Facility location and distribution decisions in supply chains with fleet sizing considering both tangible and intangible criteria

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Abstract. This paper addresses location and distribution decisions in supply chains where decisions for locating distribution centers (DCs) and determining the best strategy for shipping products from plants to customers through DCs are taken into consideration. The goal is to select locations of DCs to be opened and to design the network flow of products so that all customers’ demands are satisfied at minimum total cost of the distribution network, while total value of locating DCs and shipping products is maximized. Unlike most of the past research that ignores availability of different transporters with different capacities and costs at each node of the supply chain network, our study takes into account different transportation modes that are available at each potential DC location as well as at each plant, and determines the fleet size on each arc of the network. Furthermore, in addition to opening and operating costs of each potential DC, the decision maker (DM) also takes into account the value of each potential DC location which is calculated through a multiple criteria decision making (MCDM) method. We develop a two-phase approach to deal with this problem. In the first phase, the decision maker scores and ranks all potential DC locations with regard to a set of criteria, and in the second phase through a multi-objective mixed integer programming (MOMIP) model final location and distribution decisions, which incorporates selection of transportation modes and their associated loads, are made.

Keywords: Facility location; Distribution; Supply Chain Management; Transportation, MCDM

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1. Introduction

Nowadays, determination of the best locations for intermediate warehouses, or distribution centers (DCs) is considered as a very important strategic issue in the design and operation of distribution networks in
supply chain systems. In a supply chain environment, DCs function as consolidation points where the product shipments from plants are sorted and combined for shipment to customers (retailers) to satisfy the demands for different products at these sites. In addition, the use of DCs provides a company with flexibility to more quickly respond to changes in the marketplace and allows it to take advantage of the economies of scale in transportation or shipping costs (Amiri, 2006; Keskin and Üster, 2007). The common goal in the designing of a distribution network, which is sometimes referred to as the production-distribution system design (PDSD) problem (Keskin and Üster, 2007), is to make location and distribution decisions at the least cost of the network such that the demands of all customers are satisfied without exceeding the capacities of both plants and DCs. The costs are usually composed of: (1) costs of opening and operating the DCs (and sometimes plants), and (2) transportation costs between plants and DCs as well as between DCs and customers (Pirkul and Jayaraman, 1998).

The facility location and shipment allocation problem has attracted attentions of many researchers such that there is a large body of research on distribution network design (or PDSD) problems. For recent reviews on these models and solution approaches the reader is referred to Francis et al. (1983), Aikens (1985), Thomas and Griffin (1996), Avella et al. (1998), Geoffrion and Powers (1995), and Goetschalckx et al. (2002). A very important characteristic of PDSD studies is the broad view that researchers had in defining and modelling the problem, such that different assumptions and considerations have resulted in relatively different models. In some of the research in this field, attention is paid to locating DCs only, on the other hand, some researchers only determine the optimal locations for plants, and some consider both. In some studies the number of facilities to be opened is limited, while in others there is not such a limitation. Another consideration is capacity limitations at different levels: at plants only, at DCs only, or at both. Distribution sourcing is another important feature in PDSD studies, which can be either single-sourcing or multi-sourcing. In single sourcing, each customer is assigned to only one DC at the upper level, but in multi-sourcing, there is not such a restriction (Keskin and Uster, 2007). Finally, network design problems can be classified under single- and multi-product categories. Tragantalarngsak et al. (2000), considered a single-product, two-echelon facility location problem in which the facilities in the
first echelon are uncapacitated, and the facilities in the second echelon are capacitated. The goal is to determine the number and locations of facilities in both echelons in order to satisfy customer demand. They developed a Lagrangian relaxation based branch-and-bound algorithm to solve the problem. Klose (1999), and Marin and Pelegrin (1997) developed LP-based and branch-and-bound algorithms, respectively, for the facility location and distribution problem in two stages. Multi-product PDSD network design problems are also well studied in the literature (for example, Geoffrion and Graves, 1974; Kaufman et al., 1977; Ro and Tcha, 1984; Pirkul and Jayaraman, 1996). Jayaraman (1998) studied the capacitated warehouse location problem that involves locating a given number of warehouses to satisfy customer demands for different products. Pirkul and Jayaraman (1998) extended the previous problem by taking into consideration locating also a given number of plants. They formulated the problem as a mixed integer model and developed a Lagrangian-based heuristic solution procedure. Keskin and Uster (2007) considered a multi-product production/distribution system design problem where a fixed number of capacitated distribution centers are to be located. They provided metaheuristic procedures, including scatter search, local, and tabu search for the solution of the problem.

However, almost all of the previous research in this domain suffers from two major shortcomings: 1) only the quantitative factors (usually costs) are considered in making the location and distribution decisions, which in turn, results in construction of some type of single-objective mathematical model. 2) Most of the researchers only consider one type of transportation facility available at plants and DCs, and ignore that, in the real world, there are usually multiple transportation options to be considered for shipping products in the network. These shortcomings motivated the current research to better model the situation as it is in the real world. This is done through building a mathematical program with the support of decision aiding tools to 1) include other crucial factors which have received little attention in the literature (due to their less tangible characteristics), but have considerable effects on the facility location decisions in the supply chains, and 2) account for this fact that in reality there are different transportation modes with different and limited capacities and fixed costs for shipping products in the network.
Therefore, in this paper, we address a capacitated two-stage multi-product supply chain network design (SCND) problem in which, in addition to quantitative factors, qualitative and intangible criteria and preferences of the DM toward each possible DC location for making location-distribution decisions are also taken into consideration. In addition, we make decisions regarding selection of transporter types and fleet size at plants and DCs with regard to different fixed rental costs, transportation costs, capacities, and availability of each transporter at plants as well as at DCs. The supply chain network under study in this paper is shown in Figure 1.

A two-phase approach is proposed to deal with this problem in which, in the first phase, using the technique for order preference by similarity to ideal solution (TOPSIS), different potential DC locations are evaluated and ranked. These ratings are then included in the multiple objective mixed-integer programming (MOMIP) model developed in the second phase of the approach to obtain the optimal solution for the problem. We assume that capacities of plants and DCs as well as demands of customers for different products are known in advance and that there is an overall production capacity at plants for all products. Moreover, the availability of each transportation mode and its associated capacity at each plant as well as each DC is known in advance.

The remainder of this paper is organized as follows. In Section 2, the first phase of the proposed approach which tries to capture the less tangible criteria, along with the DM’s judgments and preferences about each DC location in the decision process, is discussed. Mathematical formulation of this supply chain network design problem is presented in Section 3. In Section 4, a numerical example for this problem is presented and results are discussed. Finally, conclusions and directions for future research are given in Section 5.
2. Phase 1: Scoring different potential DC locations

In the first phase of the approach the decision maker (DM), through a multiple criteria decision making (MCDM) model, evaluates and scores different potential DC locations. Through this phase the DM is able to include the less tangible and well defined data and parameters, and also his/her personal judgments, in the decision process.

Normally, MCDM methods are used to rank and select the best alternative(s). In this phase with respect to different and usually conflicting criteria in location decisions, we try to calculate the value of each potential DC location.

2.1. Determining criteria in location decisions

Many criteria can be considered in location decisions, but in the case of locating DCs we arrive at the following as the most critical ones:

- Earthquake possibility (E)
- Fire history (F)
• Access to infrastructures (I)
• Reliability in operations (R)
• Closeness to market (M)
• Expert personnel availability (P)

However, it should be noted that according to the DM’s priorities, and also the context of a given DC location problem, some of these criteria can be changed. Using the above criteria, we can introduce intangible factors into the decision making process for location selection. The hierarchical structure for evaluating different DC locations regarding the above criteria is shown in Figure 2.

2.2. Scoring different alternative DC Locations

There are numerous MCDM techniques to calculate ratings of alternatives. However, in this paper we use the TOPSIS technique (Hwang and Yoon, 1981) for this purpose. TOPSIS is one of the most favorable MCDM techniques among researchers. This is because, aside from its relatively less demanding
calculations in comparison with methods like Analytic Hierarch Process (AHP) (Saaty, 1980) and ELimination and (Et) Choice Translating REality (ELECTRE) (Roy, 1968), its logic is sound and its application is more straightforward. In addition, the TOPSIS method, unlike AHP and ELECTRE, suffers less from problems like rank-reversal.

3. Phase 2: The Multi-objective MIP Model

We consider a location-distribution system which consists of two stages. In the first stage, products are shipped from plants to DCs, and in the second stage, they are shipped from DCs to customers. Customers’ demands for multiple-products should be satisfied by plants through DCs. In addition, demand of each customer for each product is deterministic and is known in advance. There are capacity limitations associated with both the plants and DCs. In addition, the available quantity of different transporters at each plant as well as at each DC is limited. Now, the problem is where to locate \(p\) DCs and how to ship products from plants to customers, through intermediate DCs, such that the total location and transportation costs are minimized while the overall utility of opening DCs is maximized. The following notation is used in the formulation of the model.

\[
\begin{align*}
I & \quad \text{set of customers, } i=1,\ldots,m \\
J & \quad \text{set of potential DCs, } j=1,\ldots,n \\
K & \quad \text{set of plants, } k=1,\ldots,K \\
L & \quad \text{set of products, } l=1,\ldots,L \\
T & \quad \text{set of transportation modes, } t=1,\ldots,T \\
p & \quad \text{number of DCs to be located} \\
D_{il} & \quad \text{demand of customer } i \text{ for product } l \\
W_j & \quad \text{capacity limit at DC } j \\
P_k & \quad \text{capacity limit at plant } k \\
\end{align*}
\]
The decision variables are:

- \( z_j \) 1 if DC at location \( j \) is opened, 0 otherwise, \( \forall j \)
- \( x_{ijl} \) amount of product \( l \) shipped from DC \( j \) to customer \( i \) using transportation mode \( t \)
- \( y_{jkl} \) amount of product \( l \) shipped from plant \( k \) to DC \( j \) using transportation mode \( t \)
- \( g^D_{ijl} \) number of transportation vehicles of type \( t \) used to transport products from DC \( j \) to customer \( i \)
- \( g^P_{jkl} \) number of transportation vehicles of type \( t \) used to transport products from plant \( k \) to DC \( j \)

Then, the problem can be formulated as follows:

\[
\begin{align*}
\text{Min} & \quad \sum_{j} f_j z_j + \sum_{i,j} \sum_{l} \sum_{t} c^D_{ijl} x_{ijl} + \sum_{j} \sum_{k} \sum_{l} \sum_{t} c^P_{jkl} y_{jkl} \\
& \quad + \sum_{i,j} \sum_{l} a_i g^D_{ijl} + \sum_{j,k} \sum_{l} a_k g^P_{jkl}
\end{align*}
\]
Max \[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} b_{ijl} x_{ijlt} \]

s.t.
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} x_{ijlt} = D_{il}, \quad \forall i \in I, \forall l \in L, \quad (1) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} x_{ijlt} \leq W_{jl} z_j, \quad \forall j \in J, \quad (2) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} x_{ijlt} = \sum_{k \in K} \sum_{l \in L} \sum_{t \in T} y_{jktl}, \quad \forall j \in J, \quad (3) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} y_{jktl} \leq P_k, \quad \forall k \in K, \quad (4) \]
\[ \sum_{j \in J} z_j = P, \quad (5) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} x_{ijlt} \leq S_{ijl} z_j, \quad \forall i \in I, \forall j \in J, \forall l \in L, \quad (6) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} y_{jktl} \leq R_{jlt} z_j, \quad \forall j \in J, \forall k \in K, \quad (7) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} x_{ijlt} \leq g_{ijlt}^0 Q_t, \quad \forall i \in I, \forall j \in J, \forall t \in T \quad (8) \]
\[ \sum_{i \in I} g_{ijlt}^0 \leq q_{jlt}^0, \quad \forall j \in J, \forall t \in T \quad (9) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} y_{jktl} \leq g_{jktl}^p Q_t, \quad \forall j \in J, \forall k \in K, \forall t \in T \quad (10) \]
\[ \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} g_{jktl}^p \leq q_{jktl}^p, \quad \forall k \in K, \forall t \in T \quad (11) \]
\[ x_{ijlt} \geq 0, \quad y_{jktl} \geq 0, \quad \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L, \forall t \in T, \quad (12) \]
\[ z_j \in \{0,1\}, \quad \forall j \in J, \quad (13) \]
\[ g_{ijlt}^0 \in \mathbb{Z}^+, \quad \forall i \in I, \forall j \in J, \forall t \in T, \quad (14) \]
\[ g_{jktl}^p \in \mathbb{Z}^+, \quad \forall j \in J, \forall k \in K, \forall t \in T, \quad (15) \]

The first objective function minimizes total costs composed of: 1) fixed costs of opening and operating
the DCs, 2) total transportation costs from DCs to customers, 3) total transportation costs from plants to DCs, 4) fixed costs associated with using different transportation modes for shipping products from DCs to customers, and 5) fixed costs associated with using different transportation modes for shipping products from plants to DCs. The second objective function maximizes total utility of flow of materials between DCs and customers. The utility of flow is the sum, over all DCs, of utility of each DC location (i.e., \( b_i \)) weighted its total shipments to customers. Constraints (1) ensure that demand for each product \( l \) at each customer site \( i \) is satisfied. Constraints (2) and (4) impose the capacity restrictions at the DCs and plants, respectively. There is no specific capacity requirement for each product at the plants and DCs. Constraints (3) are for flow conservation at each DC. Constraint (5) represents the number of DCs to be located. Constraint sets (6) indicate that a customer can only be assigned to an open DC. Constraint sets (7) are added to obtain a stronger formulation and are implied by constraints (2), (4) and (5). Constraints (8) determine the fleet size of each type of transporters used between each DC and each customer. Constraints (9) ensure that limitations imposed on the available transporters at DCs are not violated. Constraints (10) determine the fleet size of each type of transporters used between each plant and each DC. Constraints (11) ensure that limitations imposed on the available transporters at plants are not violated. Finally, constraints (12)-(15) are non-negativity and integrality constraints.

Since the proposed model is a multi-objective MIP problem, some multiple objective decision making (MODM) method should be applied. Here, we use the compromise programming (CP) method to deal with this MODM problem. Compromise programming tries to find a solution that comes “as close as possible” to the ideal values (Zeleny, 1982). Here “Closeness” is defined by the \( L_p \) distance metric as follows:

\[
L_p = \left[ \sum_{i=1}^{k} \gamma_i \left( \frac{f_i - f_i^*}{f_i^*} \right)^p \right]^{1/p}
\]

for \( p = 1, 2, \ldots, \infty \).
in which \( f_1, f_2, \ldots, f_k \) are the different objectives. \( f^*_i = \min(f_i) \) subject to the set of constraints, ignoring all other objectives, is called the ideal value for the \( i \)th objective, and \( \gamma_i \) is the weight of the \( i \)th objective function.

The \( x^* \) is called a compromise solution if it minimizes \( L_p \) by considering \( \gamma_i > 0, \sum \gamma_i = 1 \), and \( p \in \{1, 2, \ldots, \infty\} \). Different efficient solutions can be obtained by considering different values for parameter \( p \). As \( p \) increases, larger deviations get more weight, such that for \( p=\infty \), the largest deviation completely dominates the distance determination. However, the most common values are \( p = 1, 2, \) and \( \infty \).

4. A numerical example

The developed model in the previous section is the extension of the well-known p-median problem which itself is known to be NP-hard (Kariv and Hakimi, 1979). Since solving this problem to optimality for realistic problem sizes is highly inefficient, or even sometimes impossible, here we consider a small-sized numerical example and solve it to optimality using LINGO 8.0 optimization package. It is noteworthy that in the case of realistic problem sizes, highly efficient heuristic or meta-heuristic procedures should be applied to obtain good and near-optimal solutions in short CPU times. The illustrative problem presented here consists of two plants, four potential DC locations, six customers, two products, and three transportation modes.

4.1. Constructing the two-phase approach

4.1.1 Evaluating potential DC locations using TOPSIS method
First, using the TOPSIS method we evaluate potential DCs against the set of criteria explained in section 2. For this purpose we first calculate the importance weight of criteria using the AHP method. The pairwise comparison matrix of criteria and importance weight of each criterion are presented in Table 1.

Table 1. The pair-wise comparison matrix of criteria

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>I</th>
<th>R</th>
<th>M</th>
<th>P</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>1</td>
<td>1/2</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>2</td>
<td>0.255</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>0.394</td>
</tr>
<tr>
<td>I</td>
<td>1/5</td>
<td>1/7</td>
<td>1</td>
<td>1/3</td>
<td>2</td>
<td>1/5</td>
<td>0.047</td>
</tr>
<tr>
<td>R</td>
<td>1/3</td>
<td>1/4</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1/2</td>
<td>0.107</td>
</tr>
<tr>
<td>M</td>
<td>1/7</td>
<td>1/8</td>
<td>1/2</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>0.034</td>
</tr>
<tr>
<td>P</td>
<td>1/2</td>
<td>1/3</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0.164</td>
</tr>
</tbody>
</table>

After calculating importance weights of criteria, decision maker (DM) should evaluate each potential DC location with respect to each criterion. For this purpose the bipolar scale is used and then decision matrix is normalized according to non-linear normalization method employed in the TOPSIS technique. The decision matrix \((D=[x_{ij}])\), and the normalized decision matrix \((R=[r_{ij}])\) are presented in Tables 2 and 3, respectively.

Table 2. The decision matrix for DC evaluation

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>I</th>
<th>R</th>
<th>M</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>A_2</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>A_3</td>
<td>7</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>A_4</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3. The normalized decision matrix for DC evaluation

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>I</th>
<th>R</th>
<th>M</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>0.327</td>
<td>0.167</td>
<td>0.522</td>
<td>0.289</td>
<td>0.511</td>
<td>0.481</td>
</tr>
<tr>
<td>A_2</td>
<td>0.109</td>
<td>0.833</td>
<td>0.373</td>
<td>0.481</td>
<td>0.656</td>
<td>0.289</td>
</tr>
<tr>
<td>A_3</td>
<td>0.764</td>
<td>0.167</td>
<td>0.373</td>
<td>0.481</td>
<td>0.219</td>
<td>0.674</td>
</tr>
<tr>
<td>A_4</td>
<td>0.546</td>
<td>0.5</td>
<td>0.671</td>
<td>0.674</td>
<td>0.511</td>
<td>0.481</td>
</tr>
</tbody>
</table>

The weighted normalized decision matrix \((V=[v_{ij}])\) is shown in Table 4, in which each element \(v_{ij}\) is calculated as follows
\[ v_i = w_j \times r_{ij} \quad \forall i, j \]

Table 4. The weighted normalized decision matrix for DC evaluation

<table>
<thead>
<tr>
<th></th>
<th>( E )</th>
<th>( F )</th>
<th>( I )</th>
<th>( R )</th>
<th>( M )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.083</td>
<td>0.066</td>
<td>0.025</td>
<td>0.031</td>
<td>0.017</td>
<td>0.079</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.228</td>
<td>0.328</td>
<td>0.018</td>
<td>0.051</td>
<td>0.022</td>
<td>0.047</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.195</td>
<td>0.066</td>
<td>0.018</td>
<td>0.051</td>
<td>0.007</td>
<td>0.110</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>0.139</td>
<td>0.197</td>
<td>0.032</td>
<td>0.072</td>
<td>0.022</td>
<td>0.079</td>
</tr>
</tbody>
</table>

The ideal solution \( (A^*) \) and the negative-ideal solution \( (A^-) \) are as follows

\[ A^* = \{v_1^*, \ldots, v_n^*\} = \{\max(v_{ij}) | j \in B), \min(v_{ij}) | j \in C) \} \]
\[ = \{0.028, 0.066, 0.032, 0.072, 0.022, 0.110\} \]

\[ A^- = \{v_1^-, \ldots, v_n^-\} = \{\min(v_{ij}) | j \in B), \max(v_{ij}) | j \in C) \} \]
\[ = \{0.195, 0.328, 0.018, 0.031, 0.007, 0.047\} \]

where \( B \) is associated with benefit criteria, and \( C \) is associated with cost criteria.

Distances of each alternative from the ideal solution and from the negative-ideal solution are calculated as follows

\[ D_i^* = \sqrt{\sum_j (v_{ij} - v_{ij}^*)^2} \quad \forall i, \]
\[ D_i^- = \sqrt{\sum_j (v_{ij} - v_{ij}^-)^2} \quad \forall i. \]

Finally, the closeness coefficient of alternatives to the ideal solution are calculated as follows

\[ \bar{C}_i = \frac{D_i^-}{D_i^* + D_i^-}, \quad i = 1, 2, \ldots, m \]
Distances of each potential DC location from the ideal and the negative-ideal solutions along with their closeness coefficients (represented by $b_j$’s in the MOMIP model) and their final rankings are shown in Table 5.

Table 5. Final Ranking and closeness coefficients of potential DC locations.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$D^+$</th>
<th>$D^-$</th>
<th>Closeness coefficient</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.076</td>
<td>0.287</td>
<td>0.791</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.271</td>
<td>0.169</td>
<td>0.384</td>
<td>4</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.169</td>
<td>0.271</td>
<td>0.616</td>
<td>2</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.175</td>
<td>0.153</td>
<td>0.467</td>
<td>3</td>
</tr>
</tbody>
</table>

4.1.2. Building the multi-objective MIP model

Parameters of the model are set as follows:

$$K=2, n=4, m=6, L=2, T=3, p=3;$$

demand of each customer for each product is given by the following matrix

$$D_a = \begin{pmatrix} 100 & 90 & 92 & 100 & 91 & 100 \\ 92 & 94 & 95 & 91 & 92 & 99 \end{pmatrix}.$$  

Transportation costs for different transportation modes between plants and DCs ($c^P$), and between DCs and customers ($c^D$) are obtained using uniform distribution in the interval [50,100] as follows

$$c^P = \text{Uniform} [50,100], \quad c^D = \text{Uniform} [50,100].$$

Capacities of plants ($P_h$), DCs ($W_j$) and the fixed cost of opening and operation each DC ($f_j$) are
The capacity of each transportation mode ($Q_t$) and associated fixed costs ($a_t$) are

$$Q_t = \begin{bmatrix} 120 \\ 60 \\ 30 \end{bmatrix}, \quad a_t = \begin{bmatrix} 10 \\ 5 \\ 2 \end{bmatrix}.$$ 

Finally, the availabilities of each transportation mode at each DC ($q_{jt}$) and at each plant ($o_{kt}$) are as follows

$$q_{jt}^p = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 4 & 4 \\ 3 & 6 & 4 \\ 2 & 2 & 5 \end{bmatrix}, \quad q_{kt}^o = \begin{bmatrix} 5 & 6 & 10 \\ 4 & 8 & 14 \end{bmatrix}.$$ 

### 4.2. Solution results and analysis

In order to obtain the solution for the proposed problem we use the LINGO 8.0 package. Results of running the mathematical model for each individual objective function one at a time, and for the overall compromise programming model are summarized in Table 6. Parameters for the $L_p$ distance metric in the compromise programming model are set as $\gamma_i=0.5 \ (i=1,2)$, and $p=1$. 

$$P_k = \begin{bmatrix} 609 \\ 625 \end{bmatrix}, \quad W_j = \begin{bmatrix} 619 \\ 674 \\ 736 \\ 407 \end{bmatrix}, \quad f_j = \begin{bmatrix} 1957 \\ 2122 \\ 2508 \\ 921 \end{bmatrix}.$$
Table 6. Results of running the proposed multi-objective model.

<table>
<thead>
<tr>
<th>Model #</th>
<th>Model description</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>Opened DCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Optimizing regarding objective function 1 alone</td>
<td>139212 *</td>
<td>639133</td>
<td>3, 4</td>
</tr>
<tr>
<td>2</td>
<td>Optimizing regarding objective function 2 alone</td>
<td>168254</td>
<td>804776 *</td>
<td>1, 3</td>
</tr>
<tr>
<td>3</td>
<td>Overall compromise programming model</td>
<td>139506</td>
<td>804776</td>
<td>1, 3</td>
</tr>
</tbody>
</table>

Note that, values in the Table 6 shown with asterisk (*) are optimal values for each objective function.

Regarding the solution result for the multi-objective model, we can calculate the deviation of each objective function from its optimal value ($dev_i$) as follows

\[
dev_1 = \frac{139506 - 139212}{139212} = 0.002
\]

\[
dev_2 = \frac{804776 - 804776}{804776} = 0.000
\]

With respect to the above deviations, though costs for this PDSD system is now increased by %0.2 (in comparison with the classic single-objective model that just takes into account quantitative factors for location and distribution decisions), but we have now incorporated less structured data (that in some cases could be of significant importance and may dominate quantitative factors) in the decision making process. The new solution led to enhanced satisfaction of the DM by about %26 (804776 – 639133/639133). It is noteworthy that the %0 deviation of model #3 from model #2 in Table 6 is due to the small size of the problem considered in this example (especially, the numbers of potential DC locations and of DCs to be opened) and, therefore, for realistic-sized instances of the problem, typically with up to 50 potential DC locations, up to 20 DCs to be opened and up to 100 customers, there would be a compromise between the two objective functions.

Configuration of the network along with the required fleet size on each arc of the network is depicted in Figure 3. Note that, for clarity purposes the amount of each product shipped on each arc is not shown.
5. Conclusion and directions for future research

In this paper we proposed a two-phase approach for dealing with the location and distribution decisions in a PDSD system. In the proposed approach we considered intangible as well as tangible criteria in the decision making process. In the first phase we tried to capture intangible criteria, and also DM’s vague preferences toward each potential location for establishing a DC using the TOPSIS
technique, and then used the acquired scores in the second phase of the approach. In the second phase, in conjunction with quantitative factors, we built a multi-objective decision making (MODM) model for addressing the location and distribution decisions in a PDSD system where by constructing a bi-objective mixed integer programming (MIP) model we tried to minimize the costs of the network and maximize the utility of location and flow of materials in the system. Finally, through a numerical example we showed that by considering some other factors other than the classical cost measure we can significantly increase the DM’s satisfaction for a slight increase in total costs of the system. Moreover, taking into account different transportation modes for distributing products in the network leads to more realistic decisions in the real world.

Directions for future research could be given as follows:

1) In order to more appropriately reflect the vagueness and ambiguity of intangible data and also of the DM’s preferences and judgments, applying fuzzy set theory could be taken into consideration.

2) As the proposed problem is an extension of the classical $p$-median problem, which itself is known to be NP-hard, devising heuristic or meta-heuristic algorithms for solving the MODM model efficiently can be an interesting and rich research area.

3) Since the problem modeled in this paper may cause an under utilization of transportation modes (vehicles, trains, ships, etc.) another interesting area for further research would be making routing decisions for distributing products from plants to customers through DCs.

Finally, issues such as production plans at plants, inventory, and incentives such as discount rates could be taken into consideration.

6. References


