بسم الله الرحمن الرحيم

برنامه ریزی حمل و نقل ریلی

فصل ۲۲: مدل برنامه ریزی خدمه (مدل Caprara)

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پائیز ۱۳۸۹

Crew Scheduling Model

(Caprara Model)

Definitions

Definitions

- **Crew management**
  - is concerned with building the work schedules of crews needed to cover a planned timetable.
  - This is part of **tactical planning** which concern the medium-term use of the available resources.

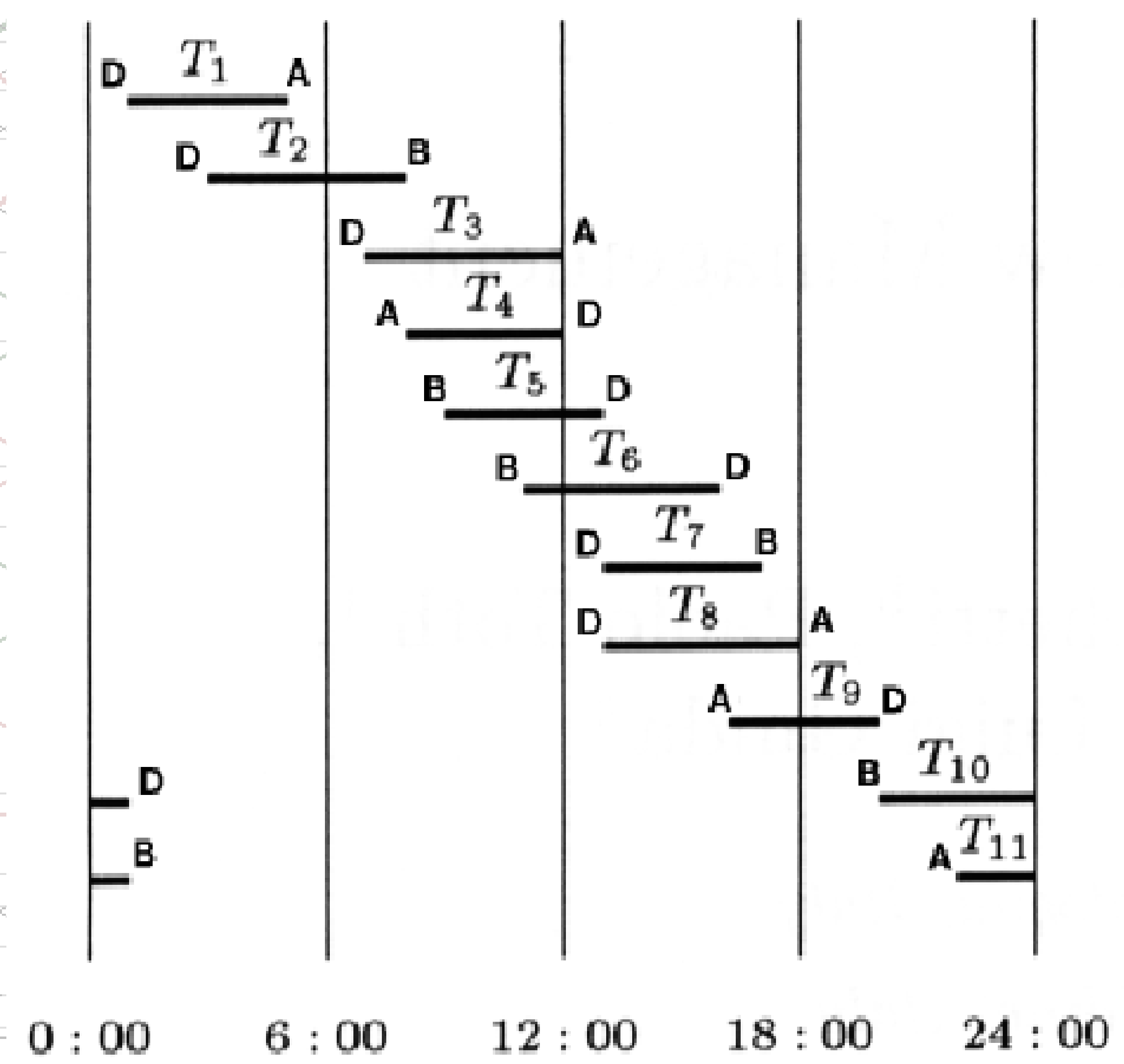
Definitions

- **Crews**
  - Personnel grouped together as crews
- **Train services**
  - We are given a planned timetable for the **train services** to be performed every day of a certain time period.
  - Train services include both the actual journeys with passengers or freight, and the transfers of empty trains or equipment between different stations.

Definitions

- **Trips**
  - Each train service has first been split into a sequence of trips
  - Trips include segments of train journeys which must be serviced by the same crew without rest.
  - Each trip is characterized by:
    - a departure time,
    - a departure station,
    - an arrival time,
    - an arrival station,
    - and possibly by additional attributes.
  - Each daily occurrence of a trip has to be performed by one crew.

Fig. 1. The trips to be covered everyday

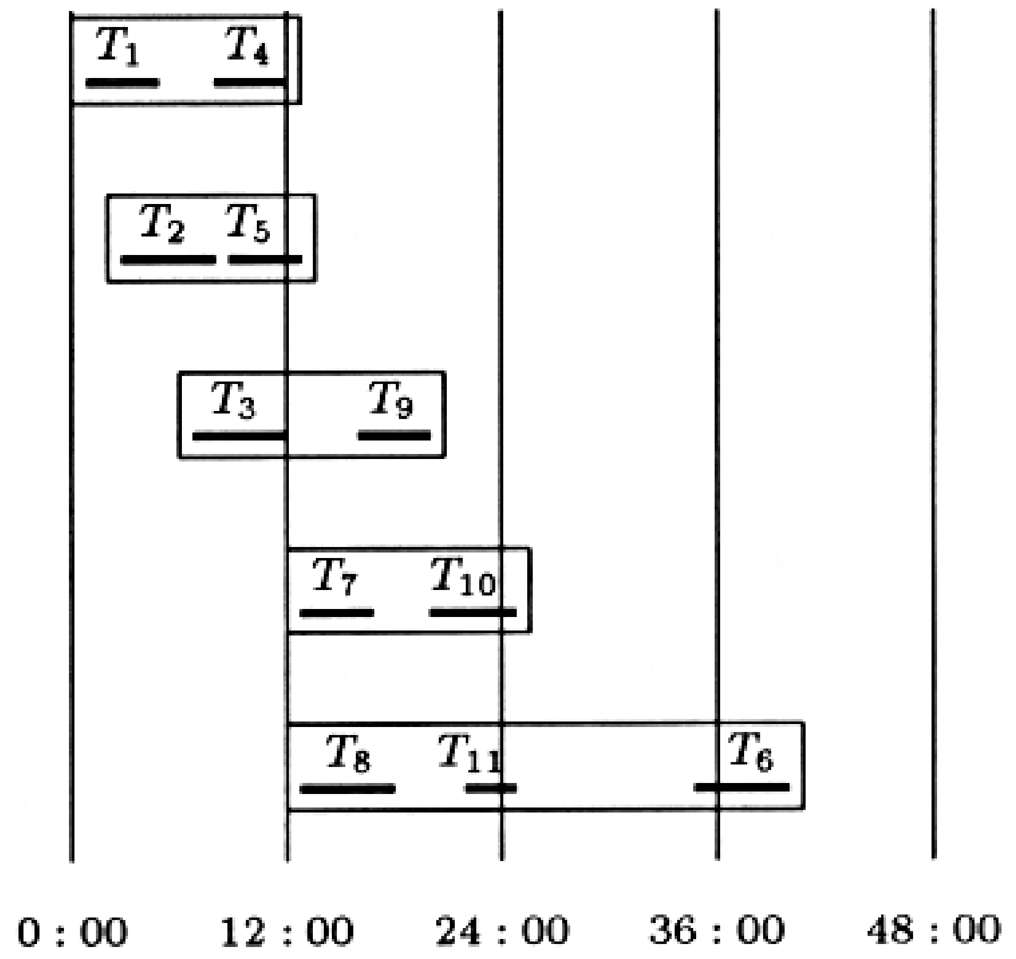


Definitions: Duty & Depot

- **Duty**
  - A sequence of trips to be covered by a single crew within a given time period that covers at most L consecutive days is called **duty** or **pairing**.
  - In railroad application, L is typically at most 2.

- **Depot**
  - A **depot** represents the starting and ending point of crew’s work segments

Fig. 2. Duties covering all the trips



Each duty overlaps at most L = 2 consecutive days

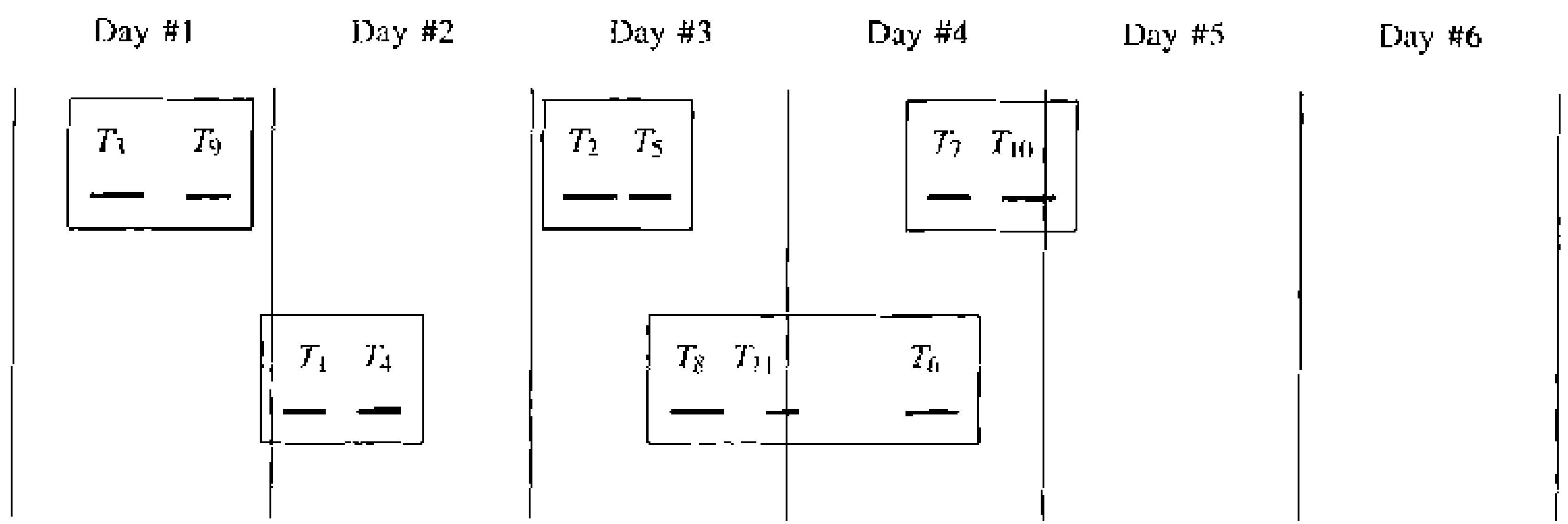
Definitions: Pairing Generation

- **Pairing Generation**
  - is a preprocessing phase of crew scheduling which all feasible duties are computed and stored.

Definitions: Roster

- **Roster**
  - **A roster** is a sequence of trips whose operational cost and feasibility depend on several rules laid down by union contracts and company regulations.
  - The roster consists of the cyclic trip sequence
  - Each crew performs a roster

Fig. 3. A Roster Covering all Trips



- The roster spans 12 days
- Each 6th day is left idle for crew rest
- According to the roster, 12 crews are needed to perform each daily occurrence of the given trips

Definitions: The Roster

- The first crew covers:
  - on calendar day d, say, trips T3 and T9,
  - on calendar day d + 1 no trip,
  - on calendar day d + 2 trips T2 and T5
  - .....
  - on calendar day d + 11 no trip,
  - on calendar day d + 12 again trips T3 and T9, and so on.

Definitions: The Roster

- On calendar day d + 1,
  - trips T3 and T9 are instead covered by the second crew,
  - which performs no trips on day d + 2,
  - trips T2 and T5 on day d + 3, and so on.
- Trips T3 and T9
  - on calendar day d + 2 are covered by crew number 3,
  - on calendar clay d + 3 by crew number 4 .....
  - on calendar day d+ 11 by crew number 12, and
  - on calendar day d + 12 by crew number 1 again.

Railway Crew Management

Railway Crew Management

- Railway crew management represents a very complex and challenging problem due to both the size of the instances to be solved and the type and number of operational constraints.
- Typical figures at the Italian railway company, are about 8,000 trains per day and a workforce of 25,000 drivers spread among several depots.
- The largest planning problems concern the inter-city and long-range passenger trains, and involve about 2,000 trains split into 5,000 trips per day.

Railway Crew Management

- The crew management problem consists of:
  - Finding a set of rosters covering every trip of the given time period, so as to satisfy all the operational constraints, with minimum cost.
  - A main objective of crew management is the minimization of the global number of crews needed to perform all the daily occurrences of the trips in the given period
- In practice, the overall crew management problem is approached in two phases:
  - **Crew scheduling (duty optimization)**
  - **Crew rostering**

Crew scheduling

- The short-term schedule of the crews is considered, and a set of duties covering all the trips is constructed.
- In the example, the trips are covered by means of the 5 duties reported in Fig. 2.
- The objective used in the crew scheduling phase mainly calls for the minimization of the number of working days corresponding to the duties.

Crew rostering

- The duties selected in phase 1 are sequenced to obtain the final rosters.
- In this step, trips are no longer taken into account explicitly, but determine the attributes of the duties which are relevant for the roster feasibility and cost.
- In Fig. 2. the 5 duties are sequenced to obtain the 12-day roster in Fig. 3.

Reasons of Decomposition

- Constraints for short term work segments are different from constraints for longer periods
  - For example, in the Italian railway company the minimum time interval between two consecutive trips in a duty is a few minutes for changing trains, whereas the time interval between two consecutive duties is 18-22 hours for home rest.
- Each crew must return within a given time to a home depot, resulting in a natural constraint for the crew scheduling phase.

Reasons of Decomposition

- The problem is much easier to solve, since typically both parts can be modeled independently, resulting in smaller problem descriptions for both phases.
- The decomposition approach fits nicely into current planners methods, especially the duty optimization can be done centrally, whereas each depot can do the rostering phase separately for its associated duties.

Crew Scheduling Model

Crew Scheduling Problem

- Crew scheduling problem requires finding min-cost ***sequences*** through a given set of ***items***.
- Items correspond to trips and sequences correspond to duties.
- A formulation of the problem in term of graphs associates a node with each item, and a directed arc with each item transition.
- A directed graph, $G = (V, A)$ having one node $j \in V$ for each trip, and an arc $(i, j) \in A$ if trip j can appear right after item i in a feasible sequence
- With this representation, the problems can be formulated as finding a min-cost collection of ***circuits*** (or ***paths***) of G covering each node once

Crew Scheduling in Urban Transit

- Consider crew scheduling in the context of urban mass transit companies
- Where duty duration (spread time) is less than 24 hours.
- Here, a minimum duty start time *b* (e.g., 2 a.m.) is given.
- Accordingly, all departure/arrival times between 0 (midnight) and b are increased by 24 hours, and an arc $(i, j) \in A$ exists only if the arrival time of trip *i* is not greater than the departure time of trip *j*.

Crew Scheduling in Railway Application

- Where duty duration (spread time) in railway applications is greater than 24 hours
- This allows an arc to connect a trip i to a trip j even if the arrival time of i is greater than the departure time of j.
- Meaning that a crew performs trips i and j on different days.
- In this case, crew scheduling calls for a min-cost collection of paths covering all the nodes once, each path satisfying a set of constraints related to the feasibility of the corresponding duty (maximum driving time, meal breaks, etc.).

Crew Scheduling in Railway Application

- As already mentioned, a basic constraint for crew scheduling is that every duty must start and end at the crew home location (depot).

- It is then natural to introduce in G a dummy node d for each depot, along with the associated arcs (d, j) (respectively, (j, d)) for each node j associated with a trip which can be the first (respectively, the last) trip in a duty assigned to depot d.

Crew Scheduling in Railway Application

- This allows one to convert each path representing a duty into a circuit by connecting the terminal nodes of the path to the depot node representing the home location of the crew.

- There are two basic ways of modeling as an integer linear program the problem of covering the nodes of a directed graph through a suitable set of circuits.

First Crew Scheduling Model

First Crew Scheduling Model

$G = (V, A)$: A directed graph, having one node $j \in V$ for each trip, and an arc $(i, j) \in A$ if trip *j* can appear right after item *i* in a feasible sequence

d: a dummy node for each depot, along with the associated arcs *(d , j)* (respectively, *(j, d)*) for each node *j* associated with a trip which can be the first (respectively, the last) trip in a duty assigned to depot $d \in D$.

$\delta^{+}(v)$: represent the set of the arcs of *G* leaving node $v \in V$.

$\delta^{-}(v)$: represent the set of the arcs of *G* entering node $v \in V$.

First Crew Scheduling Model

D:	denote the set of depot nodes $D \subset V$.
$\mathcal{P}$:	is the family of all arc subsets P.
P:	arc subset which can not be part of any feasible solution $P \in \mathcal{P}$.
c_{ij}:	the cost of each arc $(i, j) \in A$.
x_{ij}:	a binary variable with each arc $(i, j) \in A$, where $x_{ij} = 1$ if arc (i, j) is used in the optimal solution and $x_{ij} = 0$ otherwise.

First Crew Scheduling Model

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} \qquad (1)$$

Subject to

$$\sum_{(i,j)\in \delta^+(v)} x_{ij} = \sum_{(i,j)\in \delta^-(v)} x_{ij} = 1\,,\ v \in V \setminus D \qquad (2)$$

$$\sum_{(i,j)\in \delta^+(v)} x_{ij} = \sum_{(i,j)\in \delta^-(v)} x_{ij}\ ,\ v \in D \qquad (3)$$

$$\sum_{(i,j)\in P} x_{ij} \le |P| - 1,\ \ P \in \mathcal{P} \qquad (4)$$

$$x_{ij} \in \{0,1\}, \quad (i,j) \in A \qquad (5)$$

First Crew Scheduling Model

- Constraints (2) impose that the same number of arcs enter and leave each node, and that each node not associated with a depot is covered exactly once.
- Constraints (3) impose that the same number of arcs enter and leave each depot.
- Constraints (4) forbid the choice of all the arcs in any infeasible arc subset P.

First Crew Scheduling Model

- Notice that $\mathcal{P}$ contains all the arc sequences which cannot be covered by a single crew because of operational constraints.
- $|\mathcal{P}|$ may grow exponentially with $|V|$
- In addition, $\mathcal{P}$ may contain subsets of arcs which cannot all be selected because of constraints related to the infeasibility of a group of circuits; these are typically called ***crew base constraints***.

Crew Base Constraints

- **Crew base constraints** have to be fulfilled:
  - lower and upper bounds on the number of selected duties associated with each depot
  - maximum percentage of selected overnight duties for each depot
  - maximum percentage of selected duties with external rest for each depot
  - similar constraints for all duties together
- These constraints can not be part of the sequencing rules or the overall duty constraints

First Crew Scheduling Model

- This model can only be applied when the cost of the solution can be expressed as the sum of the costs associated with the arcs.
- Hence it cannot be used when the cost of a circuit depends on the overall node sequence, or on the "type" of the crew, e.g., on the home location.

A Variant of the First Model

x_{ij}^k: a binary variable with each arc $(i, j) \in A$ when performed by a crew of type *k*, where $x_{ij}^k = 1$ means that a crew of type *k* covers nodes *i* and *j* in sequence, and $x_{ij}^k = 0$ otherwise.

c_{ij}^k: the cost of each arc $(i, j) \in A$, when performed by a crew of type *k*, where $c_{ij}^k = +\infty$ if (*i*, *j*) cannot be used by a crew of type k.

K: the set of crew types.

$\mathcal{P}^k$: is the family of all arc subsets P which cannot be part of any feasible solution for the crews of type *k*.

A Variant of the First Model

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \quad (6)$$

Subject to

$$\sum_{(i,j) \in \delta^+(v)} x_{ij}^k = \sum_{(i,j) \in \delta^-(v)} x_{ij}^k, \quad v \in V, k \in K \quad (7)$$

$$\sum_{(i,j) \in P} x_{ij}^k \leq |P| - 1, \; P \in \mathcal{P}^k, \; k \in K \quad (8)$$

$$\sum_{k \in K} \sum_{(i,j) \in \delta^+(v)} x_{ij}^k = 1, \; v \in V \setminus D \quad (9)$$

$$x_{ij}^k \in \{0,1\}, \; (i,j) \in A, \quad k \in K \quad (10)$$

A Variant of the First Model

- Model (6) - (10) allows arc costs depending on the crew type.
- Moreover, infeasibility constraints of type (8) can exploit the fact that the type of crew is given, which may lead to tighter linear programming relaxations.
- An obvious drawback is the increased size of the model, in terms of both the number of variables and constraints.

Second Crew Scheduling Model

Second Crew Scheduling Model

C : denote the collection of all the simple circuits of G corresponding to a feasible duty for a crew, $C = \{C_1, \ldots., C_n\}$, and with n = $|C|$.

C_j : is a circuits of G corresponding to a feasible duty for a crew

$\mathcal{S}$: denotes the family of all sets S.

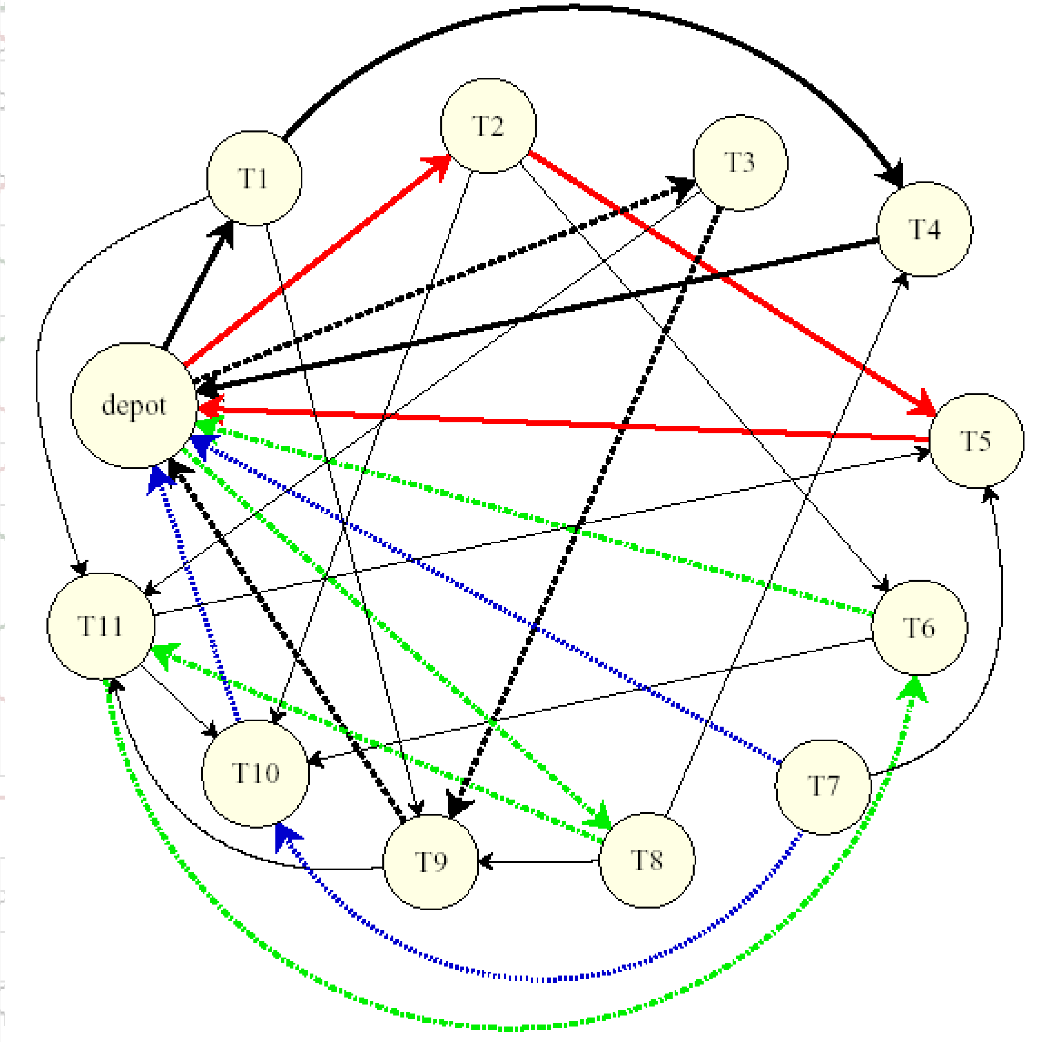
S: $S \subseteq \{1, \ldots., n\}$ with the property that no feasible solution contains all circuits C_j for $j \in S$.

c_j : the cost of C_j

I_j : the node set which is covered by duty of C_j

y_j : The binary variable, takes value 1 if C_j is part of the optimal solution, and 0 otherwise.

Fig. 4. Graph representation of the crew



Second Crew Scheduling Model

set partitioning problem with ***side constraints***

$$\min \sum_{j=1}^{n} c_j y_j \qquad (11)$$

Subject to

$$\sum_{j: v \in I_j} y_j = 1, \qquad v \in V \setminus D \qquad (12)$$

$$\sum_{j \in s} y_j \le |S| - 1, \quad S \in \mathcal{S} \qquad (13)$$

$$y_j \in \{0, 1\}, \quad j = 1, \dots\dots\dots, n \qquad (14)$$

Second Crew Scheduling Model

- Constraints (12) impose that each node not associated with a depot is covered by exactly one circuit,
- Constraints (13) model the crew base constraints.

Second Crew Scheduling Model

- A main advantage of the set partitioning model is that it allows for circuit costs depending on the whole sequence of arcs, and possibly on the crew type.
- Moreover, the feasibility constraints (13) need not take into account restrictions concerning the feasibility of a single circuit.
- The second model has a possibly exponential number of binary variables, each associated with a feasible circuit of G.

Crew Scheduling Model at the Italian Railways

Crew Scheduling at the Italian Railways

- In Italian railway applications a typical crew duty lasts no more than 24 hours and covers only a few trips.
- Heavy operational constraints affect duty feasibility.
- This makes it practical to effect the explicit generation of all feasible duties, which are computed and stored in ***pairing generation***.

Crew Scheduling at the Italian Railways

- In addition, operational rules allow a crew to be transported with no extra cost as a passenger on a trip, hence the overall solution can cover a trip more than once.
- In this situation, the ***set partitioning formulation*** (11)-(14) can be replaced by its ***set covering problem*** relaxation obtained by replacing = with $\geq$ in (12).

Crew Scheduling at the Italian Railways

- Even without side constraints (13), set covering problems arising in railway applications appear rather difficult mainly because of their size.
- Indeed, the largest instances at the Italian railways involve up to 5,000 trips and 1,000,000 duties, i.e., they are 1-2 orders of size larger than those arising in typical airline applications.

Pure Set Covering Problem (SCP)

I_j:	the node set which is covered by duty of *j th*
N:	set of duties, $N=\{1,\ldots,n\}$
c_j:	the cost of duty of *j th*
M	trip set, $M=\{1,\ldots,m\}$
i :	index for trips, $i \in M$
j :	index for duties, $j \in N$
J_i	the collection of duties which, trip of *i th* is included in them, $J_i \in \{ j \in N \mid i \in I_j \}$
y_j	$y_j = 1$ if duty *j* is selected in the optimal solution, $y_j = 0$ otherwise.

Pure Set Covering Problem (SCP)

$$\min \sum_{j \in N} c_j y_j \qquad (15)$$

Subject to

$$\sum_{j \in J_i} y_j \geq 1, \quad i \in M \qquad (16)$$

$$y_j \in \{0,1\}, \quad j \in N \qquad (17)$$

Crew Rostering Problem

Crew Rostering Problem

- A roster contains a subset of duties and spans a cyclic sequence of groups of 6 consecutive days, conventionally called **weeks**.
- The number of days in a roster is an integer multiple of 6.
- The crew rostering problem consists of finding a feasible set of rosters covering all the duties and spanning a minimum number of weeks.

Crew Rostering Problem

- The global number of crews required every day to cover all the duties is equal to 6 times the total number of weeks in the solution.
- Thus, the minimization of the number of weeks implies the minimization of the global number of crews required.

Crew Rostering Problem

- Each duty can have additional characteristics:
  - **duty with external rest**, if it includes a long rest out of the depot for the crew;
  - **long duty**, if it does not include an external rest and its working time is longer than 8 hours;
  - **overnight duty**, if it requires some working between midnight and 5 am;
  - **heavy overnight duty**, if it is an overnight duty without external rest, and requires more than 1 hour and 30 minutes' work between midnight and 5 am.

Crew Rostering Model

- There are two types of rests, conventionally called simple and double rests.
  - **Simple rests:** must be at least 48 hours long,
  - **Double rests:** must span at least two complete days, i.e., either the fifth and sixth day of a week or the sixth day of a week and the first day of the following one.

Crew Rostering Model

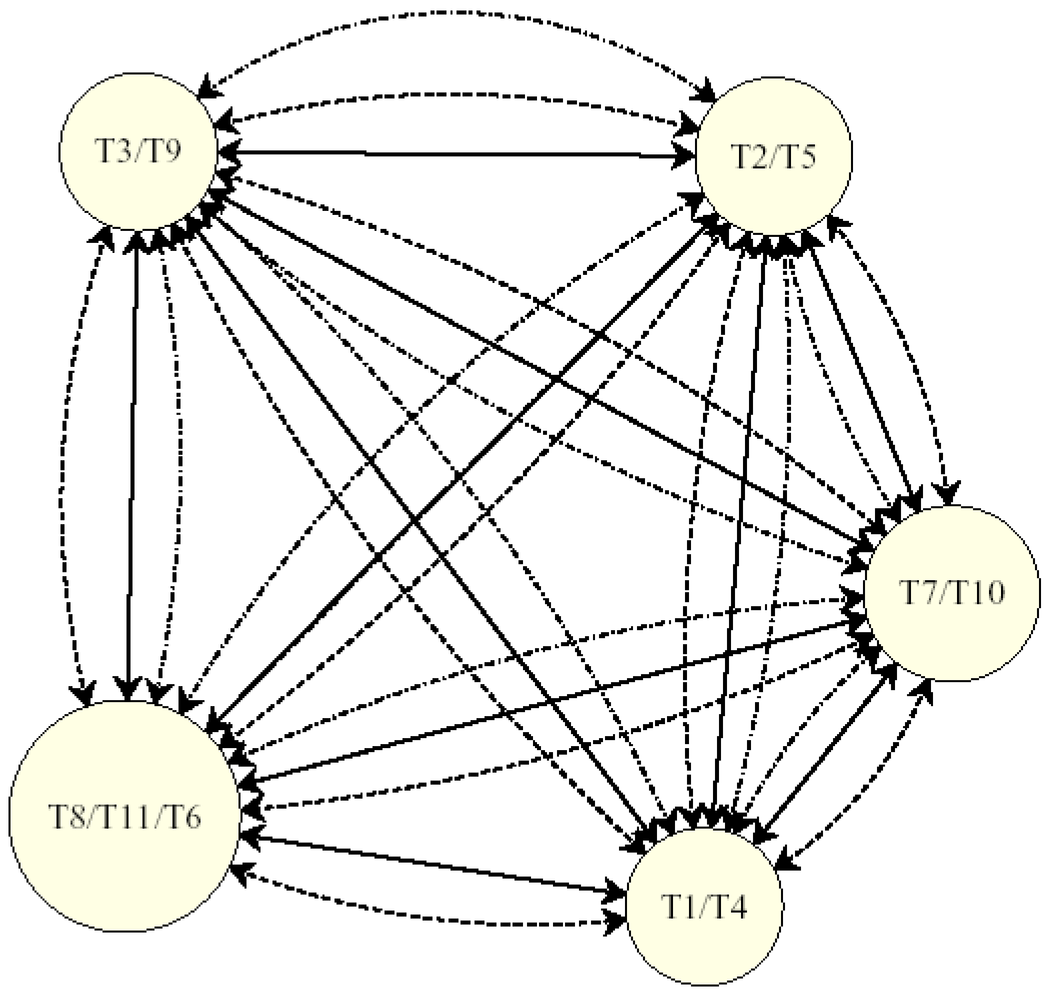
$G = (V, A)$: A directed graph, where each node in $V=\{1, ..., n\}$ is associated with a duty and the arcs represent the consecutive sequencing of duty pairs within a roster, $A=\{A_1, A_2, A_3\}$

A_1: For each pair of nodes $i, j \in V$, we have an arc $(i, j) \in A_1$, when the nodes are sequenced directly in the same week. These arcs are called ***directed arcs***.

A_2: For each pair of nodes $i, j \in V$, we have an arc $(i, j) \in A_2$, when a simple rest is imposed between them. These arcs are called ***simple-rest arcs***.

A_3: For each pair of nodes $i, j \in V$, we have an arc $(i, j) \in A_3$, when a double rest is imposed between them. These arcs are called ***double-rest arcs***.

Graph Representation



Crew Rostering Model

c_{ij}^{1} : Is the minimum time (in minutes) between the start of duty i and the start of duty j when they are sequenced directly in the same week, $(i, j) \in A_1$.

c_{ij}^{2} : Is the minimum time (in minutes) between the start of duty i and the start of duty j when a simple rest is imposed between them, $(i, j) \in A_2$

c_{ij}^{3} : Is the minimum time (in minutes) between the start of duty i and the start of duty j when a double rest is imposed between them, $(i, j) \in A_3$

Crew Rostering Model

x_{ij}^{1} : A binary variable equal to 1 if the directed arc $(i, j) \in A_1$ is in the optimal solution, and 0 otherwise.

x_{ij}^{2} : A binary variable equal to 1 if the simple-rest arc $(i, j) \in A_2$ is in the optimal solution, and 0 otherwise.

x_{ij}^{3} : A binary variable equal to 1 if the double-rest arc $(i, j) \in A_3$ is in the optimal solution, and 0 otherwise.

r An integer variable represents the minimum number of simple- or double-rest arcs in the solution,

z An integer variable represents the minimum number of double-rest arcs in the solution.

α The number of minutes in a week, 6 * 1440.

Crew Rostering Model

- Objective Function:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij}^{1} x_{ij}^{1} + c_{ij}^{2} x_{ij}^{2} + c_{ij}^{3} x_{ij}^{3}) \qquad (19)$$

Crew Rostering Model

$$\sum_{i=1}^{n}(x_{ij}^{1}+x_{ij}^{2}+x_{ij}^{3})=1, \qquad j=1,\ldots,n \tag{20}$$

$$\sum_{j=1}^{n}(x_{ij}^{1}+x_{ij}^{2}+x_{ij}^{3})=1, \qquad i=1,\ldots,n \tag{21}$$

- Constraints (20) & (21): impose that each node has exactly one entering and one leaving arc

Crew Rostering Model

$$r \geq \frac{1}{\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij}^{1}x_{ij}^{1} + c_{ij}^{2}x_{ij}^{2} + c_{ij}^{3}x_{ij}^{3}) \qquad (22)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij}^{2} + x_{ij}^{3}) \geq r \qquad (23)$$

- Constraints (22) & (23): ensure that the total number of simple- or double-rest arcs is at least the total cost of the solution, expressed in weeks.

Crew Rostering Model

$$z \geq 0.4\, r \qquad (24)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^{3} \geq z \qquad (25)$$

- Constraints (24) and (25) ensure that the total number of double-rest arcs is at least 0.4 times the total number of simple- and double-rest arcs.

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij}^{1}x_{ij}^{1} + c_{ij}^{2}x_{ij}^{2} + c_{ij}^{3}x_{ij}^{3}) \tag{19}$$

Subject to

$$\sum_{i=1}^{n}(x_{ij}^{1} + x_{ij}^{2} + x_{ij}^{3}) = 1, \quad j = 1, \ldots, n \tag{20}$$

$$\sum_{j=1}^{n}(x_{ij}^{1} + x_{ij}^{2} + x_{ij}^{3}) = 1, \quad i = 1, \ldots, n \tag{21}$$

$$r \geq \frac{1}{\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij}^{1}x_{ij}^{1} + c_{ij}^{2}x_{ij}^{2} + c_{ij}^{3}x_{ij}^{3}) \tag{22}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij}^{2} + x_{ij}^{3}) \geq r \tag{23}$$

$$z \geq 0.4r \tag{24}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}x_{ij}^{3} \geq z \tag{25}$$

$$x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3} \in \{0,1\}, \quad i, j = 1, \ldots, n \tag{26}$$

$$r, z \geq 0 \ integer. \tag{27}$$

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