}^2 \ge x_{d4}^1+25$  $1000*(1-A_{123})+x_{a3}^2 \ge x_{a3}^1+25$ 

برای بلاک اول – قطار رفت ۳ و قطار برگشت ۱:

برای بلاک دوم–قطار رفت ۳ و قطار برگشت ۲:  $4 + x_{a3}^3 \le x_{d3}^2 + 1000 * B_{232}$  $4 + x_{a2}^2 \le x_{d2}^3 + 1000 * (1 - B_{232})$ برای بلاک دوم–قطار رفت ۴ و قطار برگشت ۲:  $4 + x_{a3}^4 \le x_{d3}^2 + 1000 * B_{242}$  $4 + x_{a2}^2 \le x_{a2}^4 + 1000 * (1 - B_{242})$ برای بلاک دوم – قطار رفت ۵ و قطار برگشت ۲:  $4 + x_{a3}^5 \le x_{d3}^2 + 1000 * B_{252}$  $4 + x_{a2}^2 \le x_{d2}^5 + 1000 * (1 - B_{252})$ 

برای بلاک سوم-قطار رفت ۳ و قطار برگشت ۱:  

$$4 + x_{a4}^{3} \le x_{d4}^{1} + 1000 * B_{133}$$

$$4 + x_{a3}^{1} \le x_{d3}^{3} + 1000 * (1 - B_{133})$$

$$4 + x_{a3}^{1} \le x_{d3}^{1} + 1000 * (1 - B_{133})$$

$$4 + x_{a4}^{1} \le x_{d4}^{1} + 1000 * (1 - B_{143})$$

$$4 + x_{a3}^{1} \le x_{d3}^{4} + 1000 * (1 - B_{143})$$

$$4 + x_{a3}^{3} \le x_{d4}^{2} + 1000 * B_{233}$$

$$4 + x_{a3}^{2} \le x_{d3}^{3} + 1000 * (1 - B_{233})$$

$$x_{d3} + x_{a3} \le x_{d3}^{2} + 1000 * B_{243}$$

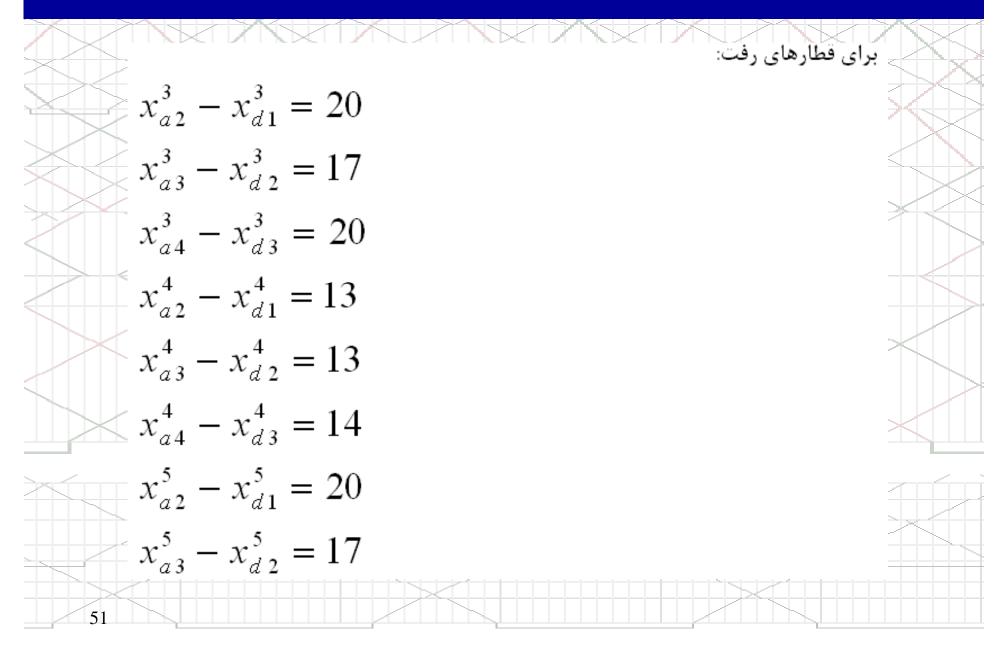
$$4 + x_{a3}^{4} \le x_{d4}^{2} + 1000 * B_{243}$$

$$4 + x_{a3}^{4} \le x_{d3}^{2} + 1000 * B_{243}$$

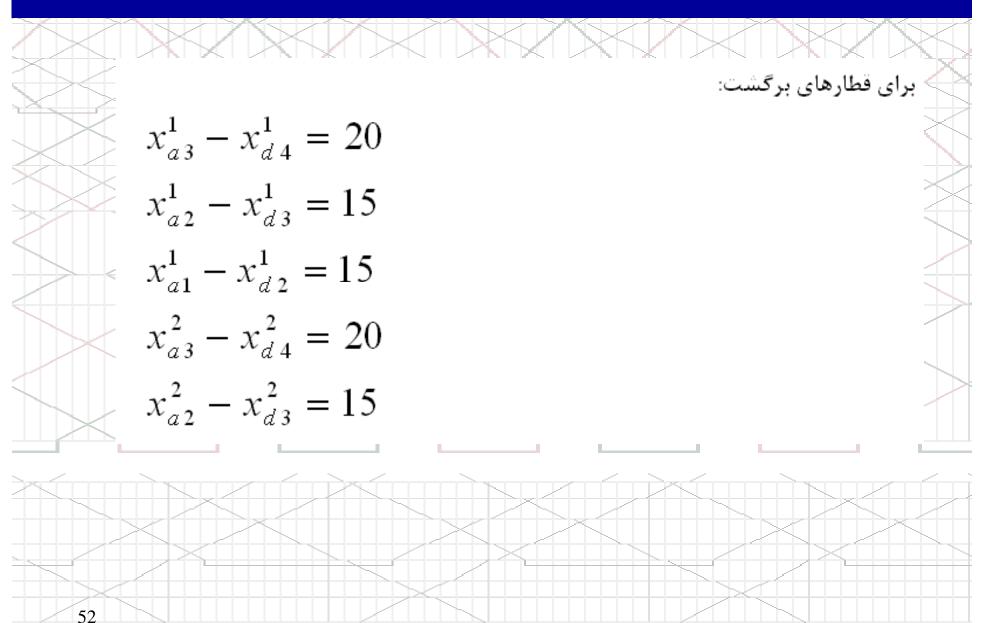
$$4 + x_{a3}^{2} \le x_{d3}^{4} + 1000 * (1 - B_{243})$$

$$50$$

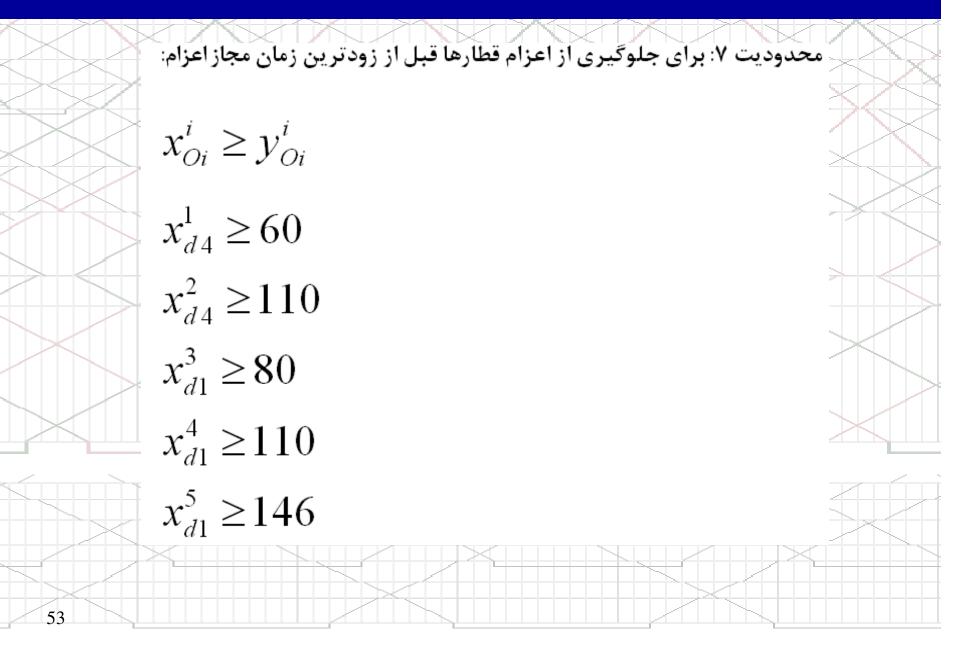
محدودیتهای سرعت در بلاک



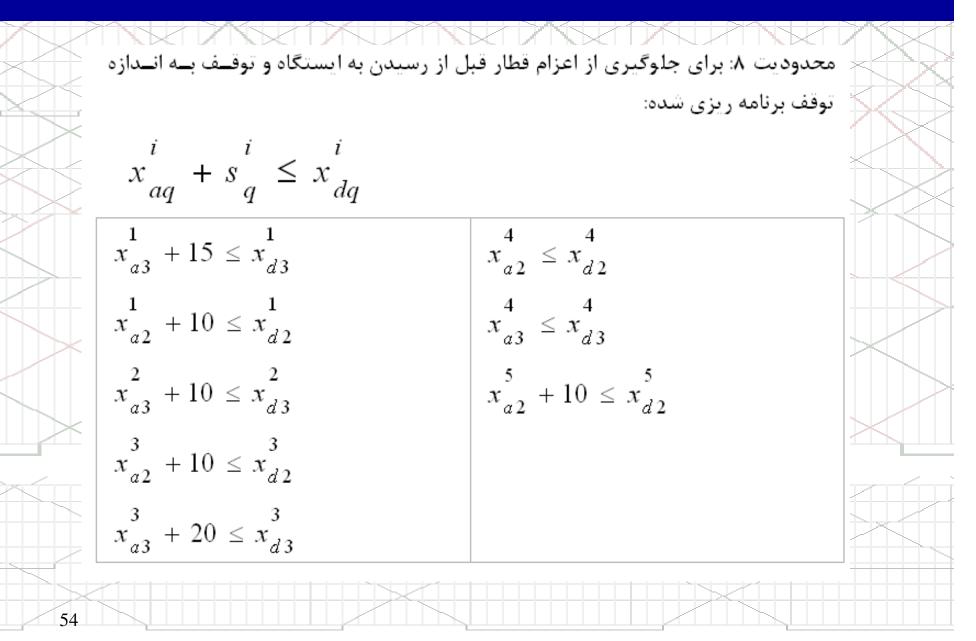
محدودیتهای سرعت در بلاک



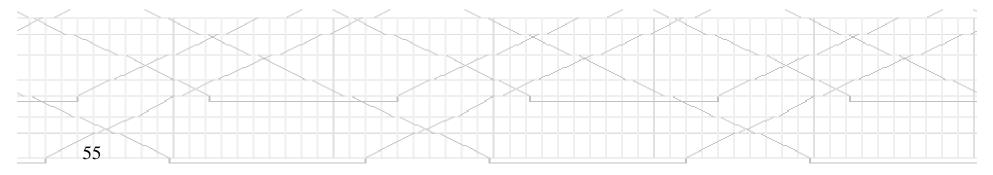




محدوديتها



# References



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